General Maxwellian Dynamics
Maxwellian solitons are Particles

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Presentation to the
Harbingers of Neophysics

Slide 2: What is a wave? (The d'Alembert wave equation)

Towne[1] states that the requirement for a physical condition to be referred to as a wave, is that its mathematical representation give rise to a partial differential equation of particular form, known as the wave equation. The classical form

$$\frac{\partial^2 w}{\partial p^2} - \frac{1}{u^2}\frac{\partial^2 w}{\partial t^2} = 0 \qquad \text{or} \qquad \nabla^2 w - \frac{1}{u^2}\frac{\partial^2 w}{\partial t^2} = 0.$$

was proposed in 1748 by d'Alembert for a one-dimensional continuum. A decade later, Euler established the equation for the three-dimensional continuum.

1 Dudley H. Towne. *Wave phenomena.* New York: Dover Publications, 1988.

Slide 3: Electromagnetic Bimodal Wave Equation & Maxwell

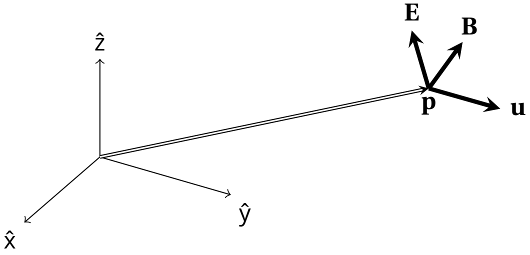


Figure 1: Illustrating the vectors of an EM-wave

Slide 4: Electromagnetic Bimodal Wave Equation & Maxwell

$$\mathcal{W}(\mathbf{p}) \xrightarrow[\text{by}]{\text{dsc}} \mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E}) = \left\{ \begin{array}{lr} \mathbf{E} = \mathbf{u} \times \mathbf{B} & (\textit{activation by } \mathbf{B}) \;\; \text{(a)} \\ \mathbf{u} = \dfrac{1}{\|\mathbf{B}\|^2}\mathbf{B} \times \mathbf{E} & (\textit{vectoring by } \mathbf{B} \times \mathbf{E}) \;\; \text{(b)} \\ \mathbf{B} = \dfrac{1}{\|\mathbf{u}\|^2}\mathbf{E} \times \mathbf{u} & (\textit{reactivation by } \mathbf{E}) \;\; \text{(c)} \end{array} \right\}$$

$\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ predicts the Maxwell equations in vacuum, that is, $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ is the fundamental mathematical explanation for the electromagnetic wave phenomena.

Slide 5: Electromagnetic Bimodal Wave Equation & Maxwell

To show that $\mathscr{M}(\mathbf{u}, \mathbf{B}, \mathbf{E})$ is the superordinated mathematical formulation for the Maxwell equations is the task we tackle now:

But, first we need to evaluate the triple vector products $\nabla \times (\mathbf{u} \times \mathbf{B})$ and $\nabla \times (\mathbf{E} \times \mathbf{u})$, which we expand using general vector analytic methods.

$$\nabla \times (\mathbf{u} \times \mathbf{B}) = \mathbf{u}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{u}) - (\mathbf{u} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{u}$$

$$\nabla \times (\mathbf{E} \times \mathbf{u}) = \mathbf{E}(\nabla \cdot \mathbf{u}) - \mathbf{u}(\nabla \cdot \mathbf{E}) - (\mathbf{E} \cdot \nabla)\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{E}$$

Slide 6: Electromagnetic Bimodal Wave Equation & Maxwell

$\nabla \cdot \mathbf{u} = 0$ because c and $\hat{\mathrm{u}}(t)$ are not functions of x, y, and z

$\nabla \cdot \mathbf{B} = 0$ because B and $\hat{\mathrm{B}}(t)$ are not functions of x, y, and z

$\nabla \cdot \mathbf{E} = 0$ ditto, because $\mathbf{E} = \mathbf{u} \times \mathbf{B}$

$(\mathbf{B} \cdot \nabla)\mathbf{u} = 0$ because $\left(B_x \frac{\partial}{\partial x} + B_y \frac{\partial}{\partial y} + B_z \frac{\partial}{\partial z} \right) c\hat{\mathrm{u}}(t) = 0$

