Maxwellian Dynamics and Particles as Maxwellian Solitons

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Ryde, United Kingdom

February 25, 2023

Presentation to the
QM Foundations & Nature of Time Seminar

Poincaré quoted by Einstein*

> …which characterizes Poincaré's standpoint. Geometry (G) predicates nothing about the relations of real things, but only geometry together with the purport (P) of physical laws can do so. Using symbols, we may say that only the sum of (G) + (P) is subject to the control of experience. Thus (G) may be chosen arbitrarily, and also parts of (P); all these laws are conventions. All that is necessary to avoid contradictions is to choose the remainder of (P) so that (G) and the whole of (P) are together in accord with experience.

* Albert Einstein. *Geometrie Und Erfahrung*. Akademie der Wissenschaften, in Kommission Bei W. De Gruyter, 1921.

Slide 3: Models or Nature?

Niels Bohr:

> The purpose of scientific theories “is not to disclose the real essence of phenomena but only to track down, so far as it is possible, relations between the manifold aspects of experience”
>
> “The ingenious formalism of quantum mechanics, which abandons pictorial representation and aims directly at a statistical account of quantum processes ...”
>
> “The formalism thus defies pictorial representation and aims directly at prediction of observations appearing under well-defined conditions”
>
> “The entire formalism is to be considered as a tool for deriving predictions of definite and statistical character ...”

Source: https://plato.stanford.edu/entries/qm-copenhagen/

Slide 4: Models or Nature?

Einstein*

> …[A]n enigma presents itself which in all ages has agitated inquiring minds. How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality? Is human reason, then, without experience, merely by taking thought, able to fathom the properties of real things.
>
> In my [Einstein's] opinion the answer to this question is, briefly, this:- As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.

* Einstein, *Geometrie Und Erfahrung*.

Poincaré

> This state of affairs may be explained in one of two ways: either everything in the universe would be of electromagnetic origin, or this aspect—shared, as it were, by all physical phenomena—would be a mere epiphenomenon, something due to our methods of measurement.

M. H. Poincaré. "*Sur la dynamique de l'électron* (Translation: On the Dynamics of the Electron)". In: Rendiconti del Circolo matematico di Palermo 21.1 (1906), pp. 129–175

https://isidore.co/misc/Physics%20papers%20and%20books/Classic%20Papers/Poincar%C3%A9's%20Re
20paper%20on%20relativity/Partial%20Translation%20of%20Poincar%C3%A9's%20Rendiconti%20Paper
20on%20Relativity.pdf

Slide 6: New type of thinking is essential

THE NEW YORK TIMES, SATURDAY, MAY 25, 1946.

Times

stic detail than hell begins to pop in and continues to pop, in the cus-endo, to the very end. ful hussy who is to be the cause of moked shenanigans that follow ap-scene rather informally. She fishes

ATOMIC EDUCATION URGED BY EINSTEIN

Scientist in Plea for $200,000 to Promote New Type of Essential Thinking

Dr. Albert Einstein, whose formula on the equivalence of mass and energy led to the discovery of the enormous amount of energy locked up within the atom, issued a personal appeal yesterday by telegram to several hundred prominent Americans. He asked contributions to a fund of $200,000 with which to carry on a nation-wide campaign "to let the people know that a new type of thinking is essential" in the atomic age "if mankind is to survive and move toward higher levels."

That was nine month after Hiroshima, but used here out of context that a new type of thinking is required in physics if QM and GR are to be unified. Two weeks ago we heard Tim say "It just shows people are not thinking out of the box for the type of physical theory we may need to explain Bell's theorem."

Slide 7: A Poincaréan Ontology

Lets adopt a Poincaréan ontology

Everything in the universe is described by Maxwellian dynamics and is not influenced by methods of observation. Here, Maxwellian dynamics is the study of interactions as nilpotent electromagnetic superpositions.

and let us see where it takes us.

Slide 8: What is a wave? (The d'Alembert wave equation)

Towne* states that the requirement for a physical condition to be referred to as a wave, is that its mathematical representation give rise to a partial differential equation of particular form, known as the wave equation. The classical form

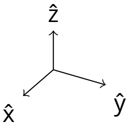
$$\frac{\partial^2 w}{\partial p^2} - \frac{1}{u^2}\frac{\partial^2 w}{\partial t^2} = 0 \qquad \text{or} \qquad \nabla^2 w - \frac{1}{u^2}\frac{\partial^2 w}{\partial t^2} = 0.$$

was proposed in 1748 by d'Alembert for a one-dimensional continuum. A decade later, Euler established the equation for the three-dimensional continuum.

* Dudley H. Towne. *Wave phenomena.* New York: Dover Publications, 1988.

Slide 9: A Cartesian reference system

The reference system whose axis are the unit vectors $\hat{x}, \hat{y}$ and $\hat{z}$



is defined by:

$$\hat{x} \cdot \hat{y} = 0 \qquad \hat{y} \cdot \hat{z} = 0 \qquad \hat{z} \cdot \hat{x} = 0$$

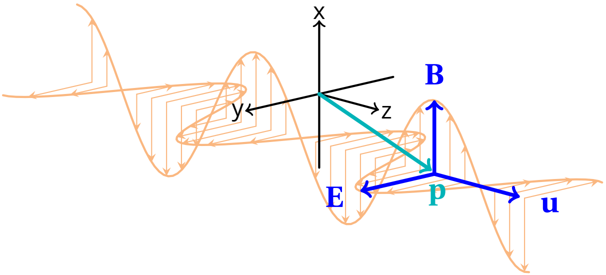
$$\hat{x} \times \hat{y} = \hat{z} \qquad \hat{y} \times \hat{z} = \hat{x} \qquad \hat{z} \times \hat{x} = \hat{y}$$

Consider the indefinite series

$$\mathbf{z}_1 = \mathbf{x}_0 \times \mathbf{y}_0, \;\; \mathbf{x}_1 = \mathbf{y}_0 \times \mathbf{z}_1, \;\; \mathbf{y}_1 = \mathbf{z}_1 \times \mathbf{x}_1, \;\; \mathbf{z}_2 = \mathbf{x}_1 \times \mathbf{y}_1 \;\; \ldots \;\; \mathbf{z}_n = \mathbf{x}_{n-1} \times \mathbf{y}_{n-1} \;\; \ldots \;\; \ldots$$

and what needs to be done so that $\mathbf{x}_0 = \mathbf{x}_n$, $\mathbf{y}_0 = \mathbf{y}_n$, $\mathbf{z}_0 = \mathbf{z}_n$ which gives us a simultaneous vector cross product equation set which has defined solutions.

Slide 10: One Plane of an Electromagnetic Wave



$$\left\{ \mathbf{E} = \mathbf{u} \times \mathbf{B}, \quad \mathbf{u} = \frac{1}{\|\mathbf{B}\|^2} \mathbf{B} \times \mathbf{E}, \quad \mathbf{B} = \frac{1}{\|\mathbf{u}\|^2} \mathbf{E} \times \mathbf{u} \right\} \qquad \mathbf{p} = \int \mathbf{u} \, \mathrm{d}t$$

$$\mathbf{p} = \int \mathbf{u} \, \mathrm{d}t$$

$$\left\{ \mathbf{E} = \mathbf{u} \times \mathbf{B}, \quad \mathbf{u} = \frac{1}{\|\mathbf{B}\|^2} \mathbf{B} \times \mathbf{E}, \quad \mathbf{B} = \frac{1}{\|\mathbf{u}\|^2} \mathbf{E} \times \mathbf{u} \right\}$$

Is a reformulation of the Maxwell equations.

To show this we apply a curl left-right curl operations $\nabla \times \mathbf{E} = \nabla \times (\mathbf{u} \times \mathbf{B})$ and $\nabla \times \mathbf{B} = \nabla \times (\mathbf{E} \times \mathbf{u}) \|\mathbf{u}\|^{-2}$ we need to evaluate the triple vector products, which we expand using general vector analytic methods.

$$\nabla \times (\mathbf{u} \times \mathbf{B}) = \mathbf{u}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{u}) - (\mathbf{u} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{u}$$

$$\nabla \times (\mathbf{E} \times \mathbf{u}) = \mathbf{E}(\nabla \cdot \mathbf{u}) - \mathbf{u}(\nabla \cdot \mathbf{E}) - (\mathbf{E} \cdot \nabla)\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{E}$$

Slide 12: Electromagnetic Bimodal Wave Equation & Maxwell

$\nabla\cdot\mathbf{u}=0$ because c and $\hat{\mathrm{u}}(t)$ are not functions of x, y, and z

$\nabla\cdot\mathbf{B}=0$ because B and $\hat{\mathrm{B}}(t)$ are not functions of x, y, and z

$\nabla\cdot\mathbf{E}=0$ ditto, because $\mathbf{E}=\mathbf{u}\times\mathbf{B}$

$(\mathbf{B}\cdot\nabla)\mathbf{u}=0$ because $\left(B_x\frac{\partial}{\partial x}+B_y\frac{\partial}{\partial y}+B_z\frac{\partial}{\partial z}\right)c\hat{\mathrm{u}}(t)=0$

$(\mathbf{E}\cdot\nabla)\mathbf{u}=0$ ditto

$(\mathbf{u}\cdot\nabla)\mathbf{B}=?$ What is a convective operator of a velocity vector $\mathbf{u}$

$(\mathbf{u}\cdot\nabla)\mathbf{E}=?$

Slide 13: Convective operator

The usual interpretation for $(\mathbf{u}\cdot\nabla)\mathbf{A}$ is:

$$(\mathbf{u}\cdot\nabla)\mathbf{A} = \begin{pmatrix} u_x\dfrac{\partial A_x}{\partial x} + u_y\dfrac{\partial A_x}{\partial y} + u_z\dfrac{\partial A_x}{\partial z} \\ u_x\dfrac{\partial A_y}{\partial x} + u_y\dfrac{\partial A_y}{\partial y} + u_z\dfrac{\partial A_y}{\partial z} \\ u_x\dfrac{\partial A_z}{\partial x} + u_y\dfrac{\partial A_z}{\partial y} + u_z\dfrac{\partial A_z}{\partial z} \end{pmatrix} \begin{pmatrix} \hat{\mathsf{x}} \\ \hat{\mathsf{y}} \\ \hat{\mathsf{z}} \end{pmatrix}$$

$$= \frac{\partial(A_x, A_y, A_z)}{\partial(x, y, z)}\mathbf{u} \qquad \text{which is the Jacobian matrix}$$

which does not help us any further.

Slide 14: Electromagnetic Bimodal Wave Equation & Maxwell

But, $\mathbf{u}\cdot\nabla = \dfrac{\partial}{\partial t}$ because $\mathbf{u}\cdot\nabla = \dfrac{\partial x}{\partial t}\dfrac{\partial}{\partial x} + \dfrac{\partial y}{\partial t}\dfrac{\partial}{\partial y} + \dfrac{\partial z}{\partial t}\dfrac{\partial}{\partial z} = \dfrac{\partial}{\partial t}$

and that leaves us with

$$\nabla\times(\mathbf{u}\times\mathbf{B}) = \cancel{\mathbf{u}(\nabla\cdot\mathbf{B})} - \cancel{\mathbf{B}(\nabla\cdot\mathbf{u})} + \cancel{(\mathbf{B}\cdot\nabla)\mathbf{u}} - (\mathbf{u}\cdot\nabla)\mathbf{B} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times(\mathbf{E}\times\mathbf{u}) = \cancel{\mathbf{E}(\nabla\cdot\mathbf{u})} - \cancel{\mathbf{u}(\nabla\cdot\mathbf{E})} + (\mathbf{u}\cdot\nabla)\mathbf{E} - \cancel{(\mathbf{E}\cdot\nabla)\mathbf{u}} = \frac{\partial\mathbf{E}}{\partial t}$$

Slide 15: Electromagnetic Bimodal Wave Equation & Maxwell

Applying a 'left and right side' curl operation on

$$\mathbf{E} = (\mathbf{u} \times \mathbf{B}) \quad \text{and} \quad \mathbf{B} = \frac{1}{\|\mathbf{u}\|^2} \mathbf{E} \times \mathbf{u} \quad \text{gives}$$

$$\nabla \times \mathbf{E} = \nabla \times (\mathbf{u} \times \mathbf{B}) = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{\|\mathbf{u}\|^2} \nabla \times (\mathbf{E} \times \mathbf{u}) = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

and on slide 12 we established $\nabla \cdot \mathbf{B} = 0$ and $\nabla \cdot \mathbf{E} = 0$.

Therefore we have the Maxwell equations provided we can prove mathematically that $c^2 = 1/\epsilon_0 \mu_0$

Slide 16: EM-Waves; Hierarchical structure of the wave equation

(1) $$\left\{ \mathbf{E} = \mathbf{u} \times \mathbf{B}, \quad \mathbf{u} = \frac{1}{\|\mathbf{B}\|^2} \mathbf{B} \times \mathbf{E}, \quad \mathbf{B} = \frac{1}{\|\mathbf{u}\|^2} \mathbf{E} \times \mathbf{u} \right\}$$

(2) $$\begin{Bmatrix} \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}, & \nabla \cdot \mathbf{E} = 0 \\ \nabla \times \mathbf{B} = \dfrac{1}{c^2} \dfrac{\partial \mathbf{E}}{\partial t}, & \nabla \cdot \mathbf{E} = 0 \end{Bmatrix}$$

(3) $$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \text{and} \quad \nabla^2 \mathbf{B} - \frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

By definition, any solution of (1) is a solution of (3)

But a solution of (3) is not necessarily a solution of (1)

Questions or Comments

Slide 17: Electromagnetic Bimodal Wave Equation & Maxwell

Nomenclature:

Let $\mathcal{W}(\mathbf{p})$ describe a Bimodal Wave and it is a solution of $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$.
A solution of $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$ is parameterised by $\mathscr{P}(\mathbf{u},\mathbf{B},\mathbf{E})$

$$\mathcal{W}(\mathbf{p}) \xrightarrow[\text{of}]{\text{solution}} \mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E}) \xrightarrow{\text{defines}} \left\{ \begin{array}{lll} \mathbf{E}=\mathbf{u}\times\mathbf{B} & (\textit{activation by } \mathbf{B}) & \text{(a)} \\ \mathbf{u}=\dfrac{1}{\|\mathbf{B}\|^2}\mathbf{B}\times\mathbf{E} & (\textit{vectoring by } \mathbf{B}\times\mathbf{E}) & \text{(b)} \\ \mathbf{B}=\dfrac{1}{\|\mathbf{u}\|^2}\mathbf{E}\times\mathbf{u} & (\textit{reactivation by } \mathbf{E}) & \text{(c)} \end{array} \right\}$$

$$\mathcal{W}(\mathbf{p}) \xrightarrow[\text{of}]{\text{solution}} \mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E}) \xrightarrow[\text{by}]{\text{par}} \mathscr{P}(\mathbf{u},\mathbf{B},\mathbf{E})$$

which we simplify to $\mathcal{W}(\mathbf{p}) \xrightarrow[\text{by}]{\text{par}} \mathscr{P}(\mathbf{u},\mathbf{B},\mathbf{E})$ where $\mathbf{p}=\int \mathbf{u}\,\mathrm{d}t$

Nomenclature:

$\mathbf{u}$	is a velocity vector $\mathbf{u} = c\hat{\mathrm{u}}$, where
$\hat{\mathrm{u}}$	that is, $\hat{\mathrm{u}} \mapsto \hat{\mathrm{u}}(t)$, is a unitless unit vector function of time only, and
c	is the speed of light.
$\mathbf{B}$	is the magnetic field $\mathbf{B} = B\hat{\mathrm{B}}$, and where
$\hat{\mathrm{B}}$	that is, $\hat{\mathrm{B}} \mapsto \hat{\mathrm{B}}(t)$, is a unitless unit vector function of time only, and is orthogonal to $\hat{\mathrm{u}}$ hence $\hat{\mathrm{u}} \cdot \hat{\mathrm{B}} = 0$, and
B	scales the magnetic field and provides the physical units.
$\mathbf{E}$	is the electric field and (a) gives $\mathbf{E} = cB(\hat{\mathrm{u}} \times \hat{\mathrm{B}}) = cB\hat{\mathrm{E}}$, with $\hat{\mathrm{E}} = \hat{\mathrm{u}} \times \hat{\mathrm{B}}$.
$\mathbf{p}$	the position of the origin for $\mathbf{u}$, $\mathbf{B}$, and $\mathbf{E}$; thus $\mathbf{p} = \int \mathbf{u}\,\mathrm{d}t$.

To prove that $\mathscr{M}(\mathbf{u}, \mathbf{B}, \mathbf{E})$ is also a reformulation of the Maxwell equations, we must prove $c^2 = 1/\epsilon_0\mu_0$ using $\mathscr{M}(\mathbf{u}, \mathbf{B}, \mathbf{E})$ and the two assertions below.

All waves have action, therefore we assert:

a) An elementary EM-wave $\mathcal{W}$ exhibits power h/t^2, where h is the Planck constant and $t = 1$ second. This requires $\mathbf{B}$ to be an elementary field.

b) An elementary wave transports an electric charge e every one second which is a *wave current*. (This is nothing new; it is another way of describing the displacement current $\partial\mathbf{E}/\partial t$ that Maxwell had identified in varying electric fields.)

Slide 20: Properties of Vacuum

$$\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E}) \xrightarrow{\text{defines}} \left\{ \text{(a)}\ \mathbf{E} = \mathbf{u}\times\mathbf{B}, \quad \text{(b)}\ \mathbf{u} = \frac{1}{\|\mathbf{B}\|^2}\mathbf{B}\times\mathbf{E}, \quad \text{(c)}\ \mathbf{B} = \frac{1}{\|\mathbf{u}\|^2}\mathbf{E}\times\mathbf{u} \right\}$$

From the definitions (Slide-18) we have $\|\mathbf{B}\| = B$ which we substitute into $\mathscr{M}(\mathbf{u},\mathbf{B},\mathbf{E})$(b) and then multiply by it by the quantised action h, because $\mathbf{B}\times\mathbf{E}$ is indicative of the wave action (Poynting vector), obtaining

$$\|h\mathbf{u}\| = \left\|\frac{h}{B^2}\mathbf{B}\times\mathbf{E}\right\| \quad \text{to get} \quad hc = \left(\frac{h}{B^2}\right)\left(|\mathbf{B}||\mathbf{E}|\right) \quad \text{gives} \quad h = \left[\frac{h}{l^4B^2c}\right]l^4\left(|\mathbf{B}||\mathbf{E}|\right)$$

after defining an elementary distance $l = ct$, and multiplying and dividing the above-left by l^4 but that also requires that $\mathbf{B}$ and $\mathbf{E}$ are elementary fields.

Here the square brackets indicate the development of a physical constant, which we want to determine by eliminating B and l by expressing B and l in terms of elementary action h and elementary charge e.

Slide 21: Elementary Electromagnetic Action (Definition)

Let's define the elementary electromagnetic action as:

$h_e = \varrho h$ where $\varrho = 1\ \mathrm{C\,kg^{-1}}$.

Action is momentum (ec) times distance (l), hence

$h_e = \varrho h = \kappa l e c$ where l is the elementary distance, and κ is a dimensionless proportionality constant of unknown value scaling lec to the EM-action h_e. We can also postulate that the electromagnetic action is proportional to the product of **B** and the elementary volume which the wave occupies

$h_e = \varrho h = \chi l^3 |\mathbf{B}|$ where χ is a constant with units and scaling to be determined.

Therefore we have $h_e = \kappa l e c = \chi l^3 |\mathbf{B}|$ which gives

$$|\mathbf{B}| = \frac{\kappa e c}{\chi l^2}$$

Slide 22: Properties of Vacuum

From Slide-42

$$\|h\mathbf{u}\| = \left\|\frac{h}{B^2}\mathbf{B}\times\mathbf{E}\right\| \quad \text{to get} \quad hc = \left(\frac{h}{B^2}\right)\left(|\mathbf{B}||\mathbf{E}|\right) \quad \text{gives} \quad h = \left[\frac{h}{l^4 B^2 c}\right] l^4 \left(|\mathbf{B}||\mathbf{E}|\right)$$

We substitute $|\mathbf{B}| = \dfrac{\kappa e c}{\chi l^2}$ from previous slide into the rightmost of above, to give

$$h = \left[\frac{h}{l^4 B^2 c}\right] l^4 \left(\frac{\kappa e c}{\chi l^2}|\mathbf{E}|\right) \qquad \text{but, } |\mathbf{E}| = cB \text{ which giveslarge}$$

$$h = \left[\frac{h}{l^4 B^2 c}\right]\left[\frac{1}{\chi}\right] \kappa l^2 e c^2 B \qquad \text{we define } B = \frac{h}{\kappa l^2 e}$$

but only if

$$1 = \left[\frac{h}{l^4 B^2 c}\right]\left[\frac{1}{\chi}\right] c^2$$

but only if

$$1 = \left[\frac{h}{l^4 B^2 c}\right]\left[\frac{1}{\chi}\right]c^2 \qquad \text{and replacing } B = \frac{h}{\kappa l^2 e} \text{ gives}$$

$$1 = \left[\frac{\kappa^2 e^2}{hc}\right]\left[\frac{1}{\chi}\right]c^2 \qquad \text{which requires } \frac{1}{\chi} = \frac{h}{\kappa^2 e^2 c}\text{, hence}$$

$$1 = \left[\frac{\kappa^2 e^2}{hc}\right]\left[\frac{h}{\kappa^2 e^2 c}\right]c^2 \quad \text{all that remains is to set } \kappa^2 = \frac{1}{2\alpha}\text{, thus}$$

$$1 = \left[\frac{e^2}{2\alpha hc}\right]\left[\frac{2\alpha h}{e^2 c}\right]c^2 = \epsilon_0 \mu_0 c^2$$

This concludes the proof that the equation set $\mathscr{M}(\mathbf{u}, \mathbf{B}, \mathbf{E})$ is a mathematical reformulation of the Maxwell equations, because now we can replace $1/c^2$ in (2), Slide-16, with $\epsilon_0 \mu_0$ as we have derived it independently.

The fine structure constant α is said to quantify the strength of the electromagnetic interaction between elementary charged particles; the modern view also includes the coupling of the electromagnetic force to the other three forces*; these are the strong, weak and gravitational forces. Repeating the equation for electromagnetic momentum, $\varrho h = \kappa l e c$, here the constant $\kappa = 1/\sqrt{2\alpha}$ is a coupling constant relating the electric charge momentum to mechanical momentum.

* *Current advances: The fine-structure constant and quantum Hall effect*. 2021. URL: `https://physics.nist.gov/cuu/Constants/alpha.html` (visited on 2021-10-30).

Slide 25: Discussion: Oliver Heaviside : What if. . .

Oliver Heaviside* in 1892 presented us with vector algebra,

Planck proposed the quantity h in the year 1900 and

Millikan published in 1913 the electric charge as 1.592×10^{-19} C.

What if Heaviside discovered $\mathscr{M}(\mathbf{u}, \mathbf{B}, \mathbf{E})$?

A Heaviside constant $\kappa = 8.277$ would have been proposed,

The magnetic permeability could have transitioned from the fixed constant $4\pi \times 10^{-7}$ to the expression $h/(\kappa^2 e^2 c)$, or at least that relationship would have been known.

Then in 1916 Sommerfeld would have established

$$\alpha^{-1} = 2\kappa^2.$$

* Oliver Heaviside. *Electromagnetic Theory*. Chelsea Publishing co., New York, 1893.

Nonetheless, $\varrho h = \kappa lec$ now provides a key to determine the values for the elementary length and time. Using the 2018 CODATA values we get:

$\kappa = 8.277\,559\,999\,29(62)$	Heaviside constant	
$l_0 = 1.666\,566\,299\,11(12) \times 10^{-24}$	metres	
$t_0 = 5.559\,066\,796\,49(42) \times 10^{-33}$	seconds	using $l = ct$.

The magnetic flux quantum is defined as

$$\phi_0 = h/(2e)$$

it was observed experimentally in 1961 by Deaver* in hollow superconducting cylinders, and Shankar† shows how to derive it by analysing the Aharonov–Bohm effect.

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* Bascom S. Deaver and William M. Fairbank. "Experimental Evidence for Quantized Flux in Superconducting Cylinders". In: *Physical Review Letters* 7.2 (1961-07), pp. 43–46. URL: `https://link.aps.org/doi/10.1103/PhysRevLett.7.43` (visited on 2022-06-15).

† R. Shankar. *Principles of Quantum Mechanics, 2nd edition.* SPlenum Press, 1994. 676 pp.

Slide 28: Discussion: Magnetic field quanta?

But we found the magnetic field of an elementary EM-wave $\mathcal{W}$ as $B = h/(\kappa l^2 e)$ (Slide 22) which implies a magnetic flux for the elementary EM-wave

$$\tilde{\phi} = h/(\kappa e) = \sqrt{2\alpha}\, h/e$$

which is clearly smaller than the magnetic flux quantum established by measurement and quantum theory. I offer no opinion whether the irrational $\tilde{\phi} = \sqrt{2\alpha}\, h/e$ is a quantum or not, other than to comment that when scaled this way we have

$$\mathbf{S} = \mu_0^{-1} \|\mathbf{B} \times \mathbf{E}\| = h\frac{c^2}{l^4} \qquad \text{energy per (area} \times \text{time)}$$

which confirms the first of the assertions on Slide-19.

‘*Who was first? The chicken or the egg*’

The values for the permittivity ϵ_0 and permeability μ_0 were derived using the velocity c defined previously in $\mathscr{M}(\mathbf{u}, \mathbf{B}, \mathbf{E})$, and by the two assertions which introduced the unit action h and the elementary charge e. Therefore:

The relation

$$c = 1/\sqrt{\epsilon_0 \mu_0}$$

does not define the speed of light from first principles!

Proposition: The vacuum has additional characteristics which defines the *transportivity* $\mathcal{T} = c^2$. As an analog to fluids, the transportivity could be a ratio of two elementary quantities, yet undiscovered, which are not functions of the speed of light.

For example, the speed of a sound wave in a material is dependent on the material properties. In fluids $c^2 = K_s/\rho$ where K_s is a coefficient of stiffness and ρ the fluid's density. Alternatively, we can also express it as $c^2 = \partial P/\partial \rho$ where P is pressure. But do take note of the fact that none of K_s, ρ and P are defined in terms of the speed of sound within that medium.

Questions or Comments

Slide 31: Travelling plane waves

Every authoritative book, for example Jackson*, describes an EM-field of a circular polarised travelling plane wave as

$$\mathbf{B}(\mathbf{z},t) = B_0(\hat{\mathrm{x}}\cos(k\mathbf{n}\cdot\mathbf{z}-\omega t)+\hat{\mathrm{y}}\sin(k\mathbf{n}\cdot\mathbf{z}-\omega t))$$

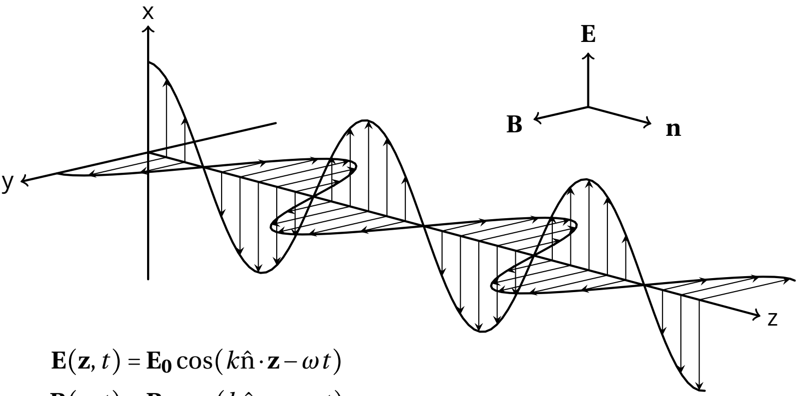
with the phase velocity defined by the wave vector $k\mathbf{n}$. But the norm of the wave vector $\|k\mathbf{n}\| = \omega z/c$ therefore

$$\mathbf{B}(\mathbf{z},t) = B_0\left[\hat{\mathrm{x}}\cos\left(\frac{\omega(z+z_0)}{c}-\omega t\right)+\hat{\mathrm{y}}\sin\left(\frac{\omega(z+z_0)}{c}-\omega t\right)\right]\Bigg|_{\mathbf{z}=\hat{\mathrm{z}}(ct+z_0)}$$

$$= B_0[\hat{\mathrm{x}}\cos\theta_0+\hat{\mathrm{y}}\sin\theta_0]\Big|_{\mathbf{z}=\hat{\mathrm{z}}(ct+z_0)}$$

* John David Jackson. *Classical Electrodynamics, 2nd. Ed.* John Wiley & Sons Inc, 1975.

Slide 32: Classical interpretation of an Electromagnetic Plain Wave



$$\mathbf{E}(\mathbf{z},t) = \mathbf{E_0}\cos(k\hat{\mathrm{n}}\cdot\mathbf{z}-\omega t)$$

$$\mathbf{B}(\mathbf{z},t) = \mathbf{B_0}\cos(k\hat{\mathrm{n}}\cdot\mathbf{z}-\omega t)$$

$$\mathbf{E}_0\cdot\hat{\mathrm{n}} = 0,\ \ \mathbf{B}_0\cdot\hat{\mathrm{n}} = 0,\ \ \mathbf{B}_0 = k\hat{\mathrm{n}}\times\mathbf{E}_0,\ \ k = \frac{\omega}{c},\ \ \hat{\mathrm{n}} = \hat{\mathrm{z}}$$

Slide 33: Example: Solutions for $\left\{\mathbf{E} = \mathbf{u} \times \mathbf{B}, \quad \mathbf{u} = \|\mathbf{B}\|^{-2}(\mathbf{B} \times \mathbf{E}), \quad \mathbf{B} = \|\mathbf{u}\|^{-2}(\mathbf{E} \times \mathbf{u})\right\}$

1D: Linear propagation path along the z-axis

$$\gamma \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} \hat{\mathrm{u}}_\gamma \\ \hat{\mathrm{B}}_\gamma \\ \hat{\mathrm{E}}_\gamma \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ \cos\omega t & \sin\omega t & 0 \\ -\sin\omega t & \cos\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{pmatrix}$$

2D: Circular propagation path in the xy-plane centred at the origin

$$\odot \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} \hat{\mathrm{u}}_\odot \\ \hat{\mathrm{B}}_\odot \\ \hat{\mathrm{E}}_\odot \end{pmatrix} = \begin{pmatrix} \sin\omega t & -\cos\omega t & 0 \\ \cos\omega t & \sin\omega t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{pmatrix}$$

3D: Closed curved, or wound up, path in xyz-space centred at the origin.

$$\Theta \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} \hat{\mathrm{u}}_\Theta \\ \hat{\mathrm{B}}_\Theta \\ \hat{\mathrm{E}}_\Theta \end{pmatrix} = \begin{pmatrix} \sin\omega t \sin \grave{n}\omega t & -\sin\omega t \cos \grave{n}\omega t & -\cos\omega t \\ \cos\omega t & \sin\omega t & 0 \\ \cos\omega t \sin \grave{n}\omega t & -\cos\omega t \cos \grave{n}\omega t & \sin\omega t \end{pmatrix} \begin{pmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{pmatrix}$$

Slide 34: Travelling plane waves

We define a wave's velocity vector $\mathbf{u} = \hat{z}c$ which gives the position vector $\mathbf{p}_i$ for the i$^{\text{th}}$ travelling plane

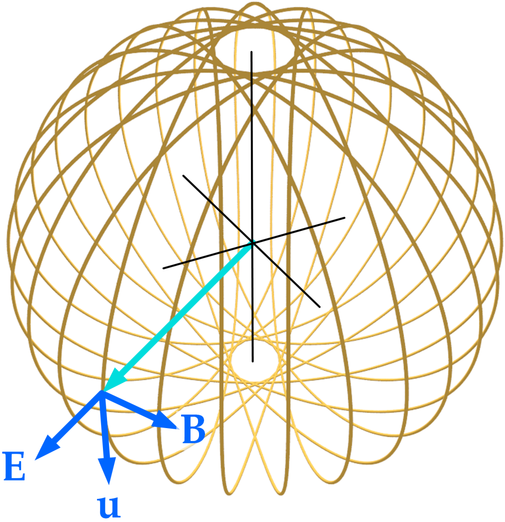
$$\mathbf{p}_i = \int \mathbf{u}\,\mathrm{d}t = \hat{z}\int c\,\mathrm{d}t = \hat{z}(z_i + ct)$$

and use it to describe a circular polarised travelling plane EM-wave $\mathcal{W}$

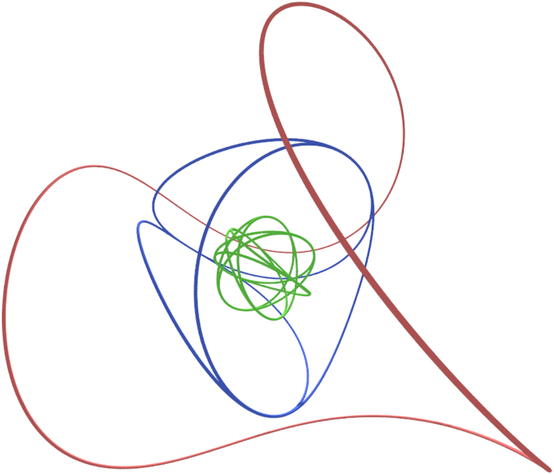
$$\mathcal{W}(\mathbf{p}_i) \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix}\mathbf{u}\\ \mathbf{B}\\ \mathbf{E}\end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ \cos\left(\omega\frac{\|\mathbf{p}_i\|}{c} - \omega t\right) & \sin\left(\omega\frac{\|\mathbf{p}_i\|}{c} - \omega t\right) & 0 \\ -\sin\left(\omega\frac{\|\mathbf{p}_i\|}{c} - \omega t\right) & \cos\left(\omega\frac{\|\mathbf{p}_i\|}{c} - \omega t\right) & 0 \end{pmatrix} \begin{pmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ \cos(\omega z_i/c) & \sin(\omega z_i/c) & 0 \\ -\sin(\omega z_i/c) & \cos(\omega z_i/c) & 0 \end{pmatrix} \begin{pmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{pmatrix}$$

Slide 35: 3D waves could describe ball lightning



Slide 36: 3D paths, each path lengths equals 2π



For a travelling object $\boldsymbol{o}$ we define a unit velocity vector as

$$\hat{\mathrm{u}}_o(t) = \hat{\mathrm{x}} \sin \omega_1 t \sin \omega_2 t + \hat{\mathrm{y}} \sin \omega_1 t \cos \omega_2 t + \hat{\mathrm{z}} \cos \omega_1 t$$

The path $\mathbf{s}_o$ that the object $\boldsymbol{o}$ follows is found by integration

$$\begin{aligned}
\mathbf{s}_o(t) &= \int c\hat{\mathrm{u}}_o \,\mathrm{d}t \\
&= \hat{\mathrm{x}} c \left(\frac{\sin(\omega_2 + \omega_1) t}{2(\omega_2 + \omega_1)} - \frac{\sin(\omega_1 - \omega_2) t}{2(\omega_1 - \omega_2)} \right) \\
&\qquad + \hat{\mathrm{y}} c \left(\frac{\cos(\omega_2 + \omega_1) t}{2(\omega_2 + \omega_1)} + \frac{\cos(\omega_1 - \omega_2) t}{2(\omega_1 - \omega_2)} \right) - \hat{\mathrm{z}} c \frac{\sin \omega_1 t}{\omega_1}
\end{aligned}$$

The path is closed, or repeats, in periods of $t = 2\pi$ and as $\| c\hat{\mathrm{u}}_o(t) \| = c$ the pathlength of $\mathbf{s}_o$ is $2\pi c$.

Slide 38: Field, Potential and Flux; Navigating the SI jungle

quantity	EM-units	Name of quantity	Unit name	SI
$\Phi = \dfrac{h}{\kappa e} = \mathbf{A}$	$\dfrac{\mathrm{Js}}{\mathrm{C}}$	*Magnetic* flux quantum	weber	$\dfrac{\mathrm{J}}{\mathrm{A}}$
$\dfrac{\Phi}{l} = \mathbf{L}$	$\dfrac{\mathrm{Js}}{\mathrm{C\,m}}$	*Magnetic* potential		$\dfrac{\mathrm{N}}{\mathrm{A}}$
$\dfrac{\Phi}{l^2} = \mathbf{B}$	$\dfrac{\mathrm{Js}}{\mathrm{C\,m^2}}$	*Magnetic* potential field	tesla	$\dfrac{\mathrm{N}}{\mathrm{A\,m}}$
$\Phi\, c = \mathbf{R}$	$\dfrac{\mathrm{Js\,m}}{\mathrm{C\,s}}$	Electric flux quantum	volt meter	$\dfrac{\mathrm{W\,m}}{\mathrm{A}}$
$\dfrac{\Phi\, c}{l} = \mathbf{V}$	$\dfrac{\mathrm{Js}}{\mathrm{C\,s}}$	Electric potential	volt	$\dfrac{\mathrm{W}}{\mathrm{A}}$
$\dfrac{\Phi\, c}{l^2} = \mathbf{E}$	$\dfrac{\mathrm{Js}}{\mathrm{C\,m\,s}}$	Electric potential field	volt per meter	$\dfrac{\mathrm{W}}{\mathrm{A\,m}}$

Slide 39: The Rotary Wave (think propeller)

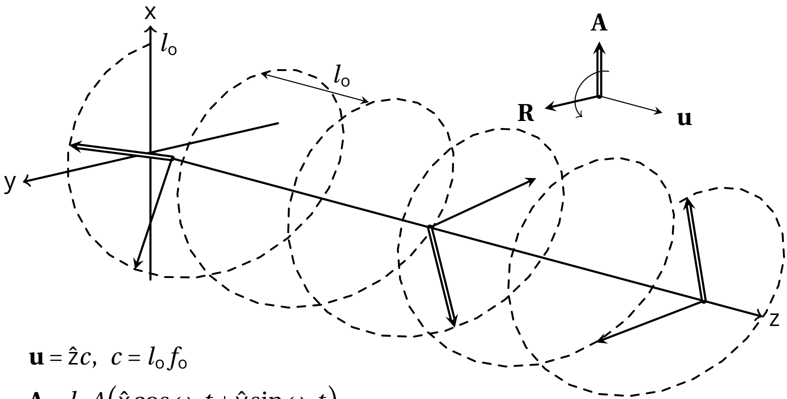
$$\left\{ \mathbf{R} = \mathbf{u} \times \mathbf{A}, \quad \mathbf{u} = \frac{1}{\|\mathbf{A}\|^2} \mathbf{A} \times \mathbf{R}, \quad \mathbf{A} = \frac{1}{\|\mathbf{u}\|^2} \mathbf{R} \times \mathbf{u} \right\}$$

where **A** and **R** are the magnetic and electric flux.

A solution is the quantised rotary wave γ

$$\gamma \xrightarrow[\text{by}]{\text{par}} \begin{cases} \mathbf{u} = \hat{z} c \\ \mathbf{A} = l_\text{o} A\left(\hat{x} \cos \grave{n} \omega_\text{o} t + \hat{y} \sin \grave{n} \omega_\text{o} t\right) \\ \mathbf{R} = c l_\text{o} A\left(-\hat{x} \sin \grave{n} \omega_\text{o} t + \hat{y} \cos \grave{n} \omega_\text{o} t\right) \end{cases}$$

Slide 40: The Rotary Wave (think propeller)



$$\mathbf{u} = \hat{z}c, \quad c = l_o f_o$$

$$\mathbf{A} = l_o A\left(\hat{x}\cos\omega_o t + \hat{y}\sin\omega_o t\right)$$

$$\mathbf{R} = c\,l_o A\left(-\hat{x}\sin\omega_o t + \hat{y}\cos\omega_o t\right)$$

$$\mathbf{R} = (\mathbf{u}\times\mathbf{A}) \quad \text{and} \quad \mathbf{u} = \frac{1}{A^2}(\mathbf{A}\times\mathbf{R}) \quad \text{and} \quad \mathbf{A} = \frac{1}{c^2}(\mathbf{R}\times\mathbf{u})$$

Slide 41: Properties of Vacuum (Two Assertions)

Referencing Slide=19 Assertions used to describe an EM-wave

a) An elementary EM-wave $\mathcal{W}$ exhibits power h/t^2

b) This elementary wave transports an electric charge e.

Assertions used to describe a Rotary wave

a) An elementary rotary wave $\mathcal{R}$ has action h. This requires $\mathbf{A}$ to be an elementary activation-flux vector.

b) This elementary rotary wave transports an elementary load ℓ.

We need to assign some units to the elementary load. I propose a new unit L, the leyden, honouring the Leyden jar.

(Hinting that the electron is not the carrier of electric charge that drives our industry.)

Slide 42: Properties of Vacuum

$$sM(\mathbf{u},\mathbf{A},\mathbf{R}) \xrightarrow{\text{defines}} \left\{ \text{(a)}\ \mathbf{R}=\mathbf{u}\times\mathbf{A}, \quad \text{(b)}\ \mathbf{u}=\frac{1}{\|\mathbf{A}\|^2}\mathbf{A}\times\mathbf{R}, \quad \text{(c)}\ \mathbf{A}=\frac{1}{\|\mathbf{u}\|^2}\mathbf{R}\times\mathbf{u} \right\}$$

From the definitions we have $\|\mathbf{A}\| = l_\text{o} A$ which we substitute into $\mathcal{M}(\mathbf{u},\mathbf{A},\mathbf{R})$(b) and then multiply by it by the quantised action h, because $\mathbf{A}\times\mathbf{R}$ is indicative of the wave action (Poynting vector), obtaining

$$\|h\mathbf{u}\| = \left\| \frac{h}{l_\text{o}^2 A^2}\mathbf{A}\times\mathbf{R} \right\| \quad \text{to get} \quad hc = \left(\frac{h}{l_\text{o}^2 A^2}\right)\left(|\mathbf{A}||\mathbf{R}|\right) \quad \text{gives} \quad h = \left[\frac{h}{l_\text{o}^2 A^2 c}\right]\left(|\mathbf{A}||\mathbf{R}|\right)$$

Earlier, when workinf with **B** and **E** we established the expression for permittivity and permeability ϵ_0 and μ_0, respectively. Here the aim is to establish the same for the fluxes, namely $\d{\epsilon}$ and $\d{\mu}$

Rotary-action is rotary momentum times the angle θ subtended, that is $\mathcal{S}_{\text{rot}} = I\omega\theta$. Hence we can formulate the quantised rotational action as

$$h_{\text{rot}} = \varrho h = \Bbbk \ell\, l_{\text{o}}^{2} \omega_{\text{o}} \theta_{\text{o}}$$

where $\Bbbk$ is a dimensionless proportionality constant, scaling $\ell\, l_{\text{o}}^{2} \omega_{\text{o}} \theta_{\text{o}}$ to the rotational-action h_{rot} and here $\varrho = 1\ \text{Lkg}^{-1}$ (leyden per kilogram) Also, in a quantised system $\theta_{\text{o}} = 2\pi$ radian. (Referenced paper needs to be corrected)

Because $l_\mathrm{o} = c\,t_\mathrm{o} = c/f_\mathrm{o}$ to obtain $\omega_\mathrm{o} = 2\pi f_\mathrm{o} = 2\pi c/l_\mathrm{o}$ hence h_rot is also expressed as:

$$h_\mathrm{rot} = \varrho h = 2\pi \hbar\!\!\!/ \ell\, l_\mathrm{o} c \theta_\mathrm{o}$$

Because the load is carried by $\mathbf{A}$ which has a magnitude $\|\mathbf{A}\| = l_\mathrm{o} A$, therefore we can also postulate the elementary rotary-action

$$h_\mathrm{rot} = \chi\, l_\mathrm{o} A \theta_\mathrm{o}$$

where χ is part of a constant to be determined. Also note that A is a quantised quantity.

Slide 45: Property of Vacuum

$$h_{\text{rot}} = \varrho h = \quad 2\pi \Bbbk \ell\, l_\text{o} c\theta_\text{o} = \chi\, l_\text{o} A\theta_\text{o}$$

thus we get $A = \dfrac{2\pi \Bbbk \ell c}{\chi}$ hence $\|\mathbf{A}\| = \dfrac{2\pi \Bbbk l_\text{o} \ell c}{\chi}$ and using above in

$$h = \left[\frac{h}{l_\text{o}^2 A^2 c}\right]\left(\|\mathbf{A}\|\,\|\mathbf{R}\|\right) \qquad \text{gives} \qquad h = \left[\frac{h}{l_\text{o}^2 A^2 c}\right]\left(\frac{2\pi \Bbbk l_\text{o} \ell c}{\chi}\|\mathbf{R}\|\right)$$

but $\|\mathbf{R}\| = c l_\text{o} A$ which gives

$$h = \left[\frac{h}{l_\text{o}^2 A^2 c}\right]\left[\frac{1}{\chi}\right] 2\pi \Bbbk \ell\, c^2 A$$

We are now in the position to define the quantised activator as

$$A = \frac{h}{2\pi \Bbbk \ell} \qquad \text{but only if}$$

but only if

$$1 = \left[\frac{h}{l_\circ^2 A^2 c}\right]\left[\frac{1}{\chi}\right]c^2$$ using $A = \dfrac{h}{2\pi \hbar \ell}$ to replace A we get

$$1 = \left[\frac{4\pi^2 \hbar^2 \ell^2}{l_\circ^2 hc}\right]\left[\frac{1}{\chi}\right]c^2$$ which requires $\chi = \dfrac{4\pi^2 \hbar^2 \ell^2 c}{l_\circ^2 h}$, hence

$$1 = \left[\frac{4\pi^2 \hbar^2 \ell^2}{l_\circ^2 hc}\right]\left[\frac{l_\circ^2 h}{4\pi^2 \hbar^2 \ell^2 c}\right]c^2 = \epsilon\mu c^2$$ from which we get

$$\epsilon = \frac{4\pi^2 \hbar^2 \ell^2}{l_\circ^2 hc} \quad \text{and} \quad \mu = \frac{l_\circ^2 h}{4\pi^2 \hbar^2 \ell^2 c}$$

Slide 47: Example: Solutions for $\left\{\mathbf{R}=\mathbf{u}\times\mathbf{A},\quad \mathbf{u}=\|\mathbf{A}\|^{-2}(\mathbf{A}\times\mathbf{R}),\quad \mathbf{A}=\|\mathbf{u}\|^{-2}(\mathbf{R}\times\mathbf{u})\right\}$

$$\gamma \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix}\hat{\mathrm{u}}_\gamma\\ \mathbf{A}_\gamma\\ \mathbf{R}_\gamma\end{pmatrix} = \begin{Bmatrix}\check{p}c\\ r l_o \check{q} A\\ \check{p}c r l_o \check{q} A\end{Bmatrix} \begin{pmatrix}0 & 0 & 1\\ \cos\check{n}\check{r}\omega_o t & \sin\check{n}\check{r}\omega_o t & 0\\ -\sin\check{n}\check{r}\omega_o t & \cos\check{n}\check{r}\omega_o t & 0\end{pmatrix} \begin{pmatrix}\hat{\mathrm{x}}\\ \hat{\mathrm{y}}\\ \hat{\mathrm{z}}\end{pmatrix}$$

$$\varphi \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix}\hat{\mathrm{u}}_\varphi\\ \mathbf{A}_\varphi\\ \mathbf{R}_\varphi\end{pmatrix} = \begin{Bmatrix}\check{p}c\\ r l_o \check{q} A\\ \check{p}c r l_o \check{q} A\end{Bmatrix} \begin{pmatrix}\sin\check{a}\omega_o t\sin\check{m}\omega_o t & -\sin\check{a}\omega_o t\cos\check{m}\omega_o t & -\cos\check{a}\omega_o t\\ \cos\check{m}\omega_o t & \sin\check{m}\omega_o t & 0\\ \cos\check{a}\omega_o t\sin\check{m}\omega_o t & -\cos\check{a}\omega_o t\cos\check{m}\omega_o t & \sin\check{a}\omega_o t\end{pmatrix} \begin{pmatrix}\hat{\mathrm{x}}\\ \hat{\mathrm{y}}\\ \hat{\mathrm{z}}\end{pmatrix}$$

$\check{m}$	$=\check{n}\check{r}/\check{w}$		$\check{a}$	$=\check{x}\check{n}\check{u}$	
$\check{n}$	$\in\mathbb{N}>0$	$E=h\check{n}f_o$	$\check{x}$	$\in\mathbb{N}$	Co-prime of $\check{n}$
$\check{r}$	±1	Rotation x–y	$\check{u}$	±1	Rotation [xy]–z
$\check{w}$	$\in\mathbb{N}>0$	path closure			
$\check{p}$	±1	Propagation direction	$\check{s}$	$=\check{r}\check{p}/\check{w}$	Spin
$\check{q}$	±1	Charge polarity	r	$\in\mathbb{R}$	linear scaling

Slide 48: Photon's action: $\left\{\mathbf{R}=\mathbf{u}\times\mathbf{A},\quad \mathbf{u}=\|\mathbf{A}\|^{-2}(\mathbf{A}\times\mathbf{R}),\quad \mathbf{A}=\|\mathbf{u}\|^{-2}(\mathbf{R}\times\mathbf{u})\right\}$

A Photon is parameterised by

$$\gamma \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} \hat{\mathrm{u}}_\gamma \\ \mathbf{A}_\gamma \\ \mathbf{R}_\gamma \end{pmatrix} = \begin{Bmatrix} \check{\mathrm{p}}c \\ r\,l_\mathrm{o}\check{\mathrm{q}}A \\ \check{\mathrm{p}}c\,r\,l_\mathrm{o}\check{\mathrm{q}}A \end{Bmatrix} \begin{pmatrix} 0 & 0 & 1 \\ \cos \text{ňř}\omega_\mathrm{o}t & \sin \text{ňř}\omega_\mathrm{o}t & 0 \\ -\sin \text{ňř}\omega_\mathrm{o}t & \cos \text{ňř}\omega_\mathrm{o}t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{pmatrix}$$

on Slide-43 a quantised rotational action was defined as

$$h_\mathrm{rot} = \mathscr{k}\,\ell\, l_\mathrm{o}^2\,\omega_\mathrm{o}\,\theta_\mathrm{o}$$

but the photon γ the activation vector subtends an angle ňř2π in 1 second, therefore $h_{\mathrm{rot}\,\gamma} = \text{ňř}\,h_\mathrm{rot}$ which requires that the definition of the action vector $\mathcal{S}_\gamma$ (Poynting vector equivalent) to be defined accordingly:

$$\boldsymbol{\mathcal{S}}_\gamma = \dot{\epsilon}\,\text{ňř}(\mathbf{A}\times\mathbf{R}) = \dot{\epsilon}\,\text{ňř}(\mathbf{A}\times(\mathbf{u}\times\mathbf{A}))$$

$$\boldsymbol{S}_\gamma = \epsilon\,\check{\mathsf{n}}\check{\mathsf{r}}\left(\mathbf{A}\times(\mathbf{u}\times\mathbf{A})\right)$$

which we evaluate

$$\|\boldsymbol{S}_\gamma\| = \epsilon\,|\check{\mathsf{n}}\check{\mathsf{r}}|\,\|\mathbf{u}\|\,\|\mathbf{A}\|^2$$

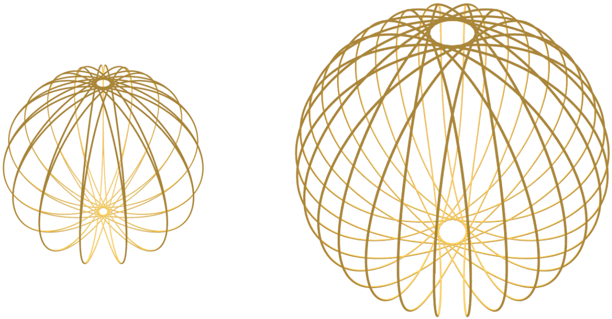
$$= |\check{\mathsf{n}}\check{\mathsf{r}}|\frac{4\pi^2 k^2 \ell^2}{l_\circ^2 hc}(c)(r\,l_\circ\check{\mathsf{q}}A)^2 \qquad \text{with } A = \frac{h}{2\pi k \ell}$$

$$= \check{\mathsf{n}}\check{\mathsf{r}}\check{\mathsf{q}}^2 r^2 l_\circ^2 h$$

and with $\check{\mathsf{r}}=\pm1$, $\check{\mathsf{q}}=\pm1$, $l_\circ=1$ and we set $r=1$ we get

$$\|\boldsymbol{S}_\gamma\| = \check{\mathsf{n}}\,h \qquad \text{therefore:} \quad E = hf$$

Slide 50: Questions or Comments



$$\varphi \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} \hat{u}_\varphi \\ \mathbf{A}_\varphi \\ \mathbf{R}_\varphi \end{pmatrix} = \left\{ \begin{matrix} \check{p}c \\ r\,l_\mathrm{o}\check{q}A \\ \check{p}c\,r\,l_\mathrm{o}\check{q}A \end{matrix} \right\} \begin{pmatrix} \sin\check{a}\omega_\mathrm{o}t\sin\check{m}\omega_\mathrm{o}t & -\sin\check{a}\omega_\mathrm{o}t\cos\check{m}\omega_\mathrm{o}t & -\cos\check{a}\omega_\mathrm{o}t \\ \cos\check{m}\omega_\mathrm{o}t & \sin\check{m}\omega_\mathrm{o}t & 0 \\ \cos\check{a}\omega_\mathrm{o}t\sin\check{m}\omega_\mathrm{o}t & -\cos\check{a}\omega_\mathrm{o}t\cos\check{m}\omega_\mathrm{o}t & \sin\check{a}\omega_\mathrm{o}t \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}$$

Left path is for $\check{m} = 1$ and $\check{a} = 17$ and for the right $\check{m} = 1/2$ and $\check{a} = 17$

Questions or Comments

Slide 51: 3D-Soliton : $\left\{\mathbf{R}=\mathbf{u}\times\mathbf{A},\quad \mathbf{u}=\|\mathbf{A}\|^{-2}(\mathbf{A}\times\mathbf{R}),\quad \mathbf{A}=\|\mathbf{u}\|^{-2}(\mathbf{R}\times\mathbf{u})\right\}$

$$\varphi \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} \hat{\mathrm{u}}_\varphi \\ \mathbf{A}_\varphi \\ \mathbf{R}_\varphi \end{pmatrix} = \begin{Bmatrix} \check{\mathrm{p}}c \\ r\,l_\mathrm{o}\check{\mathrm{q}}A \\ \check{\mathrm{p}}c\,r\,l_\mathrm{o}\check{\mathrm{q}}A \end{Bmatrix} \begin{pmatrix} \sin\check{\mathrm{a}}\omega_\mathrm{o}t\sin\check{\mathrm{m}}\omega_\mathrm{o}t & -\sin\check{\mathrm{a}}\omega_\mathrm{o}t\cos\check{\mathrm{m}}\omega_\mathrm{o}t & -\cos\check{\mathrm{a}}\omega_\mathrm{o}t \\ \cos\check{\mathrm{m}}\omega_\mathrm{o}t & \sin\check{\mathrm{m}}\omega_\mathrm{o}t & 0 \\ \cos\check{\mathrm{a}}\omega_\mathrm{o}t\sin\check{\mathrm{m}}\omega_\mathrm{o}t & -\cos\check{\mathrm{a}}\omega_\mathrm{o}t\cos\check{\mathrm{m}}\omega_\mathrm{o}t & \sin\check{\mathrm{a}}\omega_\mathrm{o}t \end{pmatrix} \begin{pmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{pmatrix}$$

The path vector p_φ is obtained by integrating $\hat{\mathrm{u}}_\varphi$

$$\begin{aligned} \mathbf{p}_\varphi &= c\int (\hat{\mathrm{x}}\sin\omega_1 t\sin\omega_2 t - \hat{\mathrm{y}}\sin\omega_1 t\cos\omega_2 t - \hat{\mathrm{z}}\cos\omega_1 t)\,\mathrm{d}t \\ &= \hat{\mathrm{x}}c\left(\frac{\sin(\omega_1-\omega_2)t}{2(\omega_1-\omega_2)} - \frac{\sin(\omega_1+\omega_2)t}{2(\omega_1+\omega_2)}\right) \\ &\quad -\hat{\mathrm{y}}c\left(\frac{\cos(\omega_1-\omega_2)t}{2(\omega_1-\omega_2)} + \frac{\cos(\omega_2+\omega_1)t}{2(\omega_2+\omega_1)}\right) - \hat{\mathrm{z}}c\frac{\sin\omega_1 t}{\omega_1} \end{aligned}$$

where $\omega_1 = \check{\mathrm{a}}\omega_\mathrm{o}$ and $\omega_2 = \check{\mathrm{m}}\omega_\mathrm{o}$ giving $\omega_1 = \check{\mathrm{x}}\check{\mathrm{m}}\omega_\mathrm{o}$
and $\check{\mathrm{x}}$ is a co-prime $\check{\mathrm{n}}$

The path's radial distance from the origin evaluates to:

$$r_\varphi = \|\mathbf{p}_\varphi\| = c\sqrt{\frac{\omega_1^4-\omega_2^2(\omega_1^2-\omega_2^2)\sin^2\omega_1 t}{\omega_1^4(\omega_1^2-\omega_2^2)}} \quad > \frac{c}{\omega_1} \quad \text{if} \quad \omega_1>\omega_2$$

and remebering $\omega_1 = \check{\mathsf{x}}\check{\mathsf{n}}\omega_2$

$$r_\varphi \gtrapprox \frac{\check{\mathsf{w}}}{\check{\mathsf{x}}\check{\mathsf{n}}}\frac{c}{\omega_\circ}$$

The activation-vector $\mathbf{A}_\varphi$ has the form as that of the photon analysed earlier, therefore from Slide-49 we get

$$\|\boldsymbol{S}_\varphi\| = \underset{\cdot}{\epsilon}|\check{\mathsf{m}}\check{\mathsf{r}}|\|\mathbf{u}\|\|\mathbf{A}\|^2 = \check{\mathsf{m}}\check{\mathsf{r}}\check{\mathsf{q}}^2 r^2 l_\circ^2 h = \check{\mathsf{m}} r^2 h$$

as $\check{\mathsf{r}}=1$, $\check{\mathsf{q}}^2=1$ and $l_\circ^2=1$

Slide 53: Energy of a 3D-Soliton

$$r_\varphi \gtrapprox \frac{\check{\mathsf{w}}}{\check{\mathsf{x}}\check{\mathsf{n}}}\frac{c}{\omega_\circ} = \frac{1}{\check{\mathsf{x}}\check{\mathsf{m}}}\frac{c}{\omega_\circ} \qquad \|\boldsymbol{S}_\varphi\| = \check{\mathsf{m}}\, r^2 h$$

let **A** not extend over the geometric centre of s_φ

$$r\, l_\circ \leqq r_\varphi \qquad \text{and using } c = \frac{l_\circ}{t_\circ} \text{ and } \omega_\circ = \frac{2\pi}{t_\circ}$$

$$\text{let } r\, l_\circ = \frac{l_\circ}{t_\circ}\frac{1}{\check{\mathsf{x}}\check{\mathsf{m}} 2\pi / t_\circ} \qquad \text{gives} \qquad r\check{\mathsf{m}} = \frac{1}{2\pi\check{\mathsf{x}}}$$

$$\therefore\ \|\boldsymbol{S}_\varphi\| = \frac{r}{2\pi\check{\mathsf{x}}} h = \hbar\frac{r}{\check{\mathsf{x}}}$$

or the energy $\mathcal{E}_\varphi = \hbar\dfrac{r}{\check{\mathsf{x}}\, t_\circ}$

Slide 54: Energy of a 3D-Soliton

We have $r_\varphi \gtrapprox \frac{\check{\mathsf{w}}}{\check{\mathsf{x}}\check{\mathsf{n}}}\frac{c}{\omega_\circ}$ that is r_φ scales inversely proportional to $\check{\mathsf{m}}$ and $\check{\mathsf{x}}$.

As action $\mathcal{S}_\varphi = \hbar\frac{r}{\check{\mathsf{x}}}$ means doubling action requires doubling r that is halving $\check{\mathsf{m}}\check{\mathsf{x}}^2$

Energy scales probationally to r or inversely to $\check{\mathsf{m}}\check{\mathsf{x}}^2$.

Slide 55: Complex space

Instead of xyz-space = $\mathbb{R}^3$ we consider xyz-space = $\mathbb{C}^3$ and with a complex *load* we note the light speed could also be complex.

$$\ell \mapsto \begin{cases} \ell\,\mathrm{e}^{\mathrm{i}\alpha} \text{ thus } A \mapsto A\mathrm{e}^{-\mathrm{i}\alpha} \text{ and } R \mapsto \begin{cases} R\mathrm{e}^{\mathrm{i}\alpha}, & \text{if } c \mapsto c\mathrm{e}^{\mathrm{i}2\alpha} \\ R\mathrm{e}^{-\mathrm{i}3\alpha}, & \text{if } c \mapsto c\mathrm{e}^{-\mathrm{i}2\alpha} \\ R\mathrm{e}^{-\mathrm{i}\alpha}, & \text{if } c \mapsto c \end{cases} \\ \qquad\qquad \text{or} \\ \ell\,\mathrm{e}^{-\mathrm{i}\alpha} \text{ thus } A \mapsto A\mathrm{e}^{\mathrm{i}\alpha} \text{ and } R \mapsto \begin{cases} R\mathrm{e}^{\mathrm{i}3\alpha}, & \text{if } c \mapsto c\mathrm{e}^{\mathrm{i}2\alpha} \\ R\mathrm{e}^{-\mathrm{i}\alpha}, & \text{if } c \mapsto c\mathrm{e}^{-\mathrm{i}2\alpha} \\ R\mathrm{e}^{\mathrm{i}\alpha}, & \text{if } c \mapsto c \end{cases} \end{cases}$$

remembering that $A = \dfrac{h}{2\pi \Bbbk \ell}$

Slide 56: 1D Plus 3D Soliton: $\mathcal{S}_\gamma = \check{\mathrm{m}} h$ and $\mathcal{S}_\varphi = \hbar r/\check{\mathrm{x}}$ and $\boldsymbol{\mathcal{S}} = \epsilon \mathbf{A} \times (\mathbf{u} \times \mathbf{A})$

$$\Theta_{\mathbb{Z}} \xrightarrow[\text{by}]{\text{par}} \begin{cases} \gamma_{\mathbb{r}} \begin{cases} \mathbf{u}_\gamma = \hat{z} c \sin\theta \\ \mathbf{A}_\gamma = \mathrm{e}^{\mathrm{i}\pi/4} \sqrt{\sec\theta} \sqrt{r/(2\pi\check{\mathrm{x}})}\, A\left(\hat{\mathrm{x}} \cos \check{\mathrm{m}}\omega_\mathrm{o} t + \hat{\mathrm{y}} \sin \check{\mathrm{m}}\omega_\mathrm{o} t\right) \\ \mathbf{R}_\gamma = \mathbf{u} \times \mathbf{A}_\varphi \end{cases} \\ \text{in superposition with} \\ \varphi_{\mathbb{i}} \begin{cases} \mathbf{u}_\varphi = \mathrm{i} c \cos\theta \left(\hat{\mathrm{x}} \sin \check{\mathrm{a}}\omega_\mathrm{o} t \sin \check{\mathrm{m}}\omega_\mathrm{o} t - \hat{\mathrm{y}} \sin \check{\mathrm{a}}\omega_\mathrm{o} t \cos \check{\mathrm{m}}\omega_\mathrm{o} t - \hat{\mathrm{z}} \cos \check{\mathrm{a}}\omega_\mathrm{o} t\right) \\ \mathbf{A}_\varphi = \mathrm{e}^{-\mathrm{i}\pi/4} \sqrt{\sec\theta}\; r A\left(\hat{\mathrm{x}} \cos \check{\mathrm{m}}\omega_\mathrm{o} t + \hat{\mathrm{y}} \sin \check{\mathrm{m}}\omega_\mathrm{o} t\right) \\ \mathbf{R}_\varphi = \mathbf{u} \times \mathbf{A}_\varphi \end{cases} \end{cases}$$

$$\mathrm{S}_\gamma = \tan\theta (\mathrm{e}^{\mathrm{i}\pi/4})^2 \left(\frac{r}{2\pi\check{\mathrm{x}}}\right) \check{\mathrm{m}} h = \mathrm{i} \tan\theta\, \check{\mathrm{m}} \frac{r}{\check{\mathrm{x}}} \hbar \text{ and is reactive}$$

$$\mathcal{S}_\varphi = \mathrm{i} \cos\theta (\mathrm{e}^{-\mathrm{i}\pi/4})^2 \sec\theta \left(\frac{r}{2\pi\check{\mathrm{x}}} h\right) = \frac{r}{\check{\mathrm{x}}} \hbar \text{ and is active}$$

Slide 57: Energy of Superpositioned 1D- and a 3D-roton

$$\mathcal{E}_\varphi = \hbar \frac{r}{\check{x}\, t_0} \qquad E_\gamma = \mathrm{i} E_\varphi \frac{\sin\theta}{\cos\theta}$$

The components of the velocity vector are

$$u_\gamma = c \sin\theta \qquad \text{and} \qquad u_\varphi = \mathrm{i} c \cos\theta = \sqrt{c^2 - u_\gamma^2}$$

and the perceived energy is

$$E_\Theta = \sqrt{E_\varphi^2 + E_\gamma^2} = E_\varphi \sqrt{\frac{c^2}{c^2 - u_\gamma^2}}$$

Slide 58: Energy of Superpositioned 1D- and a 3D-roton

Having established E_Θ, we now, by some or other means, increase the real velocity u_γ by $\mathrm{d}u_\gamma$, thus

$$E_\Theta + \mathrm{d}E_\Theta = E_\varphi \sqrt{1 + \frac{(u_\gamma + \mathrm{d}u_\gamma)^2}{c^2 - (u_\gamma + \mathrm{d}u_\gamma)^2}}$$

therefore

$$\mathrm{d}E_\Theta = E_\varphi \sqrt{1 + \frac{(u_\gamma + \mathrm{d}u_\gamma)^2}{c^2 - (u_\gamma + \mathrm{d}u_\gamma)^2}} - E_\varphi \sqrt{1 + \frac{u_\gamma^2}{c^2 - u_\gamma^2}}$$

and performing a series expansion on $\mathrm{d}E_\Theta$ gives

$$\mathrm{d}E_\Theta = E_\varphi \frac{c\, u_\gamma\, u_\gamma}{(c^2 - u_\gamma^2)^{3/2}} + \mathcal{O}[\mathrm{d}u_\gamma^2]$$

Slide 59: Energy of Superpositioned 1D- and a 3D-roton

$$\mathrm{d}E_{\Theta} = E_{\varphi}\,\frac{c\,u_{\gamma}\,u_{\gamma}}{(c^2 - u_{\gamma}^2)^{3/2}} + \mathcal{O}[\mathrm{d}u_{\gamma}^2]$$

Energy = force × distance and force is defined by Newton's second law of motion, hence we also have

$$\mathrm{d}E_{\mathrm{N}} = m_i\,\frac{\mathrm{d}u_{\gamma}}{\mathrm{d}t}\,u_{\gamma}\mathrm{d}t$$

where m_i is the inertial mass. Equating $\mathrm{d}E_{\mathrm{N}} = \mathrm{d}E_{\Theta}$ we obtain after cancelling common terms

$$m_i = E_{\varphi}\,\frac{c}{(c^2 - u_{\gamma}^2)^{3/2}}$$

Slide 60: Energy of Superpositioned 1D- and a 3D-roton

$$m_i = E_\varphi \frac{c}{(c^2 - u_\gamma^2)^{3/2}}$$

and if $u_\gamma = 0$ the above reduces to

$$E_\varphi = m_\circ c^2$$

and it then follows trivially (Slide-57) that

$$E_\Theta = \frac{m_\circ c^2}{\sqrt{1 - v^2/c^2}}$$

Slide 61: Entanglement: Kwiat *et al.*

PHYSICAL REVIEW
LETTERS

VOLUME 75 11 DECEMBER 1995 NUMBER 24

New High-Intensity Source of Polarization-Entangled Photon Pairs

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(Received 5 July 1995)

DOI: 10.1103/PhysRevLett.75.4337

Slide 62: Entanglement: Kwiat *et al.*

Using an additional birefringent phase shifter (or even slightly rotating the down-conversion crystal itself), the value of α can be set as desired, e.g., to the values 0 or π. (Somewhat surprisingly, a net phase shift of π may be obtained by a 90° rotation of a *quarter* wave plate in one of the paths.) Similarly,.a half wave plate in one path can be used to change horizontal polarization to vertical and vice versa. One can thus very easily produce any of the four EPR-Bell states,

$$|\psi^{\pm}\rangle = (|H_1, V_2\rangle \pm |V_1, H_2\rangle)/\sqrt{2},$$
$$|\phi^{\pm}\rangle = (|H_1, H_2\rangle \pm |V_1, V_2\rangle)/\sqrt{2}, \qquad (2)$$

which form the complete maximally entangled basis of the two-particle Hilbert space, and which are important in many quantum communication and quantum information schemes.

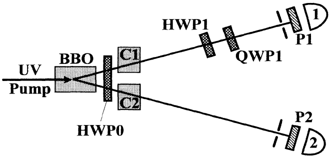
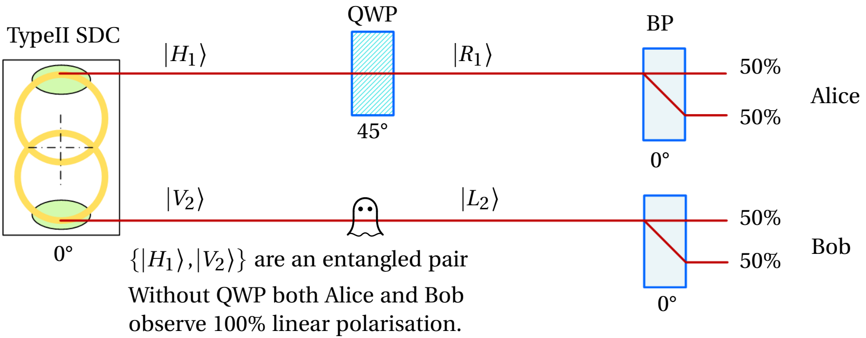


FIG. 2. Schematic of one method to produce and select the polarization-entangled state from the down-conversion crystal. The extra birefringent crystals C1 and C2, along with the half wave plate HWP0, are used to compensate the birefringent walk-off effects from the production crystal. By appropriately setting half wave plate HWP1 and quarter wave plate QWP1, one can produce all four of the orthogonal EPR-Bell states. Each polarizer P1 and P2 consisted of two stacked polarizing beam splitters preceded by a rotatable half wave plate.

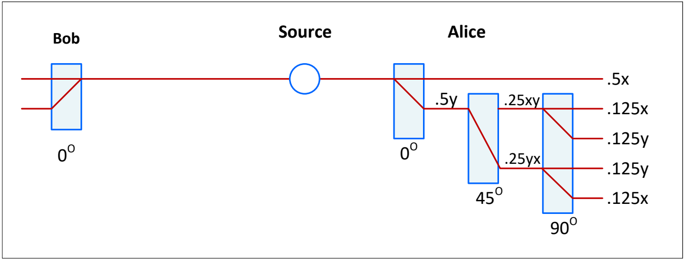
Slide 63: Entanglement: Shaving Kwiat *et al.* with Occam's razor



Here horizontal and vertical polarised photons are harvested from a type-II spontaneous down-conversion entangled photon source (SDC), but from opposing sections of the light cones. The $|H_1\rangle$ photons are converted to $|R_1\rangle$ by the quarter-wave plate (QWP). Universal state conservation () acts on Bob's beam such that $\langle L_2|V_2\rangle$ and the prediction is that both Bob and Alice observe a 50:50 polarisation distribution.

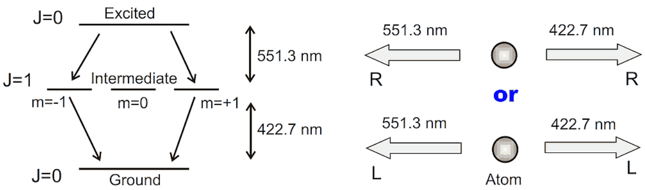
Slide 64: The Experiment

Consider an EPR experiment, the source emits circular polarised and entangled photon pairs in opposite directions to Alice and Bob. Alice uses an asymmetrical 75:25 polariser.



Prediction: Bob's polarisation distribution is skewed 25:75!!

Slide 65: Calcium light source



Simon, D.S., Jaeger, G. and Sergienko, A.V. *Quantum Metrology, Imaging, and Communication*

The production of entangled photon pairs in calcium cascades. The two-photon decay can occur via an intermediate m=+1 or m=-1 state.

Bohr says: The amplitudes for the possibilities must be added, leading to the polarisation-entangled states $|\psi\rangle = (|LL\rangle + |RR\rangle)/\sqrt{2} = (|HH\rangle + |VV\rangle)/\sqrt{2}$.

The realist says: So that the two-photon decay is nilpotent, requires the production of two circular-polarised photons that underly Maxwellian wave structure $|\psi\rangle = (|L\rangle + |L\rangle)$ or $|\psi\rangle = (|R\rangle + |R\rangle)$

We must analyse two entangled photons in the same reference frame, defined by a right handed Euclidean space $\mathbb{C}^3$ as $[\![\mathsf{xyz}]\!]$. It is a six dimensional space where each axis, that is the x, y and z axes, are a complex $\mathbb{z}$-plane. In this space we need to define the following:

- Direction of propagation is defined by $\check{\mathsf{p}} = \pm 1$.
- Direction of rotation is referenced to $[\![\mathsf{xyz}]\!]$ as $\check{\mathsf{r}} = \pm 1$.
- The helicity of the photon is given by $\check{\mathsf{s}} = \check{\mathsf{p}}\check{\mathsf{r}}$, or spin $S = \check{\mathsf{s}}\hbar$
- A polarisation direction is x, y, or none (circular).

$$\gamma \xrightarrow[\text{by}]{\text{dsc}} \begin{pmatrix} 0 & 0 & \check{\mathsf{p}} \\ \cos \check{\mathsf{r}}\omega t & \sin \check{\mathsf{r}}\omega t & 0 \\ -\check{\mathsf{p}} \sin \check{\mathsf{r}}\omega t & \check{\mathsf{p}} \cos \check{\mathsf{r}}\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathsf{x}} \\ \hat{\mathsf{y}} \\ \hat{\mathsf{z}} \end{pmatrix}$$

Slide 67: Nilpotency

⟨1⟩ The superposition of the magnetic flux vectors $\hat{\mathrm{A}}_{\mathsf{A+B}} = \hat{\mathrm{A}}_{\mathsf{A}} + \hat{\mathrm{A}}_{\mathsf{B}}$ must maintain the Maxwellian conditions, that is Universal State Conservation.

⟨2⟩ Energy conservation requires that $\left|\hat{\mathrm{P}}_{\mathsf{A}}\right| + \left|\hat{\mathrm{P}}_{\mathsf{B}}\right| = 2\left|\hat{\mathrm{A}}_{\mathsf{A+B}} \times \hat{\mathrm{R}}_{\mathsf{A+B}}\right|^{*}$ That is the sum of the energies of the waves γ_{A} and γ_{B} is equal to the energy of the superimposed waves γ_{A} plus γ_{B}.

⟨3⟩ Linear momentum conservation requires $\hat{\mathrm{P}}_{\mathsf{A}} + \hat{\mathrm{P}}_{\mathsf{B}} = 0$

⟨4⟩ Angular momentum conservation requires the helicity $\check{\mathsf{s}}_{\mathsf{A}} = \check{\mathsf{s}}_{\mathsf{B}}$.

∗ The absolute $\left|\hat{\mathrm{P}}\right|$ is calculated with $\check{\mathsf{p}} = 1$, and the factor 2 because of the unit vectors.

Slide 68: Nilpotency

Clarifying $\langle 1 \rangle$: $\hat{\mathrm{A}}_{\mathsf{A+B}} = \hat{\mathrm{A}}_{\mathsf{A}} + \hat{\mathrm{A}}_{\mathsf{B}}$ must also be a solution of

$$\left\{ \mathbf{R} = \mathbf{u} \times \mathbf{A}, \quad \mathbf{u} = \frac{1}{\|\mathbf{A}\|^2} \mathbf{A} \times \mathbf{R}, \quad \mathbf{A} = \frac{1}{\|\mathbf{u}\|^2} \mathbf{R} \times \mathbf{u} \right\}$$

that is

$$\left\{ \hat{\mathrm{R}}_{\mathsf{A+B}} = \hat{\mathrm{u}} \times \hat{\mathrm{A}}_{\mathsf{A+B}}, \quad \hat{\mathrm{u}} = \frac{1}{\left\|\hat{\mathrm{A}}_{\mathsf{A+B}}\right\|^2} \hat{\mathrm{A}}_{\mathsf{A+B}} \times \hat{\mathrm{R}}_{\mathsf{A+B}}, \quad \hat{\mathrm{A}}_{\mathsf{A+B}} = \frac{1}{\|\hat{\mathrm{u}}\|^2} \hat{\mathrm{R}}_{\mathsf{A+B}} \times \hat{\mathrm{u}} \right\}$$

Slide 69: Definition of entanglement

Proposition: Two photons γ_A and γ_B are said to be entangled when their superposition $\gamma_\mathsf{A}+\gamma_\mathsf{B}$ is nilpotent. Nilpotency is given when

$$\begin{pmatrix} 0 & 0 & \check{\mathsf{p}}_\mathsf{A} \\ \cos\check{\mathsf{r}}_\mathsf{A}\omega t & \sin\check{\mathsf{r}}_\mathsf{A}\omega t & 0 \\ -\check{\mathsf{p}}_\mathsf{A}\sin\check{\mathsf{r}}_\mathsf{A}\omega t & \check{\mathsf{p}}_\mathsf{A}\cos\check{\mathsf{r}}_\mathsf{A}\omega t & 0 \end{pmatrix}\begin{pmatrix}\hat{\mathsf{x}}\\ \hat{\mathsf{y}}\\ \hat{\mathsf{z}}\end{pmatrix} + \begin{pmatrix} 0 & 0 & \check{\mathsf{p}}_\mathsf{B} \\ \sin\check{\mathsf{r}}_\mathsf{B}\omega t & -\cos\check{\mathsf{r}}_\mathsf{B}\omega t & 0 \\ \check{\mathsf{p}}_\mathsf{B}\cos\check{\mathsf{r}}_\mathsf{B}\omega t & \check{\mathsf{p}}_\mathsf{B}\sin\check{\mathsf{r}}_\mathsf{B}\omega t & 0 \end{pmatrix}\begin{pmatrix}\hat{\mathsf{x}}\\ \hat{\mathsf{y}}\\ \hat{\mathsf{z}}\end{pmatrix}$$

and $\check{\mathsf{p}}_\mathsf{A}+\check{\mathsf{p}}_\mathsf{B}=0$ and $\check{\mathsf{r}}_\mathsf{A}+\check{\mathsf{r}}_\mathsf{B}=0$. All four conditions $\langle 1\rangle$ to $\langle 5\rangle$ are fulfilled, also the helicities or spin $\check{\mathsf{s}}_\mathsf{A}=\check{\mathsf{s}}_\mathsf{B}$ are equal.

Slide 70: Introducing polarisation

If γ_{A} is polarised in the x-orientation by a polarisation angle ϑ then that is described as follows

$$\begin{pmatrix} 0 & 0 & \check{\mathsf{p}}_{\mathsf{A}} \\ \cos \check{\mathsf{r}}_{\mathsf{A}}\omega t & \sin \check{\mathsf{r}}_{\mathsf{A}}\omega t & 0 \\ -\check{\mathsf{p}}_{\mathsf{A}} \sin \check{\mathsf{r}}_{\mathsf{A}}\omega t & \check{\mathsf{p}}_{\mathsf{A}} \cos \check{\mathsf{r}}_{\mathsf{A}}\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathsf{x}} \\ \hat{\mathsf{y}} e^{i_y \vartheta} \\ \hat{\mathsf{z}} \end{pmatrix} + \begin{pmatrix} 0 & 0 & \check{\mathsf{p}}_{\mathsf{B}} \\ \sin \check{\mathsf{r}}_{\mathsf{B}}\omega t & -\cos \check{\mathsf{r}}_{\mathsf{B}}\omega t & 0 \\ \check{\mathsf{p}}_{\mathsf{B}} \cos \check{\mathsf{r}}_{\mathsf{B}}\omega t & \check{\mathsf{p}}_{\mathsf{B}} \sin \check{\mathsf{r}}_{\mathsf{B}}\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathsf{x}} \\ \hat{\mathsf{y}} \\ \hat{\mathsf{z}} \end{pmatrix}$$

Because of the asymmetry in γ_{A} and γ_{B} we immediately recognise that the entanglement condition $\langle 1 \rangle$ is violated, because $\hat{\mathsf{A}}_{\mathsf{A+B}} = \hat{\mathsf{A}}_{\mathsf{A}} + \hat{\mathsf{A}}_{\mathsf{B}}$ is not a solution of the simultaneous algebraic equations

$$\left\{ \mathbf{R} = \mathbf{u} \times \mathbf{A}, \quad \mathbf{u} = \frac{1}{\|\mathbf{A}\|^2} \mathbf{A} \times \mathbf{R}, \quad \mathbf{A} = \frac{1}{\|\mathbf{u}\|^2} \mathbf{R} \times \mathbf{u} \right\}$$

Slide 71: Entanglement maintains nilpotency

Universal state conservation auto-polarise γ_{B}

Yes spooky action at a distance.

$$\begin{pmatrix} 0 & 0 & \check{\mathsf{p}}_{\mathsf{A}} \\ \cos\check{\mathsf{r}}_{\mathsf{A}}\omega t & \sin\check{\mathsf{r}}_{\mathsf{A}}\omega t & 0 \\ -\check{\mathsf{p}}_{\mathsf{A}}\sin\check{\mathsf{r}}_{\mathsf{A}}\omega t & \check{\mathsf{p}}_{\mathsf{A}}\cos\check{\mathsf{r}}_{\mathsf{A}}\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathsf{x}} \\ \hat{\mathsf{y}}e^{\mp i_y\vartheta} \\ \hat{\mathsf{z}} \end{pmatrix} + \begin{pmatrix} 0 & 0 & \check{\mathsf{p}}_{\mathsf{B}} \\ \sin\check{\mathsf{r}}_{\mathsf{B}}\omega t & -\cos\check{\mathsf{r}}_{\mathsf{B}}\omega t & 0 \\ \check{\mathsf{p}}_{\mathsf{B}}\cos\check{\mathsf{r}}_{\mathsf{B}}\omega t & \check{\mathsf{p}}_{\mathsf{B}}\sin\check{\mathsf{r}}_{\mathsf{B}}\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathsf{x}}e^{\pm i_x\vartheta} \\ \hat{\mathsf{y}} \\ \hat{\mathsf{z}} \end{pmatrix}$$

The superposition $\hat{\mathrm{A}}_{\mathsf{A}_X+\mathsf{B}_Y}$ is given by

$$\chi\hat{\mathrm{A}}_{\mathsf{A}_X+\mathsf{B}_Y} = \begin{pmatrix} \cos\check{\mathsf{r}}_{\mathsf{A}}\omega t & \sin\check{\mathsf{r}}_{\mathsf{A}}\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathsf{x}} \\ \hat{\mathsf{y}}e^{\mp i_y\vartheta} \\ \hat{\mathsf{z}} \end{pmatrix} + \begin{pmatrix} \sin\check{\mathsf{r}}_{\mathsf{B}}\omega t & -\cos\check{\mathsf{r}}_{\mathsf{B}}\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathsf{x}}e^{\pm i_x\vartheta} \\ \hat{\mathsf{y}} \\ \hat{\mathsf{z}} \end{pmatrix}$$

$$= \begin{pmatrix} \cos\check{\mathsf{r}}_{\mathsf{A}}\omega t + \sin\check{\mathsf{r}}_{\mathsf{B}}\omega t & \sin\check{\mathsf{r}}_{\mathsf{A}}\omega t - \cos\check{\mathsf{r}}_{\mathsf{B}}\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathsf{x}}(1+e^{\pm i_x\vartheta}) \\ \hat{\mathsf{y}}(1+e^{\mp i_y\vartheta}) \\ 2\hat{\mathsf{z}} \end{pmatrix}$$

Slide 72: Entanglement maintains nilpotency

The superposition $\hat{\mathrm{A}}_{\mathsf{A}_X+\mathsf{B}_Y}$ is given by

$$\chi\hat{\mathrm{A}}_{\mathsf{A}_X+\mathsf{B}_Y} = \begin{pmatrix} \cos\check{\mathrm{r}}_{\mathsf{A}}\omega t+\sin\check{\mathrm{r}}_{\mathsf{B}}\omega t & \sin\check{\mathrm{r}}_{\mathsf{A}}\omega t-\cos\check{\mathrm{r}}_{\mathsf{B}}\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathrm{x}}(1+e^{\pm i_x\vartheta}) \\ \hat{\mathrm{y}}(1+e^{\mp i_y\vartheta}) \\ 2\hat{\mathrm{z}} \end{pmatrix}$$

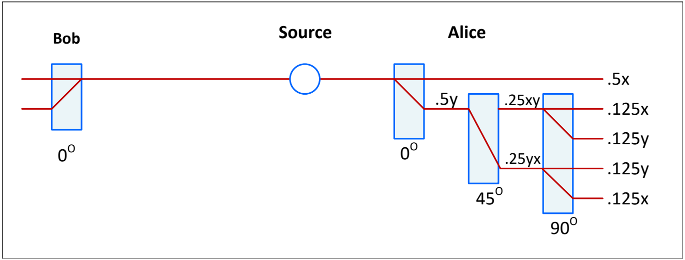
$$\hat{\mathrm{A}}_{\mathsf{A}_X+\mathsf{B}_Y} = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\check{\mathrm{r}}_{\mathsf{A}}\omega t+\sin\check{\mathrm{r}}_{\mathsf{B}}\omega t & \sin\check{\mathrm{r}}_{\mathsf{A}}\omega t-\cos\check{\mathrm{r}}_{\mathsf{B}}\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathrm{x}}(1+e^{\pm i_x\vartheta})/\|1+e^{i\vartheta}\| \\ \hat{\mathrm{y}}(1+e^{\mp i_y\vartheta})/\|1+e^{i\vartheta}\| \\ \hat{\mathrm{z}} \end{pmatrix}$$

and provides a solution to

$$\left\{ \mathbf{R}=\mathbf{u}\times\mathbf{A}, \quad \mathbf{u}=\frac{1}{\|\mathbf{A}\|^2}\mathbf{A}\times\mathbf{R}, \quad \mathbf{A}=\frac{1}{\|\mathbf{u}\|^2}\mathbf{R}\times\mathbf{u} \right\}$$

Therefore Bob observes a skewed 25:75 polarisation distribution

Yes, faster than speed of light communication is possible!



Slide 74: Entanglement: Kwiat *et al.*

overall phase shift is omitted.

Using an additional birefringent phase shifter (or even slightly rotating the down-conversion crystal itself), the value of α can be set as desired, e.g., to the values 0 or π. (Somewhat surprisingly, a net phase shift of π may be obtained by a 90° rotation of a *quarter* wave plate in one of the paths.) Similarly,.a half wave plate in one path can be used to change horizontal polarization to vertical and vice versa. One can thus very easily produce any of the four EPR-Bell states,

$$|\psi^{\pm}\rangle = (|H_1, V_2\rangle \pm |V_1, H_2\rangle)/\sqrt{2},$$
$$|\phi^{\pm}\rangle = (|H_1, H_2\rangle \pm |V_1, V_2\rangle)/\sqrt{2}, \qquad (2)$$

which form the complete maximally entangled basis of the two-particle Hilbert space, and which are important in many quantum communication and quantum information schemes.

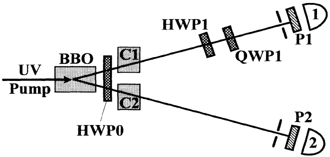


FIG. 2. Schematic of one method to produce and select the polarization-entangled state from the down-conversion crystal. The extra birefringent crystals C1 and C2, along with the half wave plate HWP0, are used to compensate the birefringent walk-off effects from the production crystal. By appropriately setting half wave plate HWP1 and quarter wave plate QWP1, one can produce all four of the orthogonal EPR-Bell states. Each polarizer P1 and P2 consisted of two stacked polarizing beam splitters preceded by a rotatable half wave plate.

Slide 75: Kwiat's surprise

The HWP and QWP act on photon A and universal state conservation on B during the photon's flight. The observer is presented with photons A" and B" which are either linearly or circularly polarised as tabled below

{A,B}	HWP	{A',B'}	QWP	{A",B"}	{A",B"}
$\{0,\frac{\pi}{2}\}$ $\{\frac{\pi}{2},0\}$	0	$\{0,\frac{\pi}{2}\}$ $\{\frac{\pi}{2},0\}$	0	$\{0,\frac{\pi}{2}\}$	$\{\frac{\pi}{2},0\}$
			$\frac{\pi}{4}$	{R,L}	{L,R}
			$\frac{\pi}{2}$	$\{\pi,\frac{3\pi}{2}\}$	$\{\frac{-\pi}{2},\pi\}$
			$\frac{3\pi}{4}$	{R,L}	{L,R}
$\{0,\frac{\pi}{2}\}$ $\{\frac{\pi}{2},0\}$	$\frac{\pi}{4}$	$\{\frac{\pi}{2},\pi\}$ $\{\pi,\frac{\pi}{2}\}$	0	$\{\frac{\pi}{2},\pi\}$	$\{\pi,\frac{\pi}{2}\}$
			$\frac{\pi}{4}$	{R,L}	{L,R}
			$\frac{\pi}{2}$	$\{\frac{3\pi}{2},0\}$	$\{0,\frac{-\pi}{2}\}$
			$\frac{3\pi}{4}$	{R,L}	{L,R}

The superposition $\hat{\mathsf{A}}_{\mathsf{A}_X+\mathsf{B}_Y}$ is given by

$$\chi\hat{\mathsf{A}}_{\mathsf{A}_X+\mathsf{B}_Y} = \begin{pmatrix} \cos\check{\mathsf{r}}_{\mathsf{A}}\omega t + \sin\check{\mathsf{r}}_{\mathsf{B}}\omega t & \sin\check{\mathsf{r}}_{\mathsf{A}}\omega t - \cos\check{\mathsf{r}}_{\mathsf{B}}\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathsf{x}}(1+e^{\pm i_x\vartheta}) \\ \hat{\mathsf{y}}(1+e^{\mp i_y\vartheta}) \\ 2\hat{\mathsf{z}} \end{pmatrix}$$

$$\hat{\mathsf{A}}_{\mathsf{A}_X+\mathsf{B}_Y} = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\check{\mathsf{r}}_{\mathsf{A}}\omega t + \sin\check{\mathsf{r}}_{\mathsf{B}}\omega t & \sin\check{\mathsf{r}}_{\mathsf{A}}\omega t - \cos\check{\mathsf{r}}_{\mathsf{B}}\omega t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathsf{x}}(1+e^{\pm i_x\vartheta})/\|1+e^{i\vartheta}\| \\ \hat{\mathsf{y}}(1+e^{\mp i_y\vartheta})/\|1+e^{i\vartheta}\| \\ \hat{\mathsf{z}} \end{pmatrix}$$

and provides a solution to

$$\left\{ \mathbf{R} = \mathbf{u}\times\mathbf{A}, \quad \mathbf{u} = \frac{1}{\|\mathbf{A}\|^2}\mathbf{A}\times\mathbf{R}, \quad \mathbf{A} = \frac{1}{\|\mathbf{u}\|^2}\mathbf{R}\times\mathbf{u} \right\}$$

Slide 77: Universal State-Space

Physical Space $\mathscr{P}$	State Space $\mathcal{S}$
$[\![\mathrm{xyz}]\!]\mathbb{C}^3$	$\langle\mathrm{pqr}\rangle\mathbb{C}^3$
$\mathscr{M}_{\mathscr{P}}(\mathbf{u},\mathbf{A},\mathbf{R})$	$\mathscr{M}_{\mathcal{S}}(\boldsymbol{t},\mathbf{P},\mathbf{Q})$
$\mathscr{M}_{\mathscr{P}}(\mathbf{u},\mathbf{L},\mathbf{V})$	$\mathbf{P}=\sum\mathbf{A}_i+\sum l_\mathrm{o}\mathbf{L}_i+\sum l_\mathrm{o}^2\mathbf{B}_i$
$\mathscr{M}_{\mathscr{P}}(\mathbf{u},\mathbf{B},\mathbf{E})$	
$\mathbf{p}=\int\mathbf{u}\,\mathrm{d}t$	$\boldsymbol{e}=\int\boldsymbol{t}\,\mathrm{d}t$

The universe's epoch $\boldsymbol{e}$ is given by $\boldsymbol{e}=\int\boldsymbol{t}\,\mathrm{d}t$ that is a position in $\mathcal{S}$, making state-vector $\boldsymbol{t}$ the arrow-of-time, and time t a variable of differentiation / integration. the arrow-of-time steers the epoch on a closed curve in $\mathcal{S}$ bringing the universe back to its original state but approached from the opposite direction (not a gravitational big shrink), thus describing a cyclic universe. The past histories are imprinted on the future microwave background; each rebirth begins with carryover information from the past.

Slide 78: Summary

- Beginning with $\mathscr{M}(\mathbf{u},\mathbf{a},\mathbf{r}) \xrightarrow{\text{defines}} \left\{ \mathbf{r} = \mathbf{u} \times \mathbf{a}, \quad \mathbf{u} = \frac{1}{\|\mathbf{a}\|^2}\mathbf{a} \times \mathbf{r}, \quad \mathbf{a} = \frac{1}{\|\mathbf{u}\|^2}\mathbf{r} \times \mathbf{u} \right\}$
- derived the Maxwell equations
- demonstrated fundamental nature by deriving ϵ_0 and μ_0
- leveraged $\mathscr{M}(\mathbf{u},\mathbf{a},\mathbf{r})$ for Maxwellian solitons for photons and particles
- derived $E = hf$ and $E = mc^2$ thus showing mass is an emergent EM-phenomenon,
- explains the phenomenon that leads to the Bell inequality,
- gives mathematical support for philosophical discussion regarding a cyclic universe

Questions or Comments