General Maxwellian Dynamics defined by a novel equation set. Particles are Maxwellian solitons.

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Abstract: Waves of all types are described mathematically using partial differential equations. Here, departing from this tradition, I describe waves using a novel system of three simultaneous vector algebraic equations: $\mathscr{M}(\mathbf{u},\mathbf{a},\mathbf{r}) = \left\{\mathbf{r} = \mathbf{u}\times\mathbf{a};\ \mathbf{u} = (\mathbf{a}\times\mathbf{r})/\|\mathbf{a}\|^2;\ \mathbf{a} = (\mathbf{r}\times\mathbf{u})/\|\mathbf{u}\|^2\right\}$ which define Maxwellian wave dynamics for any fields $\mathbf{a}$ and $\mathbf{b}$ that support wave action and $\mathbf{u}$ a velocity vector. That is $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ is a novel reformulation of the Maxwell equations in vacuum. Furthermore, the expressions for the permittivity ϵ_0, permeability μ_0 and the magnetic flux density $\mathbf{B}$, in terms of action h, elementary charge e and speed of light c, are obtained by manipulating $\mathscr{M}$ with the assumption that an EM-wave has action and transports charge. As an application of $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ I show that three dimensional spherical EM-wave structures do exist, in theory at least. They are stationary with finite dimensionality and could provide the basis for describing EM-solitons, which in turn could be used to describe many natural phenomena, including ball lightning among others. Instead of working with fields I reformulate $\mathscr{M}$ in terms of flux vectors $\mathbf{A}$ and $\mathbf{R}$. Using $\mathscr{M}(\mathbf{u},\mathbf{A},\mathbf{R})$ I describe rotary waves (propeller-like instead of ripples on a pond) and show that rotary waves could be the basis to describe particles, physically, as solitons in terms of Maxwellian wave dynamics.

Keywords: General Maxwellian Dynamics, Wave equation, Maxwell equations, EM-waves, EM-soliton, Ball lightning, Bimodal waves, Particles as waves

What is a wave? Towne [1] states that the requirement for a physical condition to be referred to as a wave, is that its mathematical representation give rise to a partial differential equation of particular form, known as the wave equation. The classical form

$$\frac{\partial^2 w}{\partial p^2} - \frac{1}{u^2}\frac{\partial^2 w}{\partial t^2} = 0 \qquad \text{or} \qquad \nabla^2 w - \frac{1}{u^2}\frac{\partial^2 w}{\partial t^2} = 0.$$

was proposed in 1748 by d'Alembert for a one-dimensional continuum. A decade later, Euler established the equation for the three-dimensional continuum. The above wave equations describe a disturbance with motion but do not indicate, in the first place, why the wave is possible at all. A physical wave is a state alternating between two domains; only one of the domains is represented in the equations above.

Here we do not concern ourself with the classical, or the d'Alembert's, wave equation. Instead I present a novel wave equation system consisting of three simultaneous vector equations. These were discovered by the fortuitous penning of the following sequence:

$$\mathbf{r}_1 = \mathbf{u}_0 \times \mathbf{a}_0, \quad \mathbf{u}_1 = \mathbf{a}_0 \times \mathbf{r}_1, \quad \mathbf{a}_1 = \mathbf{r}_1 \times \mathbf{u}_1, \quad \mathbf{r}_2 = \mathbf{u}_1 \times \mathbf{a}_1, \quad \cdots$$

and I soon realised that the sequence can continue unaltered indefinitely; that is $\mathbf{u}_n = \mathbf{u}$, $\mathbf{a}_n = \mathbf{a}$ and $\mathbf{r}_n = \mathbf{r}$, but only if normalisation is introduced:

$$\mathbf{r} = \mathbf{u} \times \mathbf{a}, \quad \mathbf{u} = \frac{1}{\|\mathbf{a}\|^2}\mathbf{a} \times \mathbf{r}, \quad \mathbf{a} = \frac{1}{\|\mathbf{u}\|^2}\mathbf{r} \times \mathbf{u}$$

Now, if the vectors are also functions of time, a new wave equation-system is born, provided that $\mathbf{u}$ is a velocity vector, and both $\mathbf{a}$ and $\mathbf{r}$ are vector domains that complement each other to facilitate the wave action. The solution of the above set of three simultaneous vector equations describes bimodal-transverse waves.

(Part I is a near repeat of the earlier article [2], Part II begins page 13.)

PART I
ELECTROMAGNETIC TRAVELLING PLANE WAVES

As a general note, throughout this article, when referring to the Maxwell equation, I refer to the Maxwell equations in vacuum:

$$\nabla \cdot \mathbf{B} = 0 \qquad\qquad \nabla \cdot \mathbf{E} = 0$$
$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \qquad\qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

1 The reformulated Maxwell equations

In an Euclidean $\mathbb{R}^3$ homogeneous space xyz, where $\hat{z} = \hat{x} \times \hat{y}$, we consider one plane $\mathcal{W}(\mathbf{p})$ of an EM-travelling plane wave and where $\mathbf{p}$ defines the position of $\mathcal{W}$. Such a wave is described (mathematically expressed: $\xrightarrow[\text{by}]{\text{dsc}}$) by the solution of

$\mathcal{M}(\mathbf{u},\mathbf{B},\mathbf{E})$, which is a system of three simultaneous vector algebraic equations:

$$\mathcal{W}(\mathbf{p}) \xrightarrow[\text{by}]{\text{dsc}} \mathcal{M}(\mathbf{u},\mathbf{B},\mathbf{E}) = \left\{ \begin{array}{lll} \mathbf{E} = \mathbf{u}\times\mathbf{B} & (\textit{activation by } \mathbf{B}) & \text{(a)} \\ \mathbf{u} = \dfrac{1}{\|\mathbf{B}\|^2}\mathbf{B}\times\mathbf{E} & (\textit{vectoring by } \mathbf{B}\times\mathbf{E}) & \text{(b)} \\ \mathbf{B} = \dfrac{1}{\|\mathbf{u}\|^2}\mathbf{E}\times\mathbf{u} & (\textit{reactivation by } \mathbf{E}) & \text{(c)} \end{array} \right\} \tag{1}$$

Here the terms activation and re-activation are synonymous with self-induction. The above equation set is a reformulation of the Maxwell equations when:

- $\mathbf{u}$ is a velocity vector $\mathbf{u} = c\hat{\mathrm{u}}$, where
- $\hat{\mathrm{u}}$ that is, $\hat{\mathrm{u}} \mapsto \hat{\mathrm{u}}(t)$, is a unitless unit vector function of time only, and
- c is the speed of light.
- $\mathbf{B}$ is the magnetic field $\mathbf{B} = B\hat{\mathrm{B}}$, and where
- $\hat{\mathrm{B}}$ that is, $\hat{\mathrm{B}} \mapsto \hat{\mathrm{B}}(t)$, is a unitless unit vector function of time only, and is orthogonal to $\hat{\mathrm{u}}$ hence $\hat{\mathrm{u}}\cdot\hat{\mathrm{B}} = 0$, and
- B scales the magnetic field and provides the physical units.
- $\mathbf{E}$ is the electric field and (1)(a) gives $\mathbf{E} = cB(\hat{\mathrm{u}}\times\hat{\mathrm{B}}) = cB\hat{\mathrm{E}}$, with $\hat{\mathrm{E}} = \hat{\mathrm{u}}\times\hat{\mathrm{B}}$.
- $\mathbf{p}$ the position of the origin for $\mathbf{u}$, $\mathbf{B}$, and $\mathbf{E}$; thus $\mathbf{p} = \int \mathbf{u}\,\mathrm{d}t$.

That (1) is a mathematical reformulation of the Maxwell equations is demonstrated as follows: First we need to evaluate the triple vector products $\nabla\times(\mathbf{u}\times\mathbf{B})$ and $\nabla\times(\mathbf{E}\times\mathbf{u})$, which we expand using general vector analytic methods.

$$\nabla\times(\mathbf{u}\times\mathbf{B}) = \mathbf{u}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{u}) + (\mathbf{B}\cdot\nabla)\mathbf{u} - (\mathbf{u}\cdot\nabla)\mathbf{B}$$
$$\nabla\times(\mathbf{E}\times\mathbf{u}) = \mathbf{E}(\nabla\cdot\mathbf{u}) - \mathbf{u}(\nabla\cdot\mathbf{E}) + (\mathbf{u}\cdot\nabla)\mathbf{E} - (\mathbf{E}\cdot\nabla)\mathbf{u}$$

Evaluating the terms

$$\nabla\cdot\mathbf{u} = 0 \quad \text{because } c \text{ and } \hat{\mathrm{u}}(t) \text{ are not functions of } x,\ y, \text{ and } z$$

$$\nabla\cdot\mathbf{B} = 0 \quad \text{because } B \text{ and } \hat{\mathrm{B}}(t) \text{ are not functions of } x,\ y, \text{ and } z \tag{2}$$

$$\nabla\cdot\mathbf{E} = 0 \quad \text{ditto, because } \mathbf{E} = \mathbf{u}\times\mathbf{B} \tag{3}$$

$$(\mathbf{B}\cdot\nabla)\mathbf{u} = 0 \quad \text{because } \left(B_x\frac{\partial}{\partial x} + B_y\frac{\partial}{\partial y} + B_z\frac{\partial}{\partial z}\right)c\hat{\mathrm{u}}(t) = 0$$

$$(\mathbf{E}\cdot\nabla)\mathbf{u} = 0 \quad \text{ditto}$$

$$\mathbf{u}\cdot\nabla = \frac{\partial}{\partial t} \text{ because } \mathbf{u}\cdot\nabla = \frac{\partial x}{\partial t}\frac{\partial}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial}{\partial y} + \frac{\partial z}{\partial t}\frac{\partial}{\partial z} = \frac{\partial}{\partial t}$$

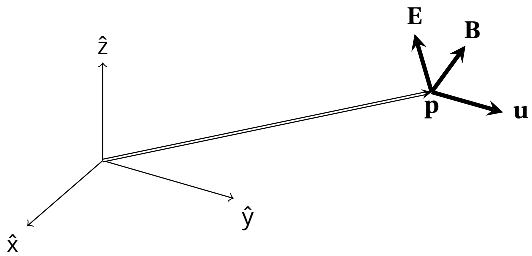


Figure 1: Illustrating the vectors used in $\mathcal{W}(\mathbf{p}) \xrightarrow[\text{by}]{\text{dsc}} \mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$.

and that leaves us with

$$\nabla \times (\mathbf{u} \times \mathbf{B}) = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times (\mathbf{E} \times \mathbf{u}) = \frac{\partial \mathbf{E}}{\partial t}$$

Applying a 'left and right side' curl operation on (1)(a) and (c) and using the above we are well on the way to recover the Maxwell equations in vacuum with (2) through (5) below.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{4}$$
$$\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \tag{5}$$

and it is well known that a further manipulation of the equations (2–5) gives the wave equations

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \text{and} \quad \nabla^2 \mathbf{B} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

proving that $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ is a new formulation for bimodal-waves such as EM-waves.

To prove that $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ is also a reformulation of the Maxwell equations, we can take the easy path and simply substitute $c^2 = 1/\epsilon_0\mu_0$ in the above. The more difficult path is to assert

a) An elementary EM-wave $\mathcal{W}$ exhibits power h/t^2, where h is the Planck constant and $t = 1$ second. This requires $\mathbf{B}$ to be an elementary field.

b) This elementary wave transports an electric charge e every one second which is a *wave current*. (This is nothing new; it is another way of describing the displacement current $\partial \mathbf{E}/\partial t$ that Maxwell had identified in varying electric fields.)

and then show that (1) together with the above assertions demands ϵ_0 and μ_0 in the form that they are known to us.