$(\mathbf{E} \cdot \nabla)\mathbf{u} = 0$ ditto

$(\mathbf{u} \cdot \nabla)\mathbf{B} = ?$

$(\mathbf{u} \cdot \nabla)\mathbf{E} = ?$

Slide 7: Electromagnetic Bimodal Wave Equation & Maxwell

$$\mathbf{u}\cdot\nabla = \frac{\partial}{\partial t} \text{ because } \mathbf{u}\cdot\nabla = \frac{\partial x}{\partial t}\frac{\partial}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial}{\partial y} + \frac{\partial z}{\partial t}\frac{\partial}{\partial z} = \frac{\partial}{\partial t}$$

and that leaves us with

$$\nabla\times(\mathbf{u}\times\mathbf{B}) = \cancel{\mathbf{u}(\nabla\cdot\mathbf{B})} - \cancel{\mathbf{B}(\nabla\cdot\mathbf{u})} + \cancel{(\mathbf{B}\cdot\nabla)\mathbf{u}} - (\mathbf{u}\cdot\nabla)\mathbf{B} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times(\mathbf{E}\times\mathbf{u}) = \cancel{\mathbf{E}(\nabla\cdot\mathbf{u})} - \cancel{\mathbf{u}(\nabla\cdot\mathbf{E})} + (\mathbf{u}\cdot\nabla)\mathbf{E} - \cancel{(\mathbf{E}\cdot\nabla)\mathbf{u}} = \frac{\partial\mathbf{E}}{\partial t}$$

Slide 8: Electromagnetic Bimodal Wave Equation & Maxwell

Applying a ‘left and right side’ curl operation on $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$(a) and (c) to obtain

$$\nabla\times\mathbf{E} = \nabla\times(\mathbf{u}\times\mathbf{B}) = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{B} = \frac{1}{\|\mathbf{u}\|^2}\nabla\times(\mathbf{E}\times\mathbf{u}) = \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}$$

and on slide 10 we established $\nabla\cdot\mathbf{B}=0$ and $\nabla\cdot\mathbf{E}=0$. Thus we have the Maxwell equations in vacuum if we can show that $c^{-2}=\epsilon_0\mu_0$.

Slide 9: Electromagnetic Bimodal Wave Equation & Maxwell

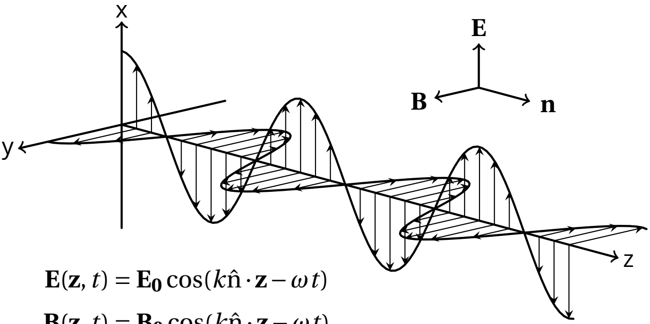
Because, $\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$ and $\nabla^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} = 0$

are derived from the Maxwell equations, proves that $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ is a new formulation for bimodal-waves as per Towne[2] (Slide 3)

$$\mathcal{W}(\mathbf{p}) \xrightarrow[\text{by}]{\text{dsc}} \mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E}) = \left\{ \begin{array}{lll} \mathbf{E} = \mathbf{u}\times\mathbf{B} & (\textit{activation by } \mathbf{B}) & \text{(a)} \\ \mathbf{u} = \frac{1}{\|\mathbf{B}\|^2}\mathbf{B}\times\mathbf{E} & (\textit{vectoring by } \mathbf{B}\times\mathbf{E}) & \text{(b)} \\ \mathbf{B} = \frac{1}{\|\mathbf{u}\|^2}\mathbf{E}\times\mathbf{u} & (\textit{reactivation by } \mathbf{E}) & \text{(c)} \end{array} \right\}$$

2 Towne, *Wave phenomena*.

Slide 10: Classical interpretation of an Electromagnetic Plain Wave



$$\mathbf{E}(\mathbf{z}, t) = \mathbf{E_0}\cos(k\hat{n}\cdot\mathbf{z} - \omega t)$$

$$\mathbf{B}(\mathbf{z}, t) = \mathbf{B_0}\cos(k\hat{n}\cdot\mathbf{z} - \omega t)$$

$$\mathbf{E}_0\cdot\hat{n} = 0,\;\; \mathbf{B}_0\cdot\hat{n} = 0,\;\; \mathbf{B}_0 = k\hat{n}\times\mathbf{E}_0,\;\; k = \frac{\omega}{c},\;\; \hat{n} = \hat{z}$$

Slide 11: The Rotary Wave (think propeller)

$$\mathcal{R}(\mathbf{p}) \xrightarrow[\text{by}]{\text{dsc}} \mathscr{M}(\mathbf{u}, \mathbf{A}, \mathbf{R}) = \begin{cases} \mathbf{R} = \mathbf{u} \times \mathbf{A} & (\textit{activation by } \mathbf{A}) \quad \text{(a)} \\ \mathbf{u} = \dfrac{1}{\|\mathbf{A}\|^2} \mathbf{A} \times \mathbf{R} & (\textit{vectoring by } \mathbf{A} \times \mathbf{R}) \quad \text{(b)} \\ \mathbf{A} = \dfrac{1}{\|\mathbf{u}\|^2} \mathbf{R} \times \mathbf{u} & (\textit{reactivation by } \mathbf{R}) \quad \text{(c)} \end{cases}$$

where $\mathbf{A} = l A \hat{\mathrm{A}}(t)$ is the activation-flux vector,

l is the length of the vector,

A an elementary quantity with units

and $\hat{\mathrm{A}}$ a unitless unity vector.

Slide 12: The Rotary Wave (think propeller)

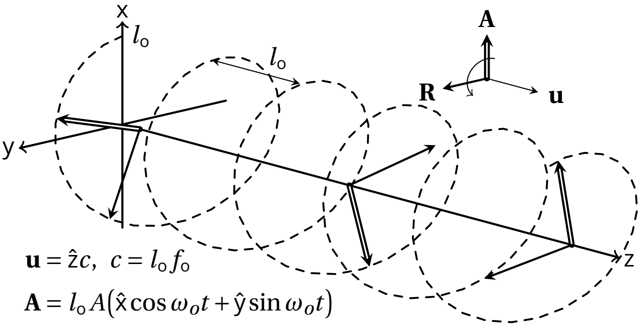
$$\nabla \cdot \mathbf{A} = 0 \qquad\qquad \nabla \cdot \mathbf{R} = 0$$

$$\nabla \times \mathbf{A} = \underset{\cdot}{\epsilon}\underset{\cdot}{\mu}\frac{\partial \mathbf{R}}{\partial t} \qquad\qquad \nabla \times \mathbf{R} = -\frac{\partial \mathbf{A}}{\partial t}$$

A solution is the quantised rotary wave γ

$$\gamma \xrightarrow[\text{by}]{\text{par}} \begin{cases} \mathbf{u} = \hat{\mathrm{z}}c \\ \mathbf{A} = r\,l_{\mathrm{o}}A\big(\hat{\mathrm{x}}\cos n\omega_{\mathrm{o}}t + \hat{\mathrm{y}}\sin \grave{n}\omega_{\mathrm{o}}t\big) \\ \mathbf{R} = cr\,l_{\mathrm{o}}A\big(-\hat{\mathrm{x}}\sin \grave{n}\omega_{\mathrm{o}}t + \hat{\mathrm{y}}\cos \grave{n}\omega_{\mathrm{o}}t\big) \end{cases}$$

Slide 13: The Rotary Wave (think propeller)



$\mathbf{u} = \hat{z}c, \;\; c = l_o f_o$

$\mathbf{A} = l_o A\left(\hat{x}\cos\omega_o t + \hat{y}\sin\omega_o t\right)$

$\mathbf{R} = c l_o A\left(-\hat{x}\sin\omega_o t + \hat{y}\cos\omega_o t\right)$

$$\mathbf{R} = (\mathbf{u}\times\mathbf{A}) \quad \text{and} \quad \mathbf{u} = \frac{1}{A^2}(\mathbf{A}\times\mathbf{R}) \quad \text{and} \quad \mathbf{A} = \frac{1}{c^2}(\mathbf{R}\times\mathbf{u})$$

Slide 14: Properties of Vacuum (Two Assertions)

Assertions used to describe an EM-wave

a) An elementary EM-wave $\mathcal{W}$ exhibits power h/t^2, where h is the Planck constant and $t = 1$ second. This requires $\mathbf{B}$ to be an elementary field.

b) This elementary wave transports an electric charge e every one second which is a *wave current*.

Slide 15: Properties of Vacuum (Two Assertions)

Assertions used to describe a Rotary wave

a) An elementary rotary wave $\mathcal{R}$ has action h. This requires **A** to be an elementary activation-flux vector.

b) This elementary rotary wave transports an elementary load ℓ.

We need to assign some units to the elementary load. I propose a new unit L, the leyden, honouring the Leyden jar.

(Hinting that the electron is not the carrier of electric charge that drives our industry.)

Slide 16: Action of a rotary wave

Let's consider a the rotary wave γ

$$\gamma(\mathbf{p}) \xrightarrow[\text{by}]{\text{dsc}} \mathscr{M}(\mathbf{u},\mathbf{A},\mathbf{R}) = \left\{ \begin{array}{ll} \mathbf{R} = \mathbf{u}\times\mathbf{A} & \text{(a)} \\ \mathbf{u} = \dfrac{1}{\|\mathbf{A}\|^2}\mathbf{A}\times\mathbf{R} & \text{(b)} \\ \mathbf{A} = \dfrac{1}{\|\mathbf{u}\|^2}\mathbf{R}\times\mathbf{u} & \text{(c)} \end{array} \right\}$$

$$\gamma \xrightarrow[\text{by}]{\text{par}} \begin{cases} \mathbf{u} = \hat{\mathrm{z}}c \\ \mathbf{A} = r l_{\mathrm{o}} A\big(\hat{\mathrm{x}}\cos n\omega_{\mathrm{o}} t + \hat{\mathrm{y}}\sin \grave{n}\omega_{\mathrm{o}} t\big) \\ \mathbf{R} = c r l_{\mathrm{o}} A\big(-\hat{\mathrm{x}}\sin \grave{n}\omega_{\mathrm{o}} t + \hat{\mathrm{y}}\cos \grave{n}\omega_{\mathrm{o}} t\big) \end{cases}$$

Slide 17: Properties of Vacuum

$$\mathbf{u} = \frac{1}{\|\mathbf{A}\|^2}\mathbf{A}\times\mathbf{R}$$

On the premise that $\mathbf{A}\times\mathbf{R}$ is indicative of the wave action, we multiply left and right by h and substitute $||\mathbf{A}|| = l_\text{o} A$

$$\|h\mathbf{u}\| = \left\| \frac{h}{l_\text{o}^2 A^2}\mathbf{A}\times\mathbf{R} \right\|$$

$$\therefore \qquad h = \left[\frac{h}{l_\text{o}^2 A^2 c} \right] \left(\|\mathbf{A}\| \, \|\mathbf{R}\| \right)$$

Slide 18: Rotary Action

Action is momentum times distance. ⇒ Therefore, rotary-action is rotary momentum times the angle θ subtended, that is $\mathcal{S}_{\mathrm{rot}} = I\omega\theta$. Hence we can formulate the quantised rotational action as

$$h_{\mathrm{rot}} = \varrho h = \Bbbk \ell\, l_{\mathrm{o}}^{2} \omega_{\mathrm{o}} \theta$$

where $\Bbbk$ is a dimensionless proportionality constant of unknown value, scaling $\ell\, l_{\mathrm{o}}^{2} \omega_{\mathrm{o}} \theta$ to the rotational-action h_{rot} and here $\varrho =$ 1 Lkg^{-1} (leyden per kilogram) a correction factor to satisfy the dimensionality of above. Also, in a quantised system $\theta = 1$ radian.

Because $l_\text{o} = c\,t_\text{o} = c/f_\text{o}$ to obtain $\omega_\text{o} = 2\pi f_\text{o} = 2\pi c/l_\text{o}$ hence h_rot is also expressed as:

$$h_\text{rot} = \varrho h = 2\pi \hbar \ell\, l_\text{o} c\theta$$

Because the load is carried by $\mathbf{A}$ which has a magnitude $\|\mathbf{A}\| = l_\text{o} A$, therefore we can also postulate the elementary rotary-action

$$h_\text{rot} = \chi\, l_\text{o} A\theta$$

where χ is part of a constant to be determined. Also note that A is a quantised quantity.

Slide 20: Property of Vacuum

$$h_{\text{rot}} = \varrho h = 2\pi \hbar \ell \, l_{\text{o}} c\theta \quad = \chi \, l_{\text{o}} A\theta$$

thus we get

$$A = \frac{2\pi \hbar \ell c}{\chi} \quad \text{hence} \quad \|\mathbf{A}\| = \frac{2\pi \hbar l_{\text{o}} \ell c}{\chi}$$

and using above in

$$h = \left[\frac{h}{l_{\text{o}}^2 A^2 c} \right] \left(\|\mathbf{A}\| \, \|\mathbf{R}\| \right) \qquad \text{gives}$$

$$= \left[\frac{h}{l_{\text{o}}^2 A^2 c} \right] \left(\frac{2\pi \hbar l_{\text{o}} \ell c}{\chi} \|\mathbf{R}\| \right)$$

Slide 21: Property of Vacuum

$$h = \left[\frac{h}{l_\circ^2 A^2 c}\right]\left(\frac{2\pi \hbar l_\circ \ell c}{\chi}\|\mathbf{R}\|\right)$$

but $\|\mathbf{R}\| = c l_\circ A$ which gives after defining a further constant $\left[\frac{l_\circ^2}{\chi}\right]$

$$h = \left[\frac{h}{l_\circ^2 A^2 c}\right]\left[\frac{l_\circ^2}{\chi}\right] 2\pi \hbar \ell c^2 A$$

We are now in the position to define the quantised activator as

$$A = \frac{h}{2\pi \hbar \ell}$$

Slide 22: Property of Vacuum

$$h = \left[\frac{h}{l_o^2 A^2 c}\right]\left(\frac{2\pi \Bbbk l_o \ell c}{\chi}\|\mathbf{R}\|\right)$$

but $\|\mathbf{R}\| = c l_o A$ which gives after defining a further constant $\left[\frac{l_o^2}{\chi}\right]$

$$h = \left[\frac{h}{l_o^2 A^2 c}\right]\left[\frac{l_o^2}{\chi}\right] 2\pi \Bbbk \ell c^2 A$$

We are now in the position to define the quantised activator as

$$A = \frac{h}{2\pi \Bbbk \ell}$$

but only if

$$1 = \left[\frac{h}{l_\circ^2 A^2 c}\right]\left[\frac{l_\circ^2}{\chi}\right]c^2$$ using $A = \dfrac{h}{2\pi \Bbbk \ell}$ to replace A to get

$$1 = \left[\frac{4\pi^2 \Bbbk^2 \ell^2}{l_\circ^2 hc}\right]\left[\frac{l_\circ^2}{\chi}\right]c^2$$ which requires $\chi = \dfrac{4\pi^2 \Bbbk^2 \ell^2 c}{h}$, hence

$$1 = \left[\frac{4\pi^2 \Bbbk^2 \ell^2}{l_\circ^2 hc}\right]\left[\frac{l_\circ^2 h}{4\pi^2 \Bbbk^2 \ell^2 c}\right]c^2 = \underset{\cdot}{\epsilon}\underset{\cdot}{\mu}c^2$$ from which we get

$$\underset{\cdot}{\epsilon} = \frac{4\pi^2 \Bbbk^2 \ell^2}{l_\circ^2 hc} \quad \text{and} \quad \underset{\cdot}{\mu} = \frac{l_\circ^2 h}{4\pi^2 \Bbbk^2 \ell^2 c}$$

Slide 24: **Roton** a soliton that underlies Maxwellian dynamics.

$$\mathbf{R} = (\mathbf{u} \times \mathbf{A}) \quad \text{and} \quad \mathbf{u} = \frac{1}{A^2}(\mathbf{A} \times \mathbf{R}) \quad \text{and} \quad \mathbf{A} = \frac{1}{c^2}(\mathbf{R} \times \mathbf{u})$$

$$\mathscr{R} \xrightarrow[\text{by}]{\text{par}} \begin{cases} \mathbf{u} = c\ \hat{u} \\ \mathbf{A} = \grave{r}\, l_o A \hat{A} \\ \mathbf{R} = c \grave{r}\, l_o A \hat{R} \end{cases}$$

where $\grave{r}$ a unitless scaling factor. The simultaneous algebraic vector equation set

$$\hat{\mathrm{R}} = \hat{u} \times \hat{\mathrm{A}} \qquad \hat{u} = \hat{\mathrm{A}} \times \hat{\mathrm{R}} \qquad \hat{\mathrm{A}} = \hat{\mathrm{R}} \times \hat{u}.$$

has infinitely many solutions, some of which can be found by a succession of Euler rotations. Each solution describes a particular roton type.

Slide 25: **1D and 2D-Roton**: $\hat{R} = \hat{u} \times \hat{A}, \quad \hat{u} = \hat{A} \times \hat{R}, \quad \hat{A} = \hat{R} \times \hat{u}.$

1D-Roton Linear propagation path along the z-axis (photon like)

$$\hat{u}_\gamma = \hat{z}$$
$$\hat{A}_\gamma = \hat{x}\cos \grave{n}\omega_o t + \hat{y}\sin \grave{n}\omega_o t$$
$$\hat{R}_\gamma = -\hat{x}\sin \grave{n}\omega_o t + \hat{y}\cos \grave{n}\omega_o t$$

2D-Roton Circular propagation path in the xy-plane centred at the origin

$$\hat{u}_\odot = \hat{x}\sin \grave{n}\omega_o t - \hat{y}\cos \grave{n}\omega_o t$$
$$\hat{A}_\odot = \hat{x}\cos \grave{n}\omega_o t + \hat{y}\sin \grave{n}\omega_o t \qquad \text{or} \quad \hat{z}$$
$$\hat{R}_\odot = \hat{z} \qquad \text{or} \quad \hat{x}\cos \grave{n}\omega_o t + \hat{y}\sin \grave{n}\omega_o t$$

where $\grave{n}$ a unitless scaling factor

3D-Roton Closed curved, or wound up, path in xyz-space centred at the origin.

$$\begin{aligned}
\hat{u}_\varphi &= \hat{\mathrm{x}} \sin\omega_1 t \sin \grave{n}\omega_\mathrm{o} t - \hat{\mathrm{y}} \sin n\omega_1 t \cos \grave{n}\omega_\mathrm{o} t - \hat{\mathrm{z}} \cos\omega_1 t \\
\hat{\mathrm{A}}_\varphi &= \hat{\mathrm{x}} \cos \grave{n}\omega_\mathrm{o} t + \hat{\mathrm{y}} \sin \grave{n}\omega_\mathrm{o} t \\
\hat{\mathrm{R}}_\varphi &= \hat{\mathrm{x}} \cos\omega_1 t \sin \grave{n}\omega_\mathrm{o} t - \hat{\mathrm{y}} \cos\omega_1 t \cos \grave{n}\omega_\mathrm{o} t + \hat{\mathrm{z}} \sin\omega_1 t
\end{aligned} \tag{1}$$

where $\omega_1 = \grave{p}\grave{n}\omega_\mathrm{o}$ and where $\grave{p}$ is a prime integer ensuring that the path is repeated in periods of t_o because $\omega_\mathrm{o} t = 2\pi$.

Slide 27: Energy of a 1D-roton

$$\gamma \xrightarrow[\text{by}]{\text{par}} \begin{cases} \mathbf{u} = \hat{\mathrm{z}}c \\ \mathbf{A} = r\, l_\text{o} A\big(\hat{\mathrm{x}} \cos n\omega_\text{o} t + \hat{\mathrm{y}} \sin \grave{n}\omega_\text{o} t\big) \\ \mathbf{R} = c r\, l_\text{o} A\big(-\hat{\mathrm{x}} \sin \grave{n}\omega_\text{o} t + \hat{\mathrm{y}} \cos \grave{n}\omega_\text{o} t\big) \end{cases}$$

Using $h_\text{rot} = \varrho h = 2\pi \hbar \ell\, l_\text{o} c\theta$ from Slide-19 we obtain the action of γ is $h_\gamma = \grave{r}^2 n h_\text{rot} = \hbar \ell\, \grave{r}^2 l_\text{o}^2 n\omega_\text{o}$. Therefore the action vector $\boldsymbol{S}_\gamma$ is given by (Slide-20)

$$\begin{aligned} \boldsymbol{S}_\gamma &= \underset{\cdot}{\epsilon}\, \grave{n}(\mathbf{A} \times \mathbf{R}) \\ &= \underset{\cdot}{\epsilon}\, \grave{n}\grave{r}^2 \big(l_\text{o} A \hat{\mathrm{A}}(t) \times l_\text{o} R \hat{\mathrm{R}}(t)\big) \end{aligned}$$

Slide 28: Energy of a 1D-roton

$$\begin{aligned}\mathcal{S}_\gamma &= \d{\epsilon}\,\grave{n}(\mathbf{A}\times\mathbf{R})\\ &= \d{\epsilon}\,\grave{n}\grave{r}^2\big(l_\text{o}A\hat{\text{A}}(t)\times l_\text{o}R\hat{\text{R}}(t)\big)\end{aligned}$$

and the norm evaluates to

$$\begin{aligned}\|\mathcal{S}_\gamma\| &= \d{\epsilon}\,\grave{n}\grave{r}^2 c\,l_\text{o}A^2\\ &= h\grave{n}\grave{r}^2\end{aligned}$$

Therefore, with $\grave{r}=1$ the rotary wave γ carries an energy content

$$\mathcal{E}_\gamma = h\frac{\grave{n}}{t_\text{o}} = hf$$

which is the Planck energy equivalence.

Slide 29: Energy of a 3D-roton

$$\hat{u}_\varphi = \hat{\mathrm{x}} \sin\omega_1 t \sin \grave{n}\omega_{\mathrm{o}} t - \hat{\mathrm{y}} \sin n\omega_1 t \cos \grave{n}\omega_{\mathrm{o}} t - \hat{\mathrm{z}} \cos\omega_1 t$$
$$\hat{\mathrm{A}}_\varphi = \hat{\mathrm{x}} \cos \grave{n}\omega_{\mathrm{o}} t + \hat{\mathrm{y}} \sin \grave{n}\omega_{\mathrm{o}} t \qquad (2)$$
$$\hat{\mathrm{R}}_\varphi = \hat{\mathrm{x}} \cos\omega_1 t \sin \grave{n}\omega_{\mathrm{o}} t - \hat{\mathrm{y}} \cos\omega_1 t \cos \grave{n}\omega_{\mathrm{o}} t + \hat{\mathrm{z}} \sin\omega_1 t$$

First we analyse the path s_ϕ on which a roton propagates; it is found by integration $s = \int \mathbf{u}\, \mathrm{d}t$.

Slide 30: Energy of a 3D-roton

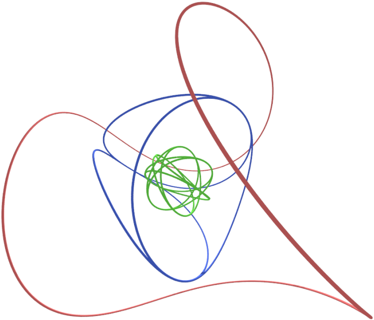
For the 3D-roton, and setting $\grave{n} = 1$ we obtain

$$s_\varphi = c\int \left(\hat{x}\sin\omega_1 t\sin\grave{n}\omega_o t - \hat{y}\sin n\omega_1 t\cos\grave{n}\omega_o t - \hat{z}\cos\omega_1 t\right)\mathrm{d}t$$

$$= \hat{x}c\left(\frac{\sin\left(\omega_1-\omega_o\right)t}{2(\omega_1-\omega_o)} - \frac{\sin\left(\omega_1+\omega_o\right)t}{2(\omega_1+\omega_o)}\right)$$

$$-\hat{y}c\left(\frac{\cos\left(\omega_1-\omega_o\right)t}{2(\omega_1-\omega_o)} + \frac{\cos\left(\omega_o+\omega_1\right)t}{2(\omega_o+\omega_1)}\right) - \hat{z}c\frac{\sin\omega_1 t}{\omega_1}$$

Slide 31: Energy of a 3D-roton



Three rotons sharing the same centre. The orbits are defined by $\{\omega_1, \omega_o\} = \{2, 1\}, \{3, 1\}, \{7, 1\}$ all path lengths are equal to 2π if $c = 1$

Slide 32: Energy of a 3D-roton

The path's radial distance from the origin evaluates to:

$$r_\varphi = c\sqrt{\frac{\omega_1^4 - \omega_{\mathrm{o}}^2(\omega_1^2 - \omega_{\mathrm{o}}^2)\sin^2\omega_1 t}{\omega_1^4(\omega_1^2 - \omega_{\mathrm{o}}^2)}} \quad > \frac{c}{\omega_1} \quad \text{if} \quad \omega_1 > \omega_{\mathrm{o}}$$

and remebering $\omega_1 = \grave{p}\grave{n}\omega_{\mathrm{o}}$, see (2), we get

$$r_\varphi \gtrapprox \frac{c}{\grave{p}\grave{n}\omega_{\mathrm{o}}}$$

Slide 33: Energy of a 3D-roton

The activation vector **A** of the 3D-roton is

$$\mathbf{A} = \grave{r}\, l_\circ A\big(\hat{\mathrm{x}}\cos n\omega_\circ t + \hat{\mathrm{y}}\sin \grave{n}\omega_\circ t\big)$$

which is the same as that of the 1D-roton. Hence the action vector $\boldsymbol{\mathcal{S}}_\varphi$ and its norm is given by

$$\boldsymbol{\mathcal{S}}_\varphi = \underset{\cdot}{\epsilon}\, \grave{n}(\mathbf{A}\times\mathbf{R})$$

and the norm evaluates to

$$\begin{aligned}\left\|\boldsymbol{\mathcal{S}}_\varphi\right\| &= \underset{\cdot}{\epsilon}\, \grave{n}\grave{r}^2 c\, l_\circ A^2\\ &= h\grave{n}\grave{r}^2\end{aligned}$$

Slide 34: Energy of a 3D-roton

$$r_\varphi \gtrapprox \frac{c}{\grave{p}\grave{n}\omega_\circ} \qquad \|\boldsymbol{S}_\varphi\| = h\grave{n}\grave{r}^2$$

let **A** not extend over the geometric centre of s_φ

$$\grave{r}\, l_\circ \leqq r_\varphi \qquad \text{and using } c = \frac{l_\circ}{t_\circ}$$

$$\leqq \frac{l_\circ}{t_\circ}\frac{1}{\grave{p}\grave{n}2\pi/t_\circ} \qquad \text{gives} \qquad \grave{r}\,\grave{n} = \frac{1}{2\pi\grave{p}}$$

$$\therefore\ \|\boldsymbol{S}_\varphi\| = h\grave{r}(\grave{n}\grave{r}) = h\grave{r}/(2\pi\grave{p}) = \hbar\frac{\grave{r}}{\grave{p}}$$

$$\text{or the energy} \qquad \mathcal{E}_\varphi = \hbar\frac{\grave{r}}{\grave{p}\, t_\circ}$$

Slide 35: Complex space

Instead of xyz-space = $\mathbb{R}^3$ we consider xyz-space = $\mathbb{C}^3$ and with a complex *load* we note the light speed could also be complex.

$$\ell \mapsto \begin{cases} \ell e^{i\alpha} \text{ thus } A \mapsto Ae^{-i\alpha} \text{ and } R \mapsto \begin{cases} Re^{i\alpha}, & \text{if } c \mapsto ce^{i2\alpha} \\ Re^{-i3\alpha}, & \text{if } c \mapsto ce^{-i2\alpha} \\ Re^{-i\alpha}, & \text{if } c \mapsto c \end{cases} \\ \qquad\qquad \text{or} \\ \ell e^{-i\alpha} \text{ thus } A \mapsto Ae^{i\alpha} \text{ and } R \mapsto \begin{cases} Re^{i3\alpha}, & \text{if } c \mapsto ce^{i2\alpha} \\ Re^{-i\alpha}, & \text{if } c \mapsto ce^{-i2\alpha} \\ Re^{i\alpha}, & \text{if } c \mapsto c \end{cases} \end{cases}$$

