A novel soliton equation system leads to a constructive electromagnetic quantum theory that describes the origin of mass and unifies the forces.

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Abstract: A novel soliton equation system (a set of simultaneous algebraic vector equations) gives rise to the Maxwell equations in vacuum. It has become evident that one must distinguish between the electromotive forces and the electrostatic forces as separate phenomena. This, together with the equation system, provides the mathematical framework in which to define a Constructive Electromagnetic Quantum Theory in $\mathbb{C}^3$. This new theory provides an explanation of all quantum phenomena, defines particles as electromagnetic solitons and identifies—most significantly—that mass and all forces are manifestations of the electromagnetic phenomenon. As an application, the theory enabled the elucidation to identify the laws that govern the atomic mass relation of the elements and their isotopes using an onion shell nucleon model. Furthermore, it also provides the fundamental understanding required to calculate the "mass gap" as $\Delta_0 \approx 2.931 \times 10^{-60}$ joules.

Keywords: Constructive Electromagnetic Quantum Theory, Electromagnetic Solitons, Quantised Electromagnetism, Origin of Mass, Atomic Onion Shell Model, Nucleon Packing, Isotope Transmutation, Unification of the Forces, Mass Gap

"In classical electrodynamics, the vector and scalar potentials were first introduced as a convenient mathematical aid for calculating the fields. It is true that in order to obtain a classical canonical formalism, the potentials are needed. Nevertheless, the fundamental equations of motion can always be expressed directly in terms of the fields alone.

In the quantum mechanics, however, the canonical formalism is necessary, and as a result, the potentials cannot be eliminated from the basic equations. Nevertheless, these equations, as well as the physical quantities, are all gauge invariant; so that it may seem that even in quantum mechanics, the potentials themselves have no independent significance."

These opening paragraphs are excerpted from Aharonov and Bohm's 1959 work [1]. Their work goes on to argue that these initial conclusions are not entirely accurate and that a deeper interpretation of the potentials is necessary within the framework of quantum mechanics.

The central thesis developed in this paper asserts that electrostatic fields and electromotive potentials are distinct yet interconnected phenomena of electromagnetic phenomenon. To provide a comprehensive description of the electromagnetic phenomenon, it is imperative to integrate the equations governing electrostatic fields with those governing electromotive potentials. This approach not only offers a classical interpretation for the quantum mechanical phenomena but also suggests that elementary particles can be conceptualised as electromagnetic solitons. The fundamental framework presented herein constructs a classical electromagnetic quantum theory.

Theorem 1: The soliton equation system. *In a space $\mathbb{C}^3$ the system of simultaneous equations*

$$\mathcal{M}(\mathbf{u},\mathbf{a},\mathbf{r}) \xrightarrow{\textit{defines}} \left\{ \mathbf{u} = \frac{1}{\mathbf{a}\cdot\mathbf{a}^*}\mathbf{a}\times\mathbf{r}, \quad \mathbf{a} = \frac{1}{\mathbf{u}\cdot\mathbf{u}^*}\mathbf{r}\times\mathbf{u}, \quad \mathbf{r} = \mathbf{u}\times\mathbf{a} \right\} \tag{1}$$

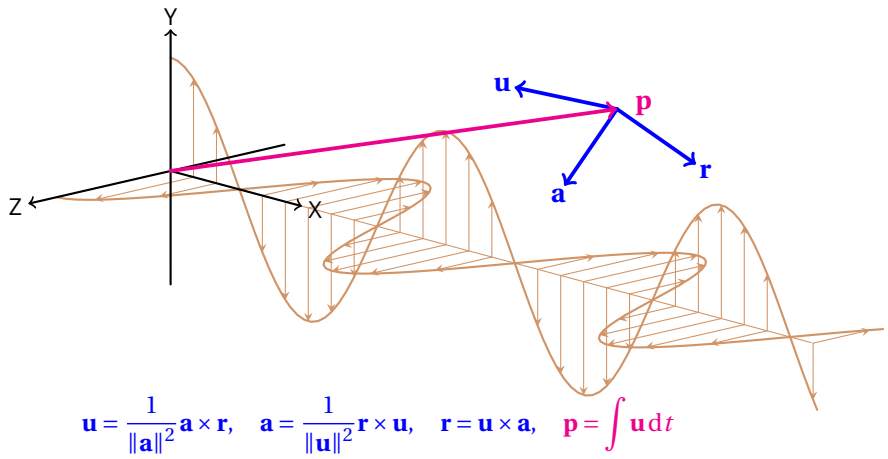

Figure 1: Illustrating the vectors **u**, **a**, **r** and **p** juxtaposed to a travelling plane wave.