From the definitions of (1) we have $\|\mathbf{B}\| = |\mathbf{B}| = B$ which we substitute into (1)(b) to obtain

$$\mathbf{u} = \frac{1}{B^2}\mathbf{B} \times \mathbf{E} \tag{6}$$

On the premise that $\mathbf{B} \times \mathbf{E}$ is indicative of the wave action, we multiply (6) by the quantised action h and evaluate the norms

$$\|h\mathbf{u}\| = \left\| \frac{h}{B^2}\mathbf{B} \times \mathbf{E} \right\|$$

$$\therefore \quad hc = \left(\frac{h}{B^2}\right)\Big(|\mathbf{B}||\mathbf{E}|\Big)$$

We define an elementary distance $l = ct$, and multiplying and dividing the above by l^4 to transform the above from a domain of fields to a domain of fluxes, gives

$$h = \left[\frac{h}{l^4 B^2 c}\right] l^4 \Big(|\mathbf{B}||\mathbf{E}|\Big) \tag{7}$$

but that also requires $\mathbf{B}$ and $\mathbf{E}$ to be elementary fields. Here the square brackets indicate the development of a physical constant, which we want to determine by eliminating B.

Let's define the elementary electromagnetic action as $h_e = \varrho h$ where $\varrho = 1$ C kg^{-1} a correction factor to satisfy the dimensionality when working with electromagnetic quantities. Action is momentum times distance. Using a mechanical analogy we can say that EM-momentum is proportional to electric charge times velocity, having units of coulomb metres per second. As the wave transports an elementary charge e (Assertion-2 above) at a velocity c, the EM-wave action is EM-momentum times distance; here we consider the elementary distance $l = ct$ measured longitudinally to the propagation direction. Therefore, the EM-wave action is also

$$h_e = \varrho h = \kappa lec \tag{8}$$

and where κ is a dimensionless proportionality constant of unknown value, scaling lec to the EM-action h_e.

Let us think about $\mathbf{B}$ in the context of the EM-wave and the above assertions;

it facilitates the transportation of the charge e. Because the charge is carried by an EM-wave we can also postulate that the electromagnetic action is proportional to the product of $\mathbf{B}$ and the elementary volume which the wave occupies

$$\varrho h = \chi l^3 |\mathbf{B}|$$

and where χ is a constant with units and scaling to be determined. Combining the above with (8) gives

$$|\mathbf{B}| = \frac{\kappa e c}{\chi l^2}$$

and we substitute $|\mathbf{B}|$ from the above into (7) to get

$$h = \left[\frac{h}{l^4 B^2 c}\right] l^4 \left(\frac{\kappa e c}{\chi l^2} |\mathbf{E}|\right)$$

but, $|\mathbf{E}| = cB$ which gives

$$h = \left[\frac{h}{l^4 B^2 c}\right] \left[\frac{1}{\chi}\right] \kappa l^2 e c^2 B$$

We are now in the position to define the expression for

$$B = \frac{h}{\kappa l^2 e} \tag{9}$$

but only if

$$1 = \left[\frac{h}{l^4 B^2 c}\right] \left[\frac{1}{\chi}\right] c^2 \qquad \text{and replacing } B \text{ using (9) gives}$$

$$1 = \left[\frac{\kappa^2 e^2}{hc}\right] \left[\frac{1}{\chi}\right] c^2 \qquad \text{which requires } \frac{1}{\chi} = \frac{h}{\kappa^2 e^2 c}\text{, hence}$$

$$1 = \left[\frac{\kappa^2 e^2}{hc}\right] \left[\frac{h}{\kappa^2 e^2 c}\right] c^2 \tag{10}$$

Now—with a bit of hindsight—all that remains is to set

$$\kappa^2 = \frac{1}{2\alpha}$$

where α is the fine structure constant. Equation (10) now gives the sought after result

$$\epsilon_0 = \frac{e^2}{2\alpha hc} \quad \text{and} \quad \mu_0 = \frac{2\alpha h}{e^2 c}$$

expressions first formulated in 1916 by Sommerfeld [3] but in a way to define the fine structure constant α.

This concludes the proof that the equation set (1); that is $\mathscr{M}(\mathbf{u}, \mathbf{B}, \mathbf{E})$, is a mathematical reformulation of the Maxwell equations, because now we can replace $1/c^2$ in (5) with $\epsilon_0\mu_0$ as we have derived it independently.

In all of the above I have not resorted to any electromagnetic theories. Admittedly I have fine tuned the last step to obtain results in accord with the accepted definitions of the physical constants, but that should not deter us; it is what experimental physicists do daily, that is, determining constants from experimental results to match theory.

Now, it is beyond any doubt that the equation set (1) together with the two assertions are fundamental to Nature. How else does one explain that the correct expressions for the electromagnetic quantities are obtained on a mathematical basis without resort to an expanse of experimental observations. This validates the raising of numerous points and questions:

- Analytical simplicity predicts new EM-waveforms

  Electromagnetic waves as described by the Maxwell equations in free space are well studied and well understood, but only in the singular context as solutions to the d'Alembert wave equation. That means that for any solution the magnetic and electric field are always expressed as a function of both position and time. Researchers then make use of the superposition to construct intricate wave structures for their analysis.

  The equation set $\mathscr{M}(\mathbf{u}, \mathbf{B}, \mathbf{E})$ provides not only a new understanding of the underlying requirements for EM-waves; it also predicts new EM-wave types because the three vectors **u**, **B**, and **E** are defined in time only and the limitations dictated by solving the d'Alembert wave equation fall away.

- From the fine structure constant to elementary time and distance

  The fine structure constant α is said to quantify the strength of the electromagnetic interaction between elementary charged particles; the modern view also includes the coupling of the electromagnetic force to the other three forces [4]; these are the strong, weak and gravitational forces. Repeating (8), $\varrho h = \kappa lec$, here the constant $\kappa = 1/\sqrt{2\alpha}$ is a coupling constant relating the electric charge momentum to mechanical momentum.

  Oliver Heaviside [5] in 1892 presented us with vector algebra; he used it to recast Maxwell's original 20 equations to the four equations that we now recognise as the Maxwell equations. Now let's suppose the events of history were different; it is conceivable that Heaviside could also have stumbled on the equation set (1). In the year 1900 Planck proposed the quantity h and the value for the electric charge e was also known; so under the above supposition it is very probable that a Heaviside constant $\kappa = 8.277$ would have been proposed and the magnetic permeability would have transitioned from the fixed constant $4\pi \times 10^{-7}$ to the expression $h/(\kappa^2 e^2 c)$, or at least that relationship would have been known. Then sixteen years later, Sommerfeld would have established $\alpha^{-1} = 2\kappa^2$.

Nonetheless, $\varrho h = \kappa lec$ now provides a key to determine the values for the elementary length and time. Using the 2018 CODATA values we get:

$$\begin{array}{lll} \kappa = 8.277\,559\,999\,29(62) & \text{Heaviside constant} & \\ l_0 = 1.666\,566\,299\,11(12) \times 10^{-24} & \text{metres} & \\ t_0 = 5.559\,066\,796\,49(42) \times 10^{-33} & \text{seconds} & \text{using } l = ct. \end{array}$$

- Poynting vector: Not $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ but $\mathbf{S} = \mathbf{H} \times \mathbf{E}$.

The Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ and the associated electromagnetic momentum $\mathbf{g} = \mathbf{E} \times \mathbf{B}$ origins are in (1)(b), that is $\mathbf{u} = \|\mathbf{B}\|^{-2}\mathbf{B} \times \mathbf{E}$, but are of opposite sign by reason of prior choices and conventions. Recapitulating Jackson [6]: "*The wave equations*

$$\nabla^2 \mathbf{E} - \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \textit{and} \quad \nabla^2 \mathbf{B} - \mu\epsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

are both solved by assuming plane wave fields

$$\mathbf{E}(\mathbf{x}, t) = \mathcal{E} e^{ik\mathbf{n}\cdot\mathbf{x} - i\omega t} \quad \textit{and} \quad \mathbf{B}(\mathbf{x}, t) = \mathcal{B} e^{ik\mathbf{n}\cdot\mathbf{x} - i\omega t}$$

where $\mathcal{E}$, $\mathcal{B}$ *and* $\mathbf{n}$ *are vectors that are constant in time and space. Each component of* $\mathbf{E}$ *and* $\mathbf{B}$ *satisfies the wave equations provided that* $k^2 \mathbf{n}\cdot\mathbf{n} = \mu\epsilon\omega^2 c^{-2}$ *and* $\mathbf{n}\cdot\mathbf{n} = 1$. *The divergence equations* $\nabla\cdot\mathbf{E} = 0$ *and* $\nabla\cdot\mathbf{B} = 0$ *demand that* $\mathbf{n}\cdot\mathcal{E} = 0$ *and* $\mathbf{n}\cdot\mathcal{B} = 0$; *this establishes the co-orthogonality of* $\mathcal{E}$, $\mathcal{B}$ *and* $\mathbf{n}$. *The curl equation* $\nabla \times \mathbf{E} = \partial\mathbf{B}/\partial t$ *demands a further restriction*

$$\mathcal{B} = \sqrt{\mu\epsilon}\, \mathbf{n} \times \mathcal{E} \tag{11}$$

implying that $\mathcal{E}$ *and* $\mathcal{B}$ *have the same phase*"—and the cross product defines the spacial orientations of $\mathcal{E}$, $\mathcal{B}$ and $\mathbf{n}$. But equally, one can choose the plane wave fields

$$\mathbf{E}(\mathbf{x}, t) = \mathcal{E} e^{ik'\mathbf{n}'\cdot\mathbf{x} - i\omega t} \quad \text{and} \quad \mathbf{B}(\mathbf{x}, t) = \mathcal{B} e^{ik'\mathbf{n}'\cdot\mathbf{x} - i\omega t}$$

where $k' = -k$ and $\mathbf{n}' = -\mathbf{n}$; now $\nabla \times \mathbf{E} = \partial \mathbf{B}/\partial t$ demands the restriction

$$\boldsymbol{\mathcal{B}} = \sqrt{\mu\epsilon}\,\mathbf{n}' \times \boldsymbol{\mathcal{E}} \;=\; -\sqrt{\mu\epsilon}\,\mathbf{n} \times \boldsymbol{\mathcal{E}} \;=\; \sqrt{\mu\epsilon}\,\boldsymbol{\mathcal{E}} \times \mathbf{n}$$

which contradicts (11). However, it orientates the vectors $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{u} = c\mathbf{n}$ in accord with (1)(b), and $\mathscr{M}(\mathbf{u}, \mathbf{B}, \mathbf{E})$ demands $\mathbf{S} = \mathbf{H} \times \mathbf{E}$.

- Magnetic field quanta.

The magnetic flux quantum is defined as $\phi_0 = h/(2e)$; it was observed experimentally in 1961 by Deaver [7] in hollow superconducting cylinders, and Shankar [8] shows how to derive it by analysing the Aharonov–Bohm effect. With (9) I found the magnetic field of an elementary EM-wave $\mathcal{W}$ as $B = h/(\kappa l^2 e)$ which implies a magnetic flux for the elementary EM-wave $\tilde{\phi} = h/(\kappa e) = \sqrt{2\alpha}\, h/e$ which is clearly smaller than the magnetic flux quantum established by measurement and quantum theory.