Slide 36: Superposition of a 1D- and a 3D-roton

$$\Theta_{\mathbb{Z}} \xrightarrow[\text{by}]{\text{par}} \begin{cases} \gamma_{\mathbb{r}} \begin{cases} \mathbf{u}_\gamma = \hat{z} c \sin\theta \\ \mathbf{A}_\gamma = e^{i\pi/4} \sqrt{\sec\theta} \sqrt{\grave{r}/(2\pi\grave{p})}\, A \left(\hat{x}\cos\omega_o t + \hat{y}\sin\omega_o t\right) \\ \mathbf{R}_\gamma = \mathbf{u} \times \mathbf{A}_\varphi \end{cases} \\ \text{in superposition with} \\ \varphi_{\mathbb{i}} \begin{cases} \mathbf{u}_\varphi = ic\cos\theta \left(\hat{x}\sin\grave{p}\omega_o t\sin\omega_o t - \hat{y}\sin\grave{p}\omega_o t\cos\omega_o t - \hat{z}\cos\grave{p}\omega \right. \\ \mathbf{A}_\varphi = e^{-i\pi/4} \sqrt{\sec\theta}\; \grave{r} A \left(\hat{x}\cos\omega_o t + \hat{y}\sin\omega_o t\right) \\ \mathbf{R}_\varphi = \mathbf{u} \times \mathbf{A}_\varphi \end{cases} \end{cases}$$

Slide 37: Superposition of a 1D- and a 3D-roton

Here we note the following

i. The absolute velocity $\|\mathbf{u}\| = \left\|\mathbf{u}_\varphi + \mathbf{u}_\gamma\right\| = c$ for all θ and at any time t.

ii. For the 3D-roton the energy content $\mathcal{E}_\varphi$ remains constant for all θ and is active.

iii. For the 1D-roton the energy content $\mathcal{E}_\gamma$ varies with θ and is reactive. (Here I use the electrical engineering terminology instead of imaginary energy.)

iv. The 1D- and the 3D-roton share a common activation vector **A** which binds the two rotons.

Slide 38: Energy of Superpositioned 1D- and a 3D-roton

$$\mathcal{E}_\varphi = \hbar \frac{\grave{r}}{\grave{p}\, t0} \qquad E_\gamma = \mathrm{i} E_\varphi \frac{\sin\theta}{\cos\theta}$$

The components of the velocity vector are

$$u_\gamma = c \sin\theta \qquad \text{and} \qquad u_\varphi = \mathrm{i} c \cos\theta = \sqrt{c^2 - u_\gamma^2}$$

and the perceived energy is

$$E_\Theta = E_\varphi \sqrt{\frac{c^2}{c^2 - u_\gamma^2}}$$

Slide 39: Energy of Superpositioned 1D- and a 3D-roton

Having established E_Θ, we now, by some or other means, increase the real velocity u_γ by $\mathrm{d}u_\gamma$, thus

$$E_\Theta + \mathrm{d}E_\Theta = E_\varphi \sqrt{1 + \frac{(u_\gamma + \mathrm{d}u_\gamma)^2}{c^2 - (u_\gamma + \mathrm{d}u_\gamma)^2}}$$

therefore

$$\mathrm{d}E_\Theta = E_\varphi \sqrt{1 + \frac{(u_\gamma + \mathrm{d}u_\gamma)^2}{c^2 - (u_\gamma + \mathrm{d}u_\gamma)^2}} - E_\varphi \sqrt{1 + \frac{u_\gamma^2}{c^2 - u_\gamma^2}}$$

and performing a series expansion on $\mathrm{d}E_\Theta$ gives

$$\mathrm{d}E_\Theta = E_\varphi \frac{c\, u_\gamma \,\mathrm{d}u_\gamma}{(c^2 - u_\gamma^2)^{3/2}} + \mathcal{O}[\mathrm{d}u_\gamma^2]$$

Energy = force × distance and force is defined by Newton's second law of motion, hence we also have

$$\mathrm{d}E_\mathsf{N} = m_i \frac{\mathrm{d}u_\gamma}{\mathrm{d}t}\, u_\gamma \mathrm{d}t$$

where m_i is the inertial mass. Equating $\mathrm{d}E_\mathsf{N} = \mathrm{d}E_\Theta$ we obtain after cancelling common terms

Slide 41: Energy of Superpositioned 1D- and a 3D-roton

$$m_i = E_\varphi \frac{c}{(c^2 - u_\gamma^2)^{3/2}}$$

and if $u_\gamma = 0$ the above reduces to

$$E_\varphi = m_\circ c^2$$

and it then follows trivially (Slide-38) that

$$E_\Theta = \frac{m_\circ c^2}{\sqrt{1 - v^2/c^2}}$$

Slide 42: Concluding Summary

1. Maxwellian dynamics describe rotons (solitons).
2. Hinting the electrostatic charge (proton-electron interaction) is different to electric charge that drives industry.
3. Rotons as photons explain Planck's $E = hf$
4. Rotons explain Newton's first law of motion in terms of a propagation of a wave.
5. Rotons explain the origin of inertial mass. (No Higgs field)
6. Rotons explain $E = mc^2$ and relativistic momentum.

Everything presented here does not contradict experience.