defines the motion of a soliton characterised by a velocity vector $\mathbf{u}(t)$ *and two co-orthogonal vector fields* $\mathbf{a}(t)$ *and* $\mathbf{r}(t)$ *that describe the disturbance in a homogenous and isotropic medium (see Figure 1).*

Here the vector quantities of **u**, **a** and **r** are complex, for example

$$\mathbf{a} = \hat{\mathrm{x}}\, a_x e^{i\alpha_x} + \hat{\mathrm{y}}\, a_y e^{i\alpha_y} + \hat{\mathrm{z}}\, a_z e^{i\alpha_z}$$
$$\mathbf{a}^* = \hat{\mathrm{x}}\, a_x e^{-i\alpha_x} + \hat{\mathrm{y}}\, a_y e^{-i\alpha_y} + \hat{\mathrm{z}}\, a_z e^{-i\alpha_z}$$

therefore $\mathbf{a}\cdot\mathbf{a}^* = a_x^2 + a_y^2 + a_z^2 = a^2 = \|\mathbf{a}\|^2$

PROOF. Performing a 'left and right side' curl operation on the second and third equations of the equation-set (1) gives

$$\nabla\times\mathbf{a} = \frac{1}{\mathbf{u}\cdot\mathbf{u}^*}\nabla\times(\mathbf{r}\times\mathbf{u}) \quad \text{and} \quad \nabla\times\mathbf{r} = \nabla\times(\mathbf{u}\times\mathbf{a}) \tag{2}$$

and to evaluate the vector triple products we use general vector analytic methods to give

$$\nabla\times(\mathbf{r}\times\mathbf{u}) = \mathbf{r}(\nabla\cdot\mathbf{u}) - \mathbf{u}(\nabla\cdot\mathbf{r}) + (\mathbf{u}\cdot\nabla)\mathbf{r} - (\mathbf{r}\cdot\nabla)\mathbf{u}$$
$$\nabla\times(\mathbf{u}\times\mathbf{a}) = \mathbf{u}(\nabla\cdot\mathbf{a}) - \mathbf{a}(\nabla\cdot\mathbf{u}) + (\mathbf{a}\cdot\nabla)\mathbf{u} - (\mathbf{u}\cdot\nabla)\mathbf{a}.$$

Because the vectors **a** and **r** are position independent, we have

$$\nabla\cdot\mathbf{a} = 0 \quad \text{and} \quad \nabla\cdot\mathbf{r} = 0. \tag{3}$$

Evaluating the terms containing $\mathbf{u} = \hat{\mathrm{x}}\,\partial x/\partial t + \hat{\mathrm{y}}\,\partial y/\partial t + \hat{\mathrm{z}}\,\partial z/\partial t$ we obtain

$$\mathbf{u}\cdot\nabla = \nabla\cdot\mathbf{u} = \frac{\partial x}{\partial t}\frac{\partial}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial}{\partial y} + \frac{\partial z}{\partial t}\frac{\partial}{\partial z} = \frac{\partial}{\partial t}$$

Because $\mathbf{a}(\mathbf{u}\cdot\nabla) = \mathbf{a}\,\partial 1/\partial t = 0$, we are left with

$$\nabla\times(\mathbf{u}\times\mathbf{a}) = -\frac{\partial\mathbf{a}}{\partial t} \quad \text{and} \quad \nabla\times(\mathbf{r}\times\mathbf{u}) = \frac{\partial\mathbf{r}}{\partial t}.$$