At this point I offer no opinion regarding whether $\tilde{\phi} = \sqrt{2\alpha}\, h/e$ is a quantum or not, other than to comment that when scaled this way we have the desirable result that

$$\mathbf{S} = \mu_0^{-1} \|\mathbf{B} \times \mathbf{E}\| = h\frac{c^2}{l^4} \qquad \text{energy per (area} \times \text{time)}$$

which confirms the first of the assertions on page-13.

- What defines the speed of light?

The above analysis also raises a '*Who was first? Chicken or egg*' situation with respect to the speed of light. The permittivity ϵ_0 and permeability μ_0 were derived using the velocity c defined previously in (1). The question is: *Does $c = 1/\sqrt{\epsilon_0\mu_0}$ define the speed of light from first principles or is there another fundamental explanation to explain the velocity c?*

For example, the speed of a sound wave in a material is dependent on the material properties. In fluids $c^2 = K_s/\rho$ where K_s is a coefficient of stiffness and ρ the fluid's density. Alternatively, we can also express it as $c^2 = \partial P/\partial \rho$ where P is pressure. But do take note of the fact that none of K_s, ρ and P are defined in terms of the speed of sound within that medium.

The inference of the above is that I consider ϵ_0 and μ_0 as derived quantities. Therefore, it can be presumed that space has additional characteristics from which the *transportivity* $\mathcal{T} = c^2$ is defined. As an analog to fluids, the transportivity could be a ratio of two properties, which are not functions of the speed of light.

- New EM-wave forms

Reformulating the Maxwell equations as a wave equation $\mathscr{M}(\mathbf{u}, \mathbf{B}, \mathbf{E})$ provides new mathematical descriptions for EM-waveforms not thought possible before as natural EM-phenomena. One such form is a three dimensional and "stationary" EM-wave, which periodically traverses a closed and curved, or wound up, path; possibly such waves provide a theoretical basis to explain ball lightning as an EM-soliton.

2 Describing EM-waves

To fully describe a wave $\mathcal{W}$ also requires a set of parameteric equations $\mathscr{P}(\mathbf{u},\mathbf{B},\mathbf{E})$ which provide the solution to $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$. The parameteric equations $\mathscr{P}(\mathbf{u},\mathbf{B},\mathbf{E})$ define the unit vectors $\hat{\mathrm{u}}(t)$, $\hat{\mathrm{B}}(t)$ and $\hat{\mathrm{E}}(t)$ all as functions of t only, with the quantifiers c, B and E providing the necessary units, or quantities, and the scaling for $\mathcal{W}$. Any set of unit vectors $\hat{\mathrm{u}}(t)$, $\hat{\mathrm{B}}(t)$ and $\hat{\mathrm{E}}(t)$ that simultaneously satisfy

$$\hat{\mathrm{E}} = \hat{\mathrm{u}} \times \hat{\mathrm{B}} \qquad \hat{\mathrm{u}} = \hat{\mathrm{B}} \times \hat{\mathrm{E}} \qquad \hat{\mathrm{B}} = \hat{\mathrm{E}} \times \hat{\mathrm{u}}.$$

provide a solution to $\mathscr{M}$. Suitable solutions can be found, among others, by a succession of Euler rotations. Now, with a solution $\mathscr{P}$ for the equation-set $\mathscr{M}$, the wave $\mathcal{W}$ is described by $(\xrightarrow[\text{by}]{\text{dsc}})$ $\mathscr{M}$ which is parameterised by $(\xrightarrow[\text{by}]{\text{par}})$ $\mathscr{P}$ and expressed as $\mathcal{W}(\mathbf{p}) \xrightarrow[\text{by}]{\text{dsc}} \mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E}) \xrightarrow[\text{by}]{\text{par}} \mathscr{P}(\mathbf{u},\mathbf{B},\mathbf{E})$, which we simply shorten to $\mathcal{W}(\mathbf{p}) \xrightarrow[\text{by}]{\text{par}} \mathscr{P}(\mathbf{u},\mathbf{B},\mathbf{E})$ thus implying $\mathcal{W}(\mathbf{p}) \xrightarrow[\text{by}]{\text{dsc}} \mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ and $\mathbf{p} = \int \mathbf{u}\,\mathrm{d}t$.

2.1 Travelling plane waves

Every authoritative book, for example Jackson[6], describes an EM-field of a circular polarised travelling plane wave as

$$\begin{aligned}\mathbf{B}(\mathbf{z},t) &= B_0\big(\hat{\mathrm{x}}\cos(k\mathbf{n}\cdot\mathbf{z}-\omega t)+\hat{\mathrm{y}}\sin(k\mathbf{n}\cdot\mathbf{z}-\omega t)\big)\\ &= B_0\left[\hat{\mathrm{x}}\cos\left(\frac{\omega z}{c}-\omega t\right)+\hat{\mathrm{y}}\sin\left(\frac{\omega z}{c}-\omega t\right)\right]\end{aligned}$$

with the phase velocity defined by the wave vector $k\mathbf{n}$.

The equation set $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ defines the wave's velocity vector $\mathbf{u} = \hat{\mathrm{z}}c$ outside the field definitions. Let us introduce a position vector

$$\mathbf{p}_i = \int \mathbf{u}\,\mathrm{d}t = \hat{\mathrm{z}}\int c\,\mathrm{d}t = \hat{\mathrm{z}}(z_i + ct)$$

and use it to describe a circular polarised travelling plane EM-wave $\mathcal{W}$ as follows:

$$\mathcal{W}(\mathbf{p}_i,t) \xrightarrow[\text{by}]{\text{par}} \mathscr{P}_{\mathcal{W}}(\mathbf{u},\mathbf{B},\mathbf{E}) = \left\{ \begin{aligned} &\mathbf{u} = \hat{z}c \\ &\mathbf{B} = B\left[\hat{x}\cos\omega\left(\frac{\mathbf{p}_i}{c} - t\right) + \hat{y}\sin\omega\left(\frac{\mathbf{p}_i}{c} - t\right)\right] \\ &\mathbf{E} = cB\left[-\hat{x}\sin\omega\left(\frac{\mathbf{p}_i}{c} - t\right) + \hat{y}\cos\omega\left(\frac{\mathbf{p}_i}{c} - t\right)\right] \end{aligned} \right\}$$

The above describes a particular travelling plane of the wave $\mathcal{W}$ evaluated at the position $\mathbf{p}$ for any initial position $-\infty < z_i < \infty$ at $t = 0$. It is the classic description for a travelling plane wave with phase velocity defined by vector $\mathbf{u}$. But why so complicated? With $p = z_i + ct$ we can simplify the above to

$$\mathcal{W}(\mathbf{p}_i,t) \xrightarrow[\text{by}]{\text{par}} \left\{ \begin{aligned} &\mathbf{u} = \hat{z}c \\ &\mathbf{B} = B\left[\hat{x}\cos(\omega z_i/c) + \hat{y}\sin(\omega z_i/c)\right] \\ &\mathbf{E} = cB\left[-\hat{x}\sin(\omega z_i/c) + \hat{y}\cos(\omega z_i/c)\right] \end{aligned} \right\} \tag{12}$$

and we know that the above parameters provide a solution to $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ and I have shown that $\mathscr{M}$ is a reformulation of the Maxwell equation in vacuum. Therefore (12) describes a particular plane of an EM-travelling plane wave.

It is well known, and corroborated by experience, that radio waves are described by the above. A radio wave is a train of infinitely many planes that make up the continuous transmitted signal. Here we must note that each travelling plane is static; nothing changes within it whilst propagating.

2.2 Proposition: Ball lightning as a three dimensional EM-soliton

Accounts of ball lightning are reported on a regular basis yet all scientific explanations have evaded general acceptance. Here I propose another explanation that ball lightning is an EM-soliton; a wave propagating on a wound up near spherical path predicted by the equation set $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$. For a travelling object w we define a unit velocity vector as

$$\hat{u}_w(t) = \hat{x}\sin\omega_1 t\sin\omega_2 t + \hat{y}\sin n\omega_1 t\cos\omega_2 t + \hat{z}\cos\omega_1 t$$

The path $\mathbf{s}_w$ that the object w follows is found by integration

$$\begin{aligned} \mathbf{s}_w(t) = \int c\hat{u}_w\,\mathrm{d}t = \hat{x}c&\left(\frac{\sin(\omega_2+\omega_1)t}{2(\omega_2+\omega_1)} - \frac{\sin(\omega_1-\omega_2)t}{2(\omega_1-\omega_2)}\right) \\ &+ \hat{y}c\left(\frac{\cos(\omega_2+\omega_1)t}{2(\omega_2+\omega_1)} + \frac{\cos(\omega_1-\omega_2)t}{2(\omega_1-\omega_2)}\right) - \hat{z}c\frac{\sin\omega_1 t}{\omega_1} \end{aligned} \tag{13}$$

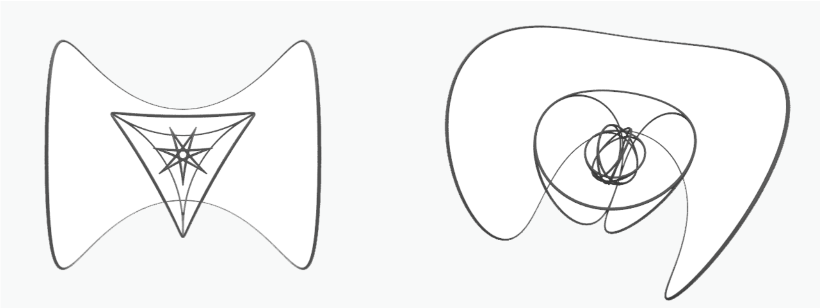


Figure 2: Two views of the path $\hat{s}_w(t)$ defined by (13) for the frequency ratios $\omega_1 : \omega_2 = 1:2$, $1:3$, and $1:7$. The path length of each curve is 2π.

.

The path is closed, or repeats, in periods of $t = 2\pi$ and as $\|c\hat{u}_w(t)\| = c$ the pathlength of $\mathbf{s}_w$ is $2\pi c$. Figure-2 sketches examples of paths defined by the above with different combinations of ω_1 and ω_2.

Let's define an EM-soliton Θ as a EM-wave that exists only on the closed path $\mathbf{p} = \int c\hat{u}_w \,\mathrm{d}t$, and at $t = 0$ it is at the position $\mathbf{p}_0 = \mathbf{s}_w(t_0)$, thus $\mathbf{u} = c\,\hat{u}_w(t_0 + t)$, hence:

$$\Theta(\mathbf{p}, t) \xrightarrow[\text{by}]{\text{dsc}} \left\{ \begin{aligned} \mathbf{u} &= c\big(\hat{x}\sin\omega_1(t_0+t)\sin\omega_2(t_0+t) \\ &\qquad + \hat{y}\sin n\omega_1(t_0+t)\cos\omega_2(t_0+t) + \hat{z}\cos\omega_1(t_0+t)\big) \\ \mathbf{B} &= B\big(\hat{x}\cos\omega_2(t_0+t) - \hat{y}\sin\omega_2(t_0+t)\big) \\ \mathbf{E} &= cB\big(\hat{x}\cos\omega_1(t_0+t)\sin\omega_2(t_0+t) \\ &\qquad + \hat{y}\cos\omega_1(t_0+t)\cos\omega_2(t_0+t) - \hat{z}\sin\omega_1(t_0+t)\big) \end{aligned} \right\} \tag{14}$$

The above satisfies the equation set $\mathscr{M}(\mathbf{u}, \mathbf{B}, \mathbf{E})$. Therefore, the above describes an EM-wave; propagating on the closed path $\mathbf{p}$ at velocity c. The above equation set (14) suggests that the wave is "trapped" by its magnetic field which forms a closed ring that precesses with the wave motion, that is when connected to the magnetic field of a point retarded by $\omega_2(t_0 - ½)$ but at a different z-elevation; and with the electric field always radiating outwards. Here I need to point out that Arnhoff [9], Chubykalo [10] and Cameron [11] presented solutions for three dimensional EM-wave structures, all of which are based on the superposition principle which allows the construction of intricate wave structures, and all contrasting with the simplicity of (14). Boerner [12] considers Cameron's proposal as the only viable explanation for ball lightning.

PART II
PARTICLES ARE MAXWELLIAN SOLITONS

I confidently assert that the equation set

$$\mathscr{M}(\mathbf{u},\mathbf{a},\mathbf{r}) = \left\{ \mathbf{r} = \mathbf{u}\times\mathbf{a}, \quad \mathbf{u} = \frac{1}{\|\mathbf{a}\|^2}\mathbf{a}\times\mathbf{r}, \quad \mathbf{a} = \frac{1}{\|\mathbf{u}\|^2}\mathbf{r}\times\mathbf{u} \right\}$$

is the basis for General Maxwellian Dynamics beyond the electromagnetic domain.

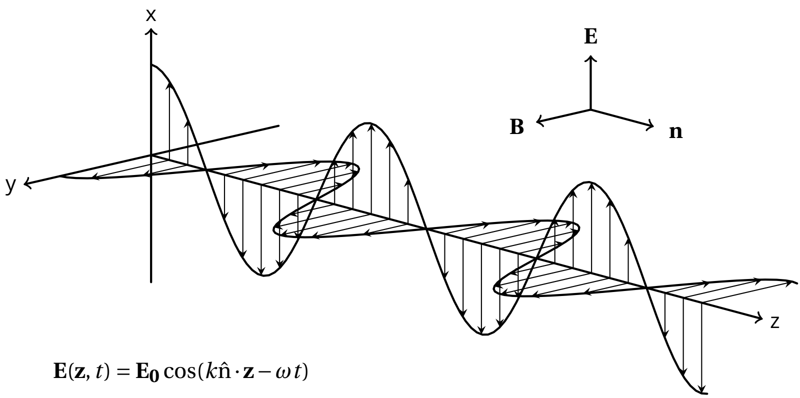
Here I propose a new wave type. Instead of being guided by waves on a pond, let's be guided by a one-bladed propeller. To visualise the wave, let the rotation axis of the propeller be aligned with the z-axis, while the propeller is rotating in the xy-plane, and propagating along the z-axis at a constant speed independent of the rotational velocity. (In Appendix-A page 25, I present a novel solution to the d'Alembert wave equation to describe such a rotary wave.) Here we use the General Maxwellian Dynamics and define

$$\mathcal{R}(\mathbf{p}) \xrightarrow[\text{by}]{\text{dsc}} \mathscr{M}(\mathbf{u},\mathbf{A},\mathbf{R}) = \left\{ \begin{array}{lll} \mathbf{R} = \mathbf{u}\times\mathbf{A} & (\textit{activation by}\ \mathbf{A}) & \text{(a)} \\ \mathbf{u} = \dfrac{1}{\|\mathbf{A}\|^2}\mathbf{A}\times\mathbf{R} & (\textit{vectoring by}\ \mathbf{A}\times\mathbf{R}) & \text{(b)} \\ \mathbf{A} = \dfrac{1}{\|\mathbf{u}\|^2}\mathbf{R}\times\mathbf{u} & (\textit{reactivation by}\ \mathbf{R}) & \text{(c)} \end{array} \right\} \tag{15}$$

where $\mathbf{A} = lA\hat{\mathrm{A}}(t)$ is the activation-flux vector, l is the length of the vector, A an elementary quantity with units and $\hat{\mathrm{A}}$ a unitless unity vector; and similarly the reactivation-flux vector $\mathbf{R} = R\hat{\mathrm{R}}(t)$. Because, $\mathscr{M}$ is a generic equation set we avoid the temptation to define $\mathbf{A}$ and $\mathbf{R}$ as the magnetic and electric flux, respectively. In addition, we now make the following assumptions:

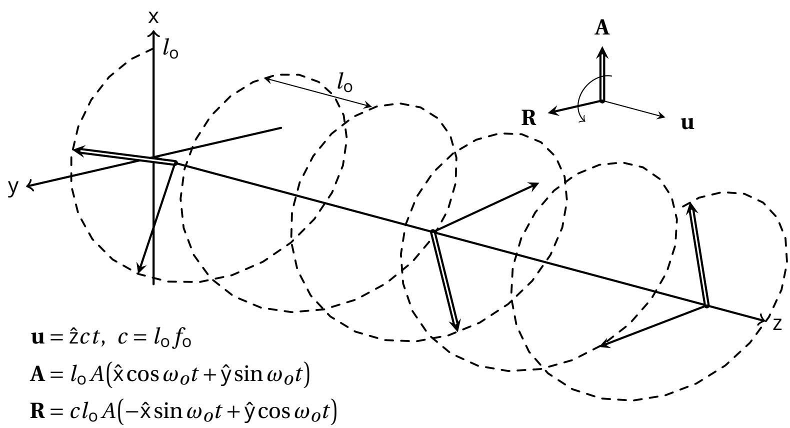
a) An elementary rotary wave $\mathcal{R}$ has action h. This requires $\mathbf{A}$ to be an elementary activation-flux vector.

b) This elementary rotary wave transports an elementary load ℓ. We need to assign some units to the elementary load. I propose a new unit L, the leyden, honouring the Leyden jar.



$$\mathbf{E}(\mathbf{z},t) = \mathbf{E_0}\cos(k\hat{\mathrm{n}}\cdot\mathbf{z} - \omega t)$$
$$\mathbf{B}(\mathbf{z},t) = \mathbf{B_0}\cos(k\hat{\mathrm{n}}\cdot\mathbf{z} - \omega t)$$
$$\mathbf{E}_0\cdot\hat{\mathrm{n}} = 0,\ \ \mathbf{B}_0\cdot\hat{\mathrm{n}} = 0,\ \ \mathbf{B}_0 = k\hat{\mathrm{n}}\times\mathbf{E}_0,\ \ k = \frac{\omega}{c},\ \ \hat{\mathrm{n}} = \hat{\mathrm{z}}$$

Figure 3: This graphic represents a transverse-travelling plane EM-wave, described by the classical solution [6] of the wave equation obtained from the Maxwell's field equations. In a travelling plane normal to **n** the fields are constant and propagate at light speed c, which is also the phase velocity of the wave. As the wave travels past a fixed point in space a varying field is observed. Here **E** and **B** are the electric and magnetic field respectively, $\hat{\mathrm{n}}$ the wave vector and k the angular wave number



$$\mathbf{u} = \hat{\mathrm{z}}ct,\ \ c = l_o f_o$$
$$\mathbf{A} = l_o A\big(\hat{\mathrm{x}}\cos\omega_o t + \hat{\mathrm{y}}\sin\omega_o t\big)$$
$$\mathbf{R} = c l_o A\big(-\hat{\mathrm{x}}\sin\omega_o t + \hat{\mathrm{y}}\cos\omega_o t\big)$$
$$\mathbf{R} = (\mathbf{u}\times\mathbf{A}) \quad \text{and} \quad \mathbf{u} = \frac{1}{A^2}(\mathbf{A}\times\mathbf{R}) \quad \text{and} \quad \mathbf{A} = \frac{1}{c^2}(\mathbf{R}\times\mathbf{u})$$

Figure 4: This graphic depicts a quantised rotary wave, here photon-like, which is a solution to the vector algebraic wave equation (15). It is shown at three random positions, at times t_1, t_2 and t_3. The rotary wave is a point-like wave in the direction of propagation. The wave does not exist before nor behind it; in the plane transverse to the propagation direction, **A** and **R** are rotating and occupy space. The helix traces **A** and is shown for illustrative purpose only. (This is not a circular polarised transverse travelling plane wave)

3 Maxwellian dynamics of rotary waves

From Section-1 we know that (15) gives the Maxwell-like equations

$$
\begin{aligned}
&\nabla\cdot\mathbf{A}=0 &\qquad\qquad &\nabla\cdot\mathbf{R}=0\\
&\nabla\times\mathbf{A}=\underset{\cdot}{\epsilon}\underset{\cdot}{\mu}\frac{\partial\mathbf{R}}{\partial t} & &\nabla\times\mathbf{R}=-\frac{\partial\mathbf{A}}{\partial t}
\end{aligned}
$$

and here the active permeability and reactive permittivity, $\underset{\cdot}{\mu}$ and $\underset{\cdot}{\epsilon}$ are different in value, and units, to the magnetic permeability and electric permittivity μ_0 and ϵ_0, respectively. But their products are equal, that is $\underset{\cdot}{\epsilon}\underset{\cdot}{\mu}=\epsilon_0\mu_0=c^{-2}$.

A solution of (15) is the quantised rotary wave γ

$$
\gamma \xrightarrow[\text{by}]{\text{par}} \begin{cases}\mathbf{u}=\hat{\mathsf{z}}\\ \mathbf{A}=r\,l_{\mathsf{o}}A\big(\hat{\mathsf{x}}\cos n\omega_{\mathsf{o}}t+\hat{\mathsf{y}}\sin\grave{n}\omega_{\mathsf{o}}t\big)\\ \mathbf{R}=crl_{\mathsf{o}}A\big(-\hat{\mathsf{x}}\sin\grave{n}\omega_{\mathsf{o}}t+\hat{\mathsf{y}}\cos\grave{n}\omega_{\mathsf{o}}t\big)\end{cases} \tag{16}
$$

where r and n are dimensionless scalars. Figure-4 sketches an elementary rotary wave defined by $r=n=1$. Here $\omega_{\mathsf{o}}=2\pi$, thus the wave propagates a distance $l_{\mathsf{o}}=ct_{\mathsf{o}}$ in a time period $t_{\mathsf{o}}=1$ while $\mathbf{A}$ does one revolution. The quantities l_{o} and t_{o} are elementary quantities. We now repeat the method of Section-1 that lead to the formulation of ϵ_0 and μ_0 to find the expression for $\underset{\cdot}{\epsilon}$ and $\underset{\cdot}{\mu}$. We continue with $r=n=1$ and develop (15)(b), and begin with the substitution $||\mathbf{A}||=l_{\mathsf{o}}A$ to give

$$
\mathbf{u}=\frac{1}{l_{\mathsf{o}}^2A^2}\mathbf{A}\times\mathbf{R} \tag{17}
$$

On the premise that $\mathbf{A}\times\mathbf{R}$ is indicative of an action, we multiply (17) by the elementary action h and evaluate the norms

$$
\begin{aligned}
\|h\mathbf{u}\| &= \left\|\frac{h}{l_{\mathsf{o}}^2A^2}\mathbf{A}\times\mathbf{R}\right\|\\
\therefore\qquad h &= \left[\frac{h}{l_{\mathsf{o}}^2A^2c}\right]\Big(\|\mathbf{A}\|\,\|\mathbf{R}\|\Big)
\end{aligned} \tag{18}
$$

and the square brackets again indicate the development of a physical constant, which we want to determine by eliminating A. Let $\mathbf{A}$ be the carrier, or transporter, of the quantised load ℓ. Action is momentum times distance, but we have a rotating system. Therefore, rotary-action is rotary momentum times the angle θ subtended. Rotary momentum is the product of the moment of inertia I and the rotational velocity ω, hence the rotational action $\mathcal{S}_{\text{rot}}=I\omega\theta$. Hence we can formulate the quantised rotational action as

$$h_{\text{rot}} = \varrho h = \Bbbk \ell \, l_{\text{o}}^2 \omega_{\text{o}} \tag{19}$$

where $\Bbbk$ is a dimensionless proportionality constant of unknown value, scaling $\ell \, l_{\text{o}}^2 \omega_{\text{o}}$ to the rotational-action h_{rot} and here $\varrho = 1$ Lkg^{-1} (leyden per kilogram) a correction factor to satisfy the dimensionality of above If (16) describes a photon then we know that the above is true, *i.e.* the above provides the mathematical explanation for the Planck energy equivalence. Because $l_{\text{o}} = c t_{\text{o}} = c/f_{\text{o}}$ to obtain $\omega_{\text{o}} = 2\pi f_{\text{o}} = 2\pi c / l_{\text{o}}$ hence h_{rot} is also expressed as:

$$h_{\text{rot}} = \varrho h = 2\pi \Bbbk \ell \, l_{\text{o}} c$$

Because the load is carried by $\mathbf{A}$ which has a magnitude $\|\mathbf{A}\| = l_{\text{o}} A$, therefore we can also postulate the elementary rotary-action

$$h_{\text{rot}} = \chi \, l_{\text{o}} A$$

where χ is part of a constant to be determined. Also note that A is a quantised quantity. From the above two equations we get

$$A = \frac{2\pi \Bbbk \ell c}{\chi} \quad \text{hence} \quad \|\mathbf{A}\| = \frac{2\pi \Bbbk l_{\text{o}} \ell c}{\chi}$$

and substituting above into (18) gives

$$h = \left[\frac{h}{l_{\text{o}}^2 A^2 c}\right] \left(\frac{2\pi \Bbbk l_{\text{o}} \ell c}{\chi} \|\mathbf{R}\|\right)$$

but $\|\mathbf{R}\| = c l_{\text{o}} A$ which gives after defining a further constant $\left[\dfrac{l_{\text{o}}^2}{\chi}\right]$

$$h = \left[\frac{h}{l_{\text{o}}^2 A^2 c}\right] \left[\frac{l_{\text{o}}^2}{\chi}\right] 2\pi \Bbbk \ell \, c^2 A$$

We are now in the position to define the quantised activator as

$$A = \frac{h}{2\pi \Bbbk \ell} \tag{20}$$

but only if

$$1 = \left[\frac{h}{l_\circ^2 A^2 c}\right]\left[\frac{l_\circ^2}{\chi}\right]c^2 \qquad \text{and using (20) to replace } A \text{ to get}$$

$$1 = \left[\frac{4\pi^2 \Bbbk^2 \ell^2}{l_\circ^2 hc}\right]\left[\frac{l_\circ^2}{\chi}\right]c^2 \qquad \text{which requires } \chi = \frac{4\pi^2 \Bbbk^2 \ell^2 c}{h}, \text{ hence}$$

$$1 = \left[\frac{4\pi^2 \Bbbk^2 \ell^2}{l_\circ^2 hc}\right]\left[\frac{l_\circ^2 h}{4\pi^2 \Bbbk^2 \ell^2 c}\right]c^2 = \underset{\cdot}{\epsilon}\underset{\cdot}{\mu} c^2$$

from which we get

$$\underset{\cdot}{\epsilon} = \frac{4\pi^2 \Bbbk^2 \ell^2}{l_\circ^2 hc} \quad \text{and} \quad \underset{\cdot}{\mu} = \frac{l_\circ^2 h}{4\pi^2 \Bbbk^2 \ell^2 c}$$

4 Rotary waves are solitons; the *roton* is a Maxwellian soliton

The term *soliton* describes self reinforcing solitary waves. Drazin [13] defined a soliton as any solution of a nonlinear equation (or a system) which:

i. *represents a wave of permanent form;*

ii. *is localised, so that it decays or approaches a constant at infinity;*

iii. *can interact strongly with other solitons and retain its identity.*

The above analysis confirms the first two points; the third is yet to be demonstrated.

Roton: Let's define a *roton* as a soliton that underlies Maxwellian dynamics. A roton need not only be the photon-like rotary-wave that propagates in a straight line. The vector algebraic equations (15) allow solutions for $\mathcal{R}$ as closed propagation paths, the simplest being a circle in a plain but 3-dimensional closed paths are also possible. The simultaneous algebraic vector equation set

$$\hat{\mathrm{R}} = \hat{u} \times \hat{\mathrm{A}} \qquad \hat{u} = \hat{\mathrm{A}} \times \hat{\mathrm{R}} \qquad \hat{\mathrm{A}} = \hat{\mathrm{R}} \times \hat{u}.$$

has infinitely many solutions, some of which can be found by a succession of Euler rotations. These solutions define the roton's spatial dimensionality and the propagation paths. The three simplest forms, here all having the same activation vector, are:

1D-roton: Linear propagation path along the z-axis (photon like)

$$\begin{aligned}\hat{u}_\gamma &= \hat{\mathrm{z}}\\ \hat{\mathrm{A}}_\gamma &= \hat{\mathrm{x}}\cos \grave{n}\omega_{\mathrm{o}} t + \hat{\mathrm{y}}\sin \grave{n}\omega_{\mathrm{o}} t\\ \hat{\mathrm{R}}_\gamma &= -\hat{\mathrm{x}}\sin \grave{n}\omega_{\mathrm{o}} t + \hat{\mathrm{y}}\cos \grave{n}\omega_{\mathrm{o}} t\end{aligned} \tag{21}$$

2D-roton: Circular propagation path in the xy-plane centred at the origin

$$\begin{aligned}\hat{u}_\odot &= \hat{\mathrm{x}}\sin \grave{n}\omega_{\mathrm{o}} t - \hat{\mathrm{y}}\cos \grave{n}\omega_{\mathrm{o}} t\\ \hat{\mathrm{A}}_\odot &= \hat{\mathrm{x}}\cos \grave{n}\omega_{\mathrm{o}} t + \hat{\mathrm{y}}\sin \grave{n}\omega_{\mathrm{o}} t\\ \hat{\mathrm{R}}_\odot &= \hat{\mathrm{z}}\end{aligned}$$

3D-roton: Closed curved, or wound up, path in xyz-space centred at the origin.

$$\begin{aligned}\hat{u}_\varphi &= \hat{\mathrm{x}}\sin \omega_1 t \sin \grave{n}\omega_{\mathrm{o}} t - \hat{\mathrm{y}}\sin n\omega_1 t \cos \grave{n}\omega_{\mathrm{o}} t - \hat{\mathrm{z}}\cos \omega_1 t\\ \hat{\mathrm{A}}_\varphi &= \hat{\mathrm{x}}\cos \grave{n}\omega_{\mathrm{o}} t + \hat{\mathrm{y}}\sin \grave{n}\omega_{\mathrm{o}} t\\ \hat{\mathrm{R}}_\varphi &= \hat{\mathrm{x}}\cos \omega_1 t \sin \grave{n}\omega_{\mathrm{o}} t - \hat{\mathrm{y}}\cos \omega_1 t \cos \grave{n}\omega_{\mathrm{o}} t + \hat{\mathrm{z}}\sin \omega_1 t\end{aligned} \tag{22}$$

where $\omega_1 = \grave{p}\grave{n}\omega_{\mathrm{o}}$ and where $\grave{p}$ is a prime integer ensuring that the path is repeated in periods of t_{o}.

4.1 Energy of a roton

Equation (16) describes a generalised 1D-rotary wave γ, a photon-like roton propagating in the z-direction, with $\mathbf{A} = \grave{r}\, l_{\mathrm{o}} A\big(\hat{\mathrm{x}}\cos \grave{n}\omega_{\mathrm{o}} t + \hat{\mathrm{y}}\sin \grave{n}\omega_{\mathrm{o}} t\big)$. From (19) it is obvious that the action of γ is $h_\gamma = \grave{r}^2 n h_{\mathrm{rot}} = \hbar\ell\, \grave{r}^2 l_{\mathrm{o}}^2 n\omega_{\mathrm{o}}$, where both $\grave{r}$ and $\grave{n}$ are unitless scalars. The action vector $\boldsymbol{\mathcal{S}}_\gamma$ is given by

$$\begin{aligned}\boldsymbol{\mathcal{S}}_\gamma &= \underset{\cdot}{\epsilon}\,\grave{n}(\mathbf{A}\times\mathbf{R})\\ &= \underset{\cdot}{\epsilon}\,\grave{n}\grave{r}^2\big(l_{\mathrm{o}} A\hat{\mathrm{A}}(t) \times l_{\mathrm{o}} R\hat{\mathrm{R}}(t)\big)\end{aligned}$$

and the norm evaluates to

$$\begin{aligned}\|\boldsymbol{\mathcal{S}}_\gamma\| &= \underset{\cdot}{\epsilon}\,\grave{n}\grave{r}^2 c\, l_{\mathrm{o}} A^2\\ &= h\grave{n}\grave{r}^2\end{aligned} \tag{23}$$

Therefore, with $\grave{r} = 1$ the rotary wave γ carries an energy content

$$\mathcal{E}_\gamma = h\frac{\grave{n}}{t_{\mathrm{o}}} = hf$$

which is the Planck energy equivalence.

Now let's analyse a 3D-roton. First we analyse the path s_ϕ on which a roton propagates; it is found by integration $s = \int \mathbf{u}\,\mathrm{d}t$. For the 3D-roton, and setting

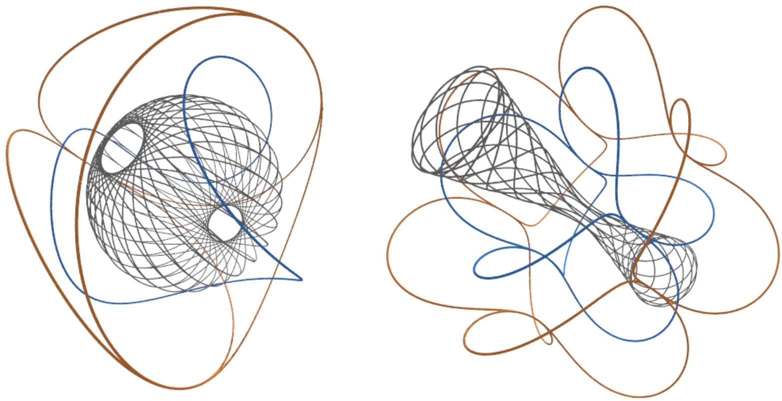


Figure 5: Each figure shows three rotons sharing the same centre. The left hand orbits are defined by $\{\omega_1, \omega_{\mathrm{o}}\} = \{29,7\},\{2,1\},\{3,1\}$. The right hand figure uses $\{\omega_1, \omega_{\mathrm{o}}\} = \{7,29,\},\{3,5,\},\{5,7\}$, with different scaling applied solely for visual aesthetics.

$\grave{n} = 1$ we obtain

$$
\begin{aligned}
s_\varphi &= c\int \left(\hat{\mathrm{x}}\sin\omega_1 t\sin\grave{n}\omega_{\mathrm{o}}t - \hat{\mathrm{y}}\sin n\omega_1 t\cos\grave{n}\omega_{\mathrm{o}}t - \hat{\mathrm{z}}\cos\omega_1 t\right)\mathrm{d}t\\
&= \hat{\mathrm{x}}c\left(\frac{\sin\left(\omega_1-\omega_{\mathrm{o}}\right)t}{2(\omega_1-\omega_{\mathrm{o}})} - \frac{\sin\left(\omega_1+\omega_{\mathrm{o}}\right)t}{2(\omega_1+\omega_{\mathrm{o}})}\right)\\
&\qquad -\hat{\mathrm{y}}c\left(\frac{\cos\left(\omega_1-\omega_{\mathrm{o}}\right)t}{2(\omega_1-\omega_{\mathrm{o}})} + \frac{\cos\left(\omega_{\mathrm{o}}+\omega_1\right)t}{2(\omega_{\mathrm{o}}+\omega_1)}\right) - \hat{\mathrm{z}}c\frac{\sin\omega_1 t}{\omega_1}
\end{aligned}
$$

and examples are sketched in Figure-5. The path's radial distance from the origin evaluates to:

$$
r_\varphi = c\sqrt{\frac{\omega_1^4 - \omega_{\mathrm{o}}^2(\omega_1^2-\omega_{\mathrm{o}}^2)\sin^2\omega_1 t}{\omega_1^4(\omega_1^2-\omega_{\mathrm{o}}^2)}} \quad > \frac{c}{\omega_1} \quad \text{if} \quad \omega_1 > \omega_{\mathrm{o}}
$$

and remebering $\omega_1 = \grave{p}\grave{n}\omega_{\mathrm{o}}$, see (22), we get

$$
r_\varphi \gtrapprox \frac{c}{\grave{p}\grave{n}\omega_{\mathrm{o}}} \tag{24}
$$

The activation vector $\mathbf{A}$ of the 3D-roton is $\mathbf{A} = \grave{r}\, l_{\mathrm{o}} A\left(\hat{\mathrm{x}}\cos n\omega_{\mathrm{o}}t + \hat{\mathrm{y}}\sin\grave{n}\omega_{\mathrm{o}}t\right)$, which is the same as that of the 1D-roton. Hence the action vector $\boldsymbol{S}_\varphi$ and its norm is given by

$$\boldsymbol{S}_\varphi = \underset{\cdot}{\epsilon}\,\grave{n}(\mathbf{A}\times\mathbf{R})$$

and the norm evaluates to

$$\begin{aligned}\left\|\boldsymbol{S}_\varphi\right\| &= \underset{\cdot}{\epsilon}\,\grave{n}\grave{r}^2 c\, l_\circ A^2 \\ &= h\grave{n}\grave{r}^2 \end{aligned} \tag{25}$$

To ensure that the activation vector **A** never extends over the volumetric centre of the 3D-roton requires that $\grave{r}\, l_\circ$ is limited to r_φ. Therefore, to double the action requires doubling $\grave{r}$ and halving $\grave{n}$ as dictated by (24), from which we can reduce product $\grave{n}\grave{r}$ to $1/(2\pi\grave{p})$ remembering that $c = l_\circ/t_\circ$. Therefore

$$\begin{aligned}\left\|\boldsymbol{S}_\varphi\right\| &= h\grave{r}(\grave{n}\grave{r}) = h\grave{r}/(2\pi\grave{p}) \\ &= \hbar\frac{\grave{r}}{\grave{p}}\end{aligned}$$

thus the energy scales proportionally with the radius of a 3D-roton

$$\mathcal{E}_\varphi = \hbar\frac{\grave{r}}{\grave{p}\,t_\circ}$$

4.2 Superposition of an 1D- and a 3D-roton

Let's suppose that the 1D-roton (21) is a representation of a photon, then we have the problem that the 3D-roton (22) does not describe a particle, it is fixed in space. Particles are not fixed in space but can move freely within space, and their momentum adheres to the relativistic laws.

Because the equation system $\left\{\mathbf{R}=\mathbf{u}\times\mathbf{A},\ \mathbf{u}=\mathbf{A}\times\mathbf{R}/\|\mathbf{A}\|^2,\ \mathbf{A}=\mathbf{R}\times\mathbf{u}/\|\mathbf{u}\|^2\right\}$ is a linear equation system we can apply the superposition principle. This allows the construct to superimpose a 1D-roton with a 3D-roton, but that also requires the introduction of the imaginary number. In other words, each axis of the 3D-space now becomes complex; this does not make the space six-dimensional. Also we adopt a complex load, that is $\ell \mapsto \ell e^{i\alpha}$, but that requires $c \in \{c, ce^{i2\alpha}, ce^{-i2\alpha}\}$ so that the simultaneous equations $\left\{\mathbf{R}=\mathbf{u}\times\mathbf{A},\ \mathbf{u}=\mathbf{A}\times\mathbf{R}/\|\mathbf{A}\|^2,\ \mathbf{A}=\mathbf{R}\times\mathbf{u}/\|\mathbf{u}\|^2\right\}$

have solutions. The possible combinations are

$$\ell \mapsto \begin{cases} \ell \mathrm{e}^{\mathrm{i}\alpha} \text{ thus } A \mapsto A\mathrm{e}^{-\mathrm{i}\alpha} \text{ and } R \mapsto \begin{cases} R\mathrm{e}^{\mathrm{i}\alpha}, & \text{if } c \mapsto c\mathrm{e}^{\mathrm{i}2\alpha} \\ R\mathrm{e}^{-\mathrm{i}3\alpha}, & \text{if } c \mapsto c\mathrm{e}^{-\mathrm{i}2\alpha} \\ R\mathrm{e}^{-\mathrm{i}\alpha}, & \text{if } c \mapsto c \end{cases} \\ \qquad\qquad \text{or} \\ \ell \mathrm{e}^{-\mathrm{i}\alpha} \text{ thus } A \mapsto A\mathrm{e}^{\mathrm{i}\alpha} \text{ and } R \mapsto \begin{cases} R\mathrm{e}^{\mathrm{i}3\alpha}, & \text{if } c \mapsto c\mathrm{e}^{\mathrm{i}2\alpha} \\ R\mathrm{e}^{-\mathrm{i}\alpha}, & \text{if } c \mapsto c\mathrm{e}^{-\mathrm{i}2\alpha} \\ R\mathrm{e}^{\mathrm{i}\alpha}, & \text{if } c \mapsto c \end{cases} \end{cases}$$

An example of the superposition of a 1D- and a 3D-roton results in the complex 1D+3D-roton $\Theta_{\mathbb{Z}}$ parameterised as follows

$$\Theta_{\mathbb{Z}} \xrightarrow[\text{by}]{\text{par}} \begin{cases} \gamma_{\mathbb{r}} \begin{cases} \mathbf{u}_\gamma = \hat{\mathrm{z}} c \sin\theta \\ \mathbf{A}_\gamma = \mathrm{e}^{\mathrm{i}\pi/4} \sqrt{\sec\theta} \sqrt{\grave{r}/(2\pi \grave{p})}\, A \left(\hat{\mathrm{x}} \cos\omega_{\mathrm{o}} t + \hat{\mathrm{y}} \sin\omega_{\mathrm{o}} t\right) \\ \mathbf{R}_\gamma = \mathbf{u} \times \mathbf{A}_\varphi \end{cases} \\ \text{in superposition with} \\ \varphi_{\mathbb{i}} \begin{cases} \mathbf{u}_\varphi = \mathrm{i} c \cos\theta \left(\hat{\mathrm{x}} \sin \grave{p}\omega_{\mathrm{o}} t \sin\omega_{\mathrm{o}} t - \hat{\mathrm{y}} \sin \grave{p}\omega_{\mathrm{o}} t \cos\omega_{\mathrm{o}} t - \hat{\mathrm{z}} \cos \grave{p}\omega_{\mathrm{o}} t\right) \\ \mathbf{A}_\varphi = \mathrm{e}^{-\mathrm{i}\pi/4} \sqrt{\sec\theta}\, \grave{r} A \left(\hat{\mathrm{x}} \cos\omega_{\mathrm{o}} t + \hat{\mathrm{y}} \sin\omega_{\mathrm{o}} t\right) \\ \mathbf{R}_\varphi = \mathbf{u} \times \mathbf{A}_\varphi \end{cases} \end{cases}$$

Here we note the following

i. The absolute velocity $\|\mathbf{u}\| = \left\|\mathbf{u}_\varphi + \mathbf{u}_\gamma\right\| = c$ for all θ and at any time t.

ii. For the 3D-roton the energy content $\mathcal{E}_\varphi$ remains constant for all θ and is active.

iii. For the 1D-roton the energy content $\mathcal{E}_\gamma$ varies with θ and is reactive. (Here I use the electrical engineering terminology instead of imaginary energy.)

iv. The 1D- and the 3D-roton share a common activation vector **A** which binds the two rotons.

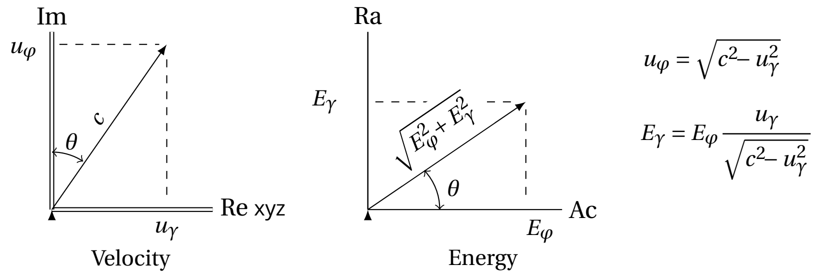


Figure 6: Both 1D-roton $\gamma_{\mathbb{r}}$ and the 3D-roton $\varphi_{\mathbb{i}}$ share the same common but complex path, because they are combined in superposition. The velocity–energy relationship for $\varphi_{\mathbb{i}} + \gamma_{\mathbb{r}}$ is also shown. (Ra=reactive, Ac=active)

Referring to (23) and (25) the energies for the two components calculate as

$$\mathcal{E}_\varphi = \hbar \frac{\grave{r}}{\grave{p}\, t0}$$

$$E_\gamma = \mathrm{i} E_\varphi \frac{\sin\theta}{\cos\theta}$$

The components of the velocity vector are

$$u_\gamma = c\sin\theta \qquad \text{and} \quad u_\varphi = \mathrm{i}c\cos\theta = \sqrt{c^2 - u_\gamma^2}$$

and the perceived energy is

$$E_\Theta = E_\varphi \sqrt{\frac{c^2}{c^2 - u_\gamma^2}}$$

Figure-6 sketches the relationship between the real and imaginary velocities as well as the active and reactive energies. Having established E_Θ, we now, by some or other means, increase the real velocity u_γ by $\mathrm{d}u_\gamma$, thus

$$E_\Theta + \mathrm{d}E_\Theta = E_\varphi \sqrt{1 + \frac{(u_\gamma + \mathrm{d}u_\gamma)^2}{c^2 - (u_\gamma + \mathrm{d}u_\gamma)^2}}$$

therefore

$$\mathrm{d}E_\Theta = E_\varphi \sqrt{1 + \frac{(u_\gamma + \mathrm{d}u_\gamma)^2}{c^2 - (u_\gamma + \mathrm{d}u_\gamma)^2}} - E_\varphi \sqrt{1 + \frac{u_\gamma^2}{c^2 - u_\gamma^2}}$$

and performing a series expansion on $\mathrm{d}E_\Theta$ gives

$$\mathrm{d}E_\Theta = E_\varphi \frac{c\, u_\gamma\, \mathrm{d}u_\gamma}{(c^2 - u_\gamma^2)^{3/2}} + \mathcal{O}[\mathrm{d}u_\gamma^2]$$

Energy = force × distance and force is defined by Newton's second law of motion, hence we also have

$$\mathrm{d}E_\mathsf{N} = m_i \frac{\mathrm{d}u_\gamma}{\mathrm{d}t}\, u_\gamma \mathrm{d}t$$

where m_i is the inertial mass. Equating $\mathrm{d}E_\mathsf{N} = \mathrm{d}E_\Theta$ we obtain after cancelling common terms

$$m_i = E_\varphi \frac{c}{(c^2 - u_\gamma^2)^{3/2}}$$

and if $u_\gamma = 0$ the above reduces to

$$E_\varphi = m_\circ c^2$$

and it then follows trivially that

$$E_\Theta = \frac{m_\circ c^2}{\sqrt{1 - v^2/c^2}} \tag{26}$$

The above discussion is not complete without mention that $E_\gamma + E_\varphi > \sqrt{E_\gamma^2 + E_\varphi^2}$ which means when $\Theta_\mathbb{Z} = \gamma_\mathbb{r} + \varphi_\mathbb{i}$ is accelerated then energy is released in some form or other, for example radiation.

5 Concluding remark

With this article I present a novel simultaneous vector-algebraic equation system, $\mathscr{M}(\mathbf{u},\mathbf{a},\mathbf{r}) = \left\{\mathbf{r} = \mathbf{u}\times\mathbf{a},\ \mathbf{u} = \mathbf{a}\times\mathbf{r}/\|\mathbf{a}\|^2,\ \mathbf{a} = \mathbf{r}\times\mathbf{u}/\|\mathbf{u}\|^2\right\}$ whose solutions describe bimodal-transverse waves in a more *aufschlussreicher*[1] way than was possible with the classic partial differential approach. When this equation set is cast into the electromagnetic domain I showed that $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ implicates the Maxwell equations in vacuum, including the required formulations for ϵ_0 and μ_0 in the form previously only derivable from seemingly unrelated atomic theories and physical observations. This gives me the confidence to assert that the equation

1 aufschlussreich: German adj., translations: enlightening, illuminating, informative, insightful, instructive, revealing, and telling.

system $\mathscr{M}(\mathbf{u},\mathbf{a},\mathbf{r})$ is the generic formulation for general Maxwellian dynamism, and that $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ superordinates the Maxwell equations in vacuum.

Casting $\mathscr{M}$ into a new physical domain described by the activation and the reactivation flux vectors **A** and **R**, respectively, I model disturbances as rotary waves that transport energy. I propose that the results and methods developed Part-II are an ansatz for modelling particles physically. This proposition is supported by the following results:

1. The Plank energy-frequency relation is derived from first principles using a 1D-roton.
2. The 1D-roton has angular momentum, that is spin, thus ideally suited to model photons.
3. Using the superposition of a 1D+3D-roton and Newton's second law, I show that such a system exhibits inertial mass according to experience. (This now raises the question, *how to reconcile* (26) *with special relativity*? In this respect I present a thought experiment in Appendix-B, page 27, showing that special relativity is, from a rigorous mathematical point of view, self-contradictory.)
4. Newton's first law of motion now has a mathematical explanation. (The only mathematical equation that describes continues motion is the d'Alembert wave equation, and here we describe particles as waves.)

A further question is *how and whether the electromagnetic domain, specified by the well known magnetic and electric quantities (defined by, among others, the elementary charge* $e = 1.602\ldots \times 10^{-19}$ *coulomb), is related to the elementary load* ℓ =??? *with units leyden*? The relevance of this question becomes apparent by my challenge to Drude's model for electric current, described in the thought experiment in Appendix-C, page 32.

Whether or not $\mathscr{M}(\mathbf{u},\mathbf{a},\mathbf{r})$ is accepted as the foundation for general Maxwellian dynamics is not for me to determine; if it does then undoubtedly many derivations of it will be developed. Whether or not it provokes a rethinking of the electromagnetic phenomenon, or whether new discoveries are made resulting from all of the above, only time will tell. Nevertheless—for me—this paper marks the beginning of new work in this subject. There is much that remains to be done; for example, extending the methods developed here to describe particle wave duality and the basic interactions of many particle systems. I have developed an interesting approach, but to bring it to conclusion requires some collaborative effort and intellectual sparring partners to review, critique and contribute towards an extended and collaborative work.

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APPENDIX

A Novel solutions to the d'Alembert wave equation.

The analytical vector solution of the wave equation was developed after developing a new, calculus based, approach to solving the d'Alembert wave equation, a second order differential equation

$$u^2 \frac{\partial^2 w}{\partial p^2} - \frac{\partial^2 w}{\partial t^2} = 0. \tag{A.1}$$

Contrary to convention, we associate the velocity term u^2 with the position function instead of the usual inverted $1/u^2$ association with the time function. Let's consider the wave w which is simultaneously described by a function of position $F(p)$, a function of time $G(t)$, and a function that is the product of the square roots of the time and position functions.

$$w = \begin{cases} F(p) = f(p)^2 & (a) \\ G(t) = g(t)^2 & (b) \\ f(p)\,g(t) & (c) \end{cases} \tag{A.2}$$

The partial derivatives, using (A.2)(c) are

$$\frac{\partial^2 w}{\partial p^2} = g(t)\frac{\mathrm{d}^2 f(p)}{\mathrm{d}p^2} \qquad \text{and} \qquad \frac{\partial^2 w}{\partial t^2} = f(p)\frac{\mathrm{d}^2 g(t)}{\mathrm{d}t^2}$$

Introducing the result into (A.1), and dividing by $f(p)g(t)$ we find

$$\frac{u^2}{f(p)}\frac{\mathrm{d}^2 f(p)}{\mathrm{d}p^2} - \frac{1}{g(t)}\frac{\mathrm{d}^2 g(t)}{\mathrm{d}t^2} \quad = 0 \tag{A.3}$$

The first and the second terms are now independent of one another. The derivatives are total because $f(p)$ and $g(t)$ are independent of one another; and each is a function of p and t respectively. Hence if (A.3) is to hold, each side must equal some constant, and in anticipation of the solution we introduce the constant $-\omega^2/4$.

$$\frac{u^2}{f(p)}\frac{\mathrm{d}^2 f(p)}{\mathrm{d}p^2} = \frac{1}{g(t)}\frac{\mathrm{d}^2 g(t)}{\mathrm{d}t^2} \quad = -\frac{\omega^2}{4} \tag{A.4}$$

For $f(p) = g(t)$ we require $p = ut$. We also recognise each term of (A.4) as the differential equation of harmonic motion, for which we know a solution in the form

$$f(p) = \mathrm{e}^{\mathrm{i}\frac{\omega p}{2c}}$$
$$g(t) = \mathrm{e}^{\mathrm{i}\frac{\omega t}{2}}$$

All that remains is to square the above, choose some arbitrary scaling constant A

and use in (A.2) to give

$$\mathcal{w} = \begin{cases} A\,\mathrm{e}^{\mathrm{i}\omega p/u} & \text{(a)} \\ A\,\mathrm{e}^{\mathrm{i}\omega t} & \text{(b)} \quad \text{and} \quad p = ut \\ A\,\mathrm{e}^{\mathrm{i}(\omega p/2u+\omega t/2)} & \text{(c)} \end{cases}$$

and (c) above, that is $\mathcal{w} = A\exp\big(\mathrm{i}(\omega p/2u + \omega t/2)\big)$, describes a travelling rotary wave as a solution for the d'Alembert wave equation.

We can transform the above into three-dimensional space, with $\mathbf{p} = \hat{z}ut$ and a rotational disturbance in the xy-plane. The rotational wave $\mathcal{w}$ is described by a rotating vector **a** whose origin is defined by the position vector **p**; formulated as follows:

$$\mathcal{w} \xrightarrow[\text{by}]{\text{dsc}} \begin{cases} \mathbf{a} = a\hat{x}\cos(\omega z/u) + a\hat{y}\sin(\omega z/u) & \text{(a)} \\ \mathbf{a} = a\hat{x}\cos(\omega t) + a\hat{y}\sin(\omega t) & \text{(b)} \quad \text{and} \quad \mathbf{p} = \hat{z}ut \\ \mathbf{a} = a\hat{x}\cos(\omega z/2u + \omega t/2) + a\hat{y}\sin(\omega z/2u + \omega t/2) & \text{(c)} \end{cases}$$

B Special theory of relativity is self-contradictory.

The assumptions adopted are:

1. All of the assumptions used in Einstein's special theory of relativity.
2. Recoil-free reflection of light by a perfect mirror, that is invoking the Mössbauer effect because it could be argued that a photon is absorbed and re-emitted.

Einstein's 1905 paper "On the Electrodynamics of Moving Bodies" [1] defines the theory of special relativity. For the purpose of this paper we recall two results:

The first result is from § 8; it concerns the energy of light L, as measured in a stationary system, and its transformation to L' when measured in a moving system[2]

$$\frac{L'}{L} = \frac{1 - \cos\phi \cdot v/c}{\sqrt{1 - v^2/c^2}} \tag{B.1}$$

Einstein remarked "*It is remarkable that the energy and the frequency of a light complex vary with the state of motion of the observer in accordance with the*

2 Note that Einstein used the symbol E in [1]. This paper uses the symbol L representing the energy of the light or photon, so that we have a compatible syntax throughout because Einstein switched to the symbol L in the second referenced paper [2].

same law." because in the previous section, § 7, he derived the same ratio for the frequency of light observed in the relative moving reference systems. Indeed, from a contemporary point of view, Einstein demonstrated that the Planck energy-frequency equivalence $E = hf$ transforms in accord with the special theory of relativity.

The second result, from § 10, is that the kinetic energy of an electron, or of any ponderable mass is expressed as

$$W = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right). \tag{B.2}$$

The iconic equation $E = mc^2$, defining the mass-energy equivalence, is derived in the fourth *annus mirabilis* paper of 1905 "Does the inertia of a body depend upon its energy content?" [2]

This paper begins with the premise that a body in a stationary system has energy E_0 and in a relative moving system has an energy H_0. This body simultaneously emits two opposing light waves, each having energy $\frac{1}{2}L$ as measured in the stationary system. After the emission of light the body has energy E_1 and H_1 in the respective reference systems. According to Einstein the following holds true:

$$E_0 = E_1 + \frac{1}{2}L + \frac{1}{2}L,$$

$$H_0 = H_1 + \frac{1}{2}L\frac{1-\frac{v}{c}\cos\phi}{\sqrt{1-v^2/c^2}} + \frac{1}{2}\mathrm{L}\frac{1+\frac{v}{c}\cos\phi}{\sqrt{1-v^2/c^2}} \tag{B.3}$$

$$= H_1 + \frac{L}{\sqrt{1-v^2/c^2}}. \tag{B.4}$$

Einstein asserts that the difference in $E_0 - H_0$ and $E_1 - H_1$ reduces to the difference in kinetic energy K_0 and K_1 of the body as measured in the two reference systems. Therefore

$$K_0 - K_1 = L\left\{\frac{1}{\sqrt{1-v^2/c^2}} - 1\right\} \tag{B.5}$$

from which, by comparing (B.2) and (B.5), it follows trivially that $E = mc^2$ The paper [2] concludes "*If the theory corresponds to the facts, radiation conveys inertia between the emitting and absorbing bodies.*"

At this point we recap and establish that Einstein states that an observed photon in the stationary reference system has an energy L, but when the same photon is observed from a moving reference system its energy increases or decreases depending on the photon's relative direction of motion to the reference system, and is expressed in (B.1) and used in (B.3).

We also conclude from (B.4) that the total energy, that is the sum of the mass energy, kinetic energy, and photon energy, transforms according to (B.2) when observed from a different moving reference system.

All of the above is a recapitulation of Einstein's work with the original conclusions; the purpose is to set the conditions, and the framework, for the thought experiment that follows.

Thought experiment: Reflections from moving mirrors

Steve, in the stationary reference frame, observes in an *experimental system* a two-photon decay of a particle of mass m. The Planck relation $E = hf$ determines the frequencies[3] of the photons which he observes, *i.e.*

$$f_{S1} = f_{S2} = f_0 = \frac{\delta mc^2}{2h}, \tag{B.6}$$

where h is the Planck constant, δm is the particle's loss of mass that is converted to photonic energy, and c is the speed of light. Let the direction of the photons be opposite to each other and be parallel to the x-axis of Steve's reference frame.

In the *experimental system* the total energy before and after the two-photon decay remains constant at

$$E = (m - \delta m)c^2 + h(f_{S1} + f_{S2}) = mc^2. \tag{B.7}$$

Monica is in a moving reference frame with constant velocity v parallel to Steve's x-axis. Applying the relativistic doppler shift § 7 of [1]—obtained by setting $\phi = 0$ in (B.3)—she observes the frequencies of the two photons as

$$f_{M1} = f_0\sqrt{\frac{c-v}{c+v}} \qquad \text{and} \qquad f_{M2} = f_0\sqrt{\frac{c+v}{c-v}}.$$

The sum of the energy of the two photons is

$$h(f_{M1} + f_{M2}) = \frac{\delta mc^2}{\sqrt{1 - v^2/c^2}}.$$

Thus in the moving reference system the total energy of the *experimental system* before and after the two-photon decay also remains constant, but transformed to

3 We use the subscript $_S$ for the stationary case, and later the subscript $_M$ for the moving case

a level as asserted in [2]

$$\begin{aligned} H &= \frac{(m-\delta m)c^2}{\sqrt{1-v^2/c^2}} + h(f_{M1}+f_{M2}) \\ &= \frac{mc^2}{\sqrt{1-v^2/c^2}}. \end{aligned} \tag{B.8}$$

At this point, we note that we have not deviated from the methods pioneered by Einstein in [2] which resulted in $E = mc^2$. We now continue the thought process using the same methods, and introduce a reflection: Monica now uses two perfect mirrors[4] to reflect the photons towards their source. She does not observe a change in energy in the photons and the mirrors remain stationary for her. In the moving system, the photon frequencies before and after the reflection are unchanged. The energy of the *experimental system*, as Monica observes it, remains

$$H' = \frac{mc^2}{\sqrt{1-v^2/c^2}}. \tag{B.9}$$

On the other hand, again by use of § 7 of [1], Steve observes the frequencies of the returned photons as

$$f'_{S1} = f_0 \frac{(c-v)}{(c+v)} \qquad \text{and} \qquad f'_{S2} = f_0 \frac{(c+v)}{(c-v)}.$$

Now recalling (B.6), that is $f_0 = \delta mc^2/2h$ we get that in the stationary system the energy of the *experimental system* after reflection calculates to

$$\begin{aligned} E' &= (m-\delta m)c^2 + hf'_{S1} + hf'_{S2} \\ &= (m-\delta m)c^2 + hf_0\left(\frac{c-v}{c+v} + \frac{c+v}{c-v}\right) \\ &= mc^2 + \frac{2\delta m v^2}{(1-v^2/c^2)}. \end{aligned} \tag{B.10}$$

Conclusion

In deriving (B.7) and (B.8), we have followed the same method that Einstein pioneered in [2] and we continued to use his method to derive (B.9) and (B.10). The principle of relativity, as asserted in axiom 1, § 2 of [1],

The laws by which the states of physical systems undergo change are not

4 A perfect mirror is characterised such that there is no loss in reflecting a photon, its mass approaches infinity thus its kinetic energy is unchanged before and after a reflection, thus the act of reflection does not change the energy of the system.

> *affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform translatory motion.*

implies that the state of the *experimental system*, before and after an interaction, changes the same for both Steve and Monica, *i.e.* if Steve does not observe a change of energy in the *experimental system* then Monica also does not.

All is well for the theoretical physicist; (B.7) and (B.8) derived by the theory are in keeping with the principle of relativity and are corroborated by experience, and he has no need to consider reflections from moving mirrors. However, the mathematical physicist who argues with mathematical rigour has a problem, because the principle of relativity is contradicted by the extended rigorous investigations that lead to the result (B.8) = (B.9) and (B.7) $\neq$ (B.10).

The principle of relativity requires that:

If (B.8) *equals* (B.9) *then* (B.7)*) must equal* (B.10)*,*

which clearly is not the case: Monica observed no change in the total energy after (i) the moving mass emitted two light waves (or photons), and after (ii) she reflected these two photons. However, Steve in the first part (i) observes no change in energy when the stationary mass emitted two photons, but after (ii) the two photons were reflected, by moving mirrors, he observes a change in the total energy, which simply put is absurd because Monica does not witness the same!

In conclusion, here I successfully demonstrated how rigorous mathematical techniques prove that the special theory of relativity is self-contradictory, because in the outcome of the above thought experiment the expression for energy (B.10) has an extra term in the stationary system which has no equivalence in the moving system. Furthermore, the fundamental law of energy conservation is violated. The only tenable conclusion for the physicist adopting mathematical rigour is that the postulate of special relativity cannot be an underlying principle for natural phenomenon. In the book "*A Mathematician's Apology*" [3] Godfrey Hardy wrote:

> *Reductio ad absurdum, which Euclid loved so much, is one of a mathematician's finest weapons. It is a far finer gambit than any chess play: a chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers the game.*

Dear Albert, checkmate! The game is over. Time has come to rethink why experience has corroborated your theories—that is, for the energy mass equivalence, inertial mass dilation, clock rates, etc, and consequently the general theory of relativity—while the rigour of mathematics shows that your methods are absurd. We need to find a new theory that explains the observed and that theory also needs to explain the gravitational and electrical forces in a unified way, as well as providing the proper framework to integrate quantum mechanics.

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C Electricity is not what we think it is.

The 18th and 19th century Natural philosophers pondered about the electric fluid. They had no better means to describe electric phenomena; Thomson's [1] 1897 discovery of the electron changed that. Today, Drude's [2] 1900 postulate with the Sommerfeld's [3] 1928 modification, remains the accepted explanation for electric current. It is thought that electrons, as carriers of charge, drift to form an electric current. For example, assuming a 1mm diameter copper wire carrying an electric current of 1 amp, the electron drift is calculated at about 0.1 mm per second. Consequently according to Drude, when charging a capacitor, the positive charged plate will have a deficiency, and the negative charged plate an equal surplus of electrons.

Maxwell, showed us that there is another form of an electric current, the displacement current equal to the rate of change of an electric field. The displacement current and electric current in a copper wire are equivalent in dimensionality and both are sources of a magnetic field. The discovery of the displacement current is heralded as a historic landmark in physics because with it the magnetic and electric domains were unified into one set of equations, the Maxwell equations.

The Maxwell equations are fundamental to Nature. In Part-I of this paper I derived purely mathematically and purely generically the Maxwell equations. Therefore, the displacement current is a mathematical necessity, or rather it is fundamental to the electromagnetic phenomena. On the other hand, Drude's model has no equivalent in fundamental mathematical description. This now raises the question whether Drude's model really describes the electric current in conductors.

Drude's model does satisfy the dimensioning of electric current; the ampere is the flow of charge and is dimensioned as coulombs per second. I know of no experiment that confirms electric current in conductors as a drift of electrons. Particle beams are parameterised by a beam current which is measured, among other ways, by the beam's magnetic field, which is identical to that generated by an equivalent electric current. But a particle beam differs from an electric current insofar as it carries kinetic energy and not electric energy. Van de Graaff in the midst of cutting edge technology at MIT would have known about Drude's theory but seemingly ignored it, as he wrote [4] "*When connected as shown, one point sprays positive and the other sprays negative electricity onto its adjacent belt.*" (see Figure-7) and we note that he does not attribute the high voltages of his generators to a lack or surplus of electrons on the domes.

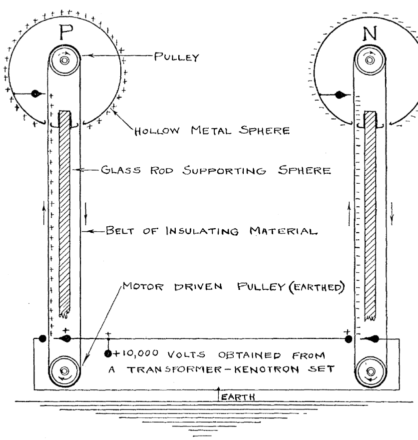

Figure 7: Van de Graaff Generator; extracted from [4]

The real question to ask is: "*Is the electrostatic charge field which is responsible for the Coulomb force the same phenomenon as the electric field (or electromotive field) responsible for the Lorentz force?*" To find an answer, I devised a thought experiment, which could be easily implemented by any competent research facility. The required apparatus is sketched in Figure-8.

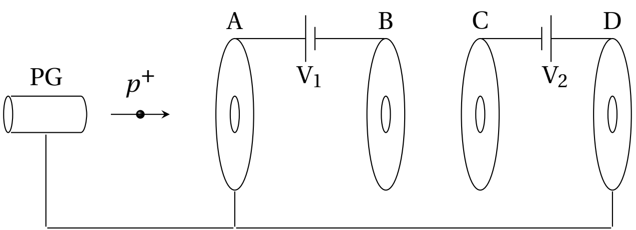

Figure 8: Thought experiment: Electrodynamic vs. static charge

High energy charged particles are emitted from the particle gun (PG) at ground potential. The particles do not gain, nor lose, any kinetic energy on the path PG–A, as A is at the same potential as PG. On the path A–B the particles gain kinetic energy as the electromotive field $\mathbf{E}_{\text{ab}}$ accelerates the particles, causing an electric current to flow in the battery V_1 which is discharged accordingly. Anything else would violate energy conservation laws.

On the path B–C there is no potential difference and the kinetic energy of the particles remains unchanged in this section. The purpose of B and C is to electrically isolate batteries V_1 and V_2.

From C to D the particles lose kinetic energy as the electromotive field $\mathbf{E}_{\text{cd}} = -\mathbf{E}_{\text{ab}}$, and here the battery V_2 is charged; anything else would violate energy conservation laws. This is the symmetrical opposite of the physics that described B–C.

The particles leave the apparatus with its original energy, and the sum of the energies stored in batteries V_1 and V_2 also has not changed, although one has gained and the other has lost energy.

The particle beam can be maintained indefinitely, meaning that the electric currents which discharge and charge the batteries can be maintained indefinitely. We can now reasonably conclude that electric current is not an electron drift as postulated by Drude, because the question: "Where do the infinite electrons to support the electric current originate from?" cannot be answered for the experimental setup.

The thought experiment does not contradict Kirchhoff's current law. My interpretation/explanation is: The electric circuit A–V_1–B is a charged capacitor, the capacitor's energy is the electromotive field energy between A and B. A charged particle interacts with a partial volume of space and discharges its electric field, (the field energy is converted to kinetic energy hence the Lorentz force) which results in a Maxwell displacement current $\mathbf{j} = \partial\mathbf{E}/\partial t$ to restore the electric equilibrium of the field. This displacement current is the same current that discharges the battery. Therefore, electric charge does flow between A and B but requires a charge carrier other than the electron. (a massless '*voltron*' that carries electric energy?)

I have shown by logical thought that the electromotive fields $\mathbf{E}_{\text{ab}}$ and $\mathbf{E}_{\text{cd}}$ cannot be the result of a differential in free electrons on A and B, and on C and D, respectively. Therefore, I conclude that the electromotive field that accelerates atomic nuclei, electrons and ionised particles is a fundamental phenomenon that is not the same as the electrostatic fields that govern atomic matter, and particle–particle interactions.

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