Therefore, the 'left and right side' curl operations (2) generate the new relations:

$$\nabla\times\mathbf{a} = \frac{1}{u^2}\frac{\partial\mathbf{r}}{\partial t} \quad \text{and} \quad \nabla\times\mathbf{r} = -\frac{\partial\mathbf{a}}{\partial t} \tag{4}$$

A further 'left and right side' curl operation on (4) gives

$$\nabla\times\nabla\times\mathbf{r} = -\frac{\partial(\nabla\times\mathbf{a})}{\partial t} \quad \text{and} \quad \nabla\times\nabla\times\mathbf{a} = \frac{1}{u^2}\frac{\partial(\nabla\times\mathbf{r})}{\partial t}$$

and because $\nabla \times \nabla \times \mathbf{r} = \nabla(\nabla \cdot \mathbf{r}) - \nabla^2 \mathbf{r}$ we recover the d'Alembert wave equations

$$\nabla^2 \mathbf{r} - \frac{1}{u^2}\frac{\partial^2 \mathbf{r}}{\partial t^2} = 0 \quad \text{and} \quad \nabla^2 \mathbf{a} - \frac{1}{u^2}\frac{\partial^2 \mathbf{a}}{\partial t^2} = 0. \tag{5}$$

This concludes the proof that the three vector algebraic equations (1) give rise to the d'Alembert wave equations (5). Therefore, the equation set $\boldsymbol{\mathcal{M}}$ is a generic bimodal-transverse soliton equation system. □

The preceding approach was purely abstract and mathematical, focusing on the derivation of the generic Maxwell equations for any pair of conceivable fields, denoted as **a** and **r**. A discerning reader will have undoubtedly noticed that by associating $\mathbf{a} \mapsto \mathbf{B}$ and $\mathbf{r} \mapsto \mathbf{E}$, equations (3) and (4) transform into the well-known Maxwell equations in vacuum. Nonetheless, to affirm this equivalence, it is imperative to show that the vector algebraic equations for $\boldsymbol{\mathcal{M}}$ yield $c^{-2} = \epsilon_0 \mu_0$. Rather than focusing on EM-fields, we extend $\boldsymbol{\mathcal{M}}$ to operate on elementary emflux quantities. In doing so, we transition fluxes from being mere mathematical visualisation tools to being fundamental phenomena that underlie electromagnetic behaviour.

1 $\boldsymbol{\mathcal{M}}(\mathbf{u}, \boldsymbol{\phi}_\mathbf{o}, \boldsymbol{\Upsilon}_\mathbf{o})$ is fundamental to electromagnetism

The discovery of $\boldsymbol{\mathcal{M}}$ enables the long-awaited quantification of electromagnetic phenomenon. To initiate this, we introduce $\boldsymbol{\phi}_\mathrm{o}$ and $\boldsymbol{\Upsilon}_\mathrm{o}$ as elementary magnetic and electric vector fluxes responsible for the electromotive phenomenon. A single pair of these elementary fluxes gives rise to an elementary solitary wave, or soliton.

Note that I specifically employ the term "elementary" rather than "quantum" so as to avoid confusion with existing definitions that utilise the words "quanta" or "quantum." This paper adopts an extended nomenclature for the express purpose of avoiding conflicts with established notational conventions, detailed in Appendix D.

Remark 1: Electrostatic *versus* Electromotive. I intentionally distinguish between the electromotive charge ℓ (represented by ℓ an ell in script font) and the elementary electrostatic charge e (represented by e in italic serif font). In the conventional interpretation of electromagnetic theory these charges are not separated, thus both charges are considered equal in magnitude. The rationale for this differentiation is clarified in Appendix A, where evidence is presented to show that electrons are not the charge carriers in an electric current.

Remark 2: Defining the *emflux*. The magnetic flux quantum is defined as $\phi_0 = h/(2e)$. Because I have purposefully separated the electromotive charge ℓ from the electrostatic charge e, a similar separation must be made for ϕ, a static quantity. Consequently, I introduce the definition of elementary magnetic momentum ϕ_o, which will henceforth be referred to as magnetic emflux (magnetic electromotive flux).

Theorem 2: Elementary Electromotive Soliton. *There exists an elementary length, denoted as l_o, and an elementary time, denoted as t_o, which are related to the speed of light c such that $l_o = c\,t_o$. In addition, a component of an electromotive wave or soliton is the magnetic electromotive flux ϕ_o which in a wave results into an* EM*-motive action h_e per elementary length of the wave front. Furthermore, an elementary electromotive soliton, defined by $\boldsymbol{\mathcal{M}}(\mathbf{u}, \boldsymbol{\phi}_o, \boldsymbol{\Upsilon}_o)$, carries an elementary electromotive charge, denoted as ℓ, and has an action h, while propagating at the speed of light.*

PROOF. The proof is structured by demonstrating that the set of simultaneous equations

$$\mathcal{M}(\mathbf{u},\boldsymbol{\phi}_o,\boldsymbol{\Upsilon}_o) \xrightarrow{\text{defines}} \left\{\mathbf{u}=\frac{1}{\|\boldsymbol{\phi}_o\|^2}\boldsymbol{\phi}_o\times\boldsymbol{\Upsilon}_o,\ \boldsymbol{\phi}_o=\frac{1}{\|\mathbf{u}\|^2}\boldsymbol{\Upsilon}_o\times\mathbf{u},\ \boldsymbol{\Upsilon}_o=\mathbf{u}\times\boldsymbol{\phi}_o\right\} \tag{6}$$

together with the theorem's assertions demands the presence of ϵ_0 and μ_0 in their known forms.

Assuming that $\boldsymbol{\phi}_o\times\boldsymbol{\Upsilon}_o$ represents wave action, we multiply the equation $\mathbf{u}=(\boldsymbol{\phi}_o\times\boldsymbol{\Upsilon}_o)\|\boldsymbol{\phi}_o\|^{-2}$ by h and evaluate its norm, yielding

$$\|h\mathbf{u}\| = \left\|\frac{h}{\|\boldsymbol{\phi}_o^2\|}\boldsymbol{\phi}_o\times\boldsymbol{\Upsilon}_o\right\| \qquad \text{to give}$$

$$h = \left[\frac{h}{c\phi_o^2}\right]\left(\|\boldsymbol{\phi}_o\|\,\|\boldsymbol{\Upsilon}_o\|\right) \tag{7}$$

where the square brackets indicate the development of a constant. The objective now is to eliminate ϕ_o in the above equation.

Theorem 2 states that for an elementary electromagnetic wave, or soliton, the magnetic electromotive flux ϕ_o, produces electromagnetic wave action h_e per elementary length of the wave front. Thus it follows that

$$h_e = \varrho h = l_o\|\boldsymbol{\phi}_o\| \tag{8}$$

where ϱ is a unit conversion constant to relate the electromagnetic action h_e to the mechanical action h.

Because the soliton transports an elementary electromotive charge ℓ, as asserted in Theorem 2, at a velocity c, the EM-wave action is proportional to the EM-momentum times distance. Here we consider the elementary distance $l_o = ct_o$. Therefore, the EM-wave action is also

$$h_e = \varrho h = \chi\kappa\ell l_o c \tag{9}$$

Here κ is a dimensionless proportionality constant of unknown value, scaling $\chi\ell c l_o$ to the EM-action h_e; and where χ represents a composite physical quantity.

Combining Equations (8) and (9) gives $\chi^{-1}\|\boldsymbol{\phi}_o\| = \kappa\ell c$. Also, $\boldsymbol{\Upsilon}_o = \mathbf{u}\times\boldsymbol{\phi}_o$ gives $\|\boldsymbol{\Upsilon}_o\| = c\phi_o$. Both of these results are introduced into (7) to get

$$h = \left[\frac{h}{c\phi_o^2}\right]\left[\frac{1}{\chi}\right]\kappa\ell c^2\phi_o$$

We are now in a position to define, purely mathematically, the expression for

$$\phi_o = \frac{h}{\kappa\ell} \tag{10}$$

but only if

$$1 = \left[\frac{h}{c\phi_o^2}\right]\left[\frac{1}{\chi}\right]c^2 \qquad \text{and replacing } \phi_o \text{ using (10) gives}$$

$$1 = \left[\frac{\kappa^2\ell^2}{ch}\right]\left[\frac{1}{\chi}\right]c^2 \qquad \text{which requires } \frac{1}{\chi} = \frac{h}{\kappa^2\ell^2 c}\text{, hence}$$

$$1 = \left[\frac{\kappa^2\ell^2}{ch}\right]\left[\frac{h}{\kappa^2\ell^2 c}\right]c^2$$

Which gives us the permittivity and permeability of the space

$$\epsilon_o = \frac{\kappa^2\ell^2}{ch} \quad \text{and} \quad \mu_o = \frac{h}{\kappa^2\ell^2 c}$$

Now, with a bit of hindsight, by equating $\kappa^{-2} = 2\alpha$, where α is the fine structure constant, and mapping $e \mapsto \ell$ and Equation the above gives the sought-after result

$$\epsilon_0 = \frac{e^2}{2\alpha hc} \quad \text{and} \quad \mu_0 = \frac{2\alpha h}{e^2 c}$$

requiring that the elementary charge e and elementary electromotive charge ℓ are equal in magnitude.

We note that $\epsilon_o \left| \boldsymbol{\phi}_\mathrm{o} \times \boldsymbol{\Upsilon}_\mathrm{o} \right| = h$ thereby having satisfied all demands of the theorem. This concludes the proof that the equation set $\boldsymbol{\mathcal{M}}(\mathbf{u}, \boldsymbol{\phi}_\mathrm{o}, \boldsymbol{\Upsilon}_\mathrm{o})$ describes EM-solitons. □

Units: A unit analysis provides following:

Qty.	Value	Units	Elementary unit
e	$1.602\,176\,634 \times 10^{-19}$	C	electrostatic charge
κ	$8.277\,559\,999\,29(62)$		
ℓ	$1.602\,176\,634 \times 10^{-19}$	C m	electromotive charge
ϕ_o	$h/\kappa\ell$	$\mathrm{kg\,m\,s^{-1}\,C^{-1}}$	magnetic electromotive flux
h_e	$6.626\,1070\,15 \times 10^{-19}$	$\mathrm{kg\,m^2\,s^{-1}\,C^{-1}}$	
ϱ	1	$\mathrm{C^{-1}}$	

Recalling $h_e = l_\mathrm{o} \left\| \boldsymbol{\phi}_\mathrm{o} \right\|$, *i. e.* (8); we are now in the position to calculate the numeric values for the elementary length and time using $\kappa^{-2} = 2\alpha$ and the 2018 CODATA values:

$$\begin{aligned}
\kappa &= 8.277\,559\,999\,29(62) && \text{which I name the Heaviside constant} \\
l_\mathrm{o} &= 1.326\,211\,321\,74(10) \times 10^{-18} && \text{elementary length in metres} \\
t_\mathrm{o} &= 4.423\,764\,795\,78(33) \times 10^{-27} && \text{elementary time in seconds} \\
\Delta_0 &= 2.931\,217\,586\,39(22) \times 10^{-60} && \text{mass gap in joules}
\end{aligned}$$

where $\Delta_0 = h t_\mathrm{o}$ is the least energy gap from a vacuum to the next lowest energy state.

Historic note: In the late 19th century Oliver Heaviside developed vector calculus, and rewrote the Maxwell works into the form commonly used today. The Heaviside constant κ is a coupling constant relating the electric charge momentum to mechanical momentum.

Question 1: What defines the speed of light?. The preceding analysis also poses a profound question reminiscent of the classic 'Who came first, the chicken or the egg?' scenario regarding the speed of light. The constants of permittivity ϵ_0 and permeability μ_0 were deduced based on the previously defined velocity c in Equation (6), or more precisely, as $\mathbf{u} = c\hat{\mathrm{u}}(t)$. This leads us to ponder: Does the expression $c = 1/\sqrt{\epsilon_0 \mu_0}$ provide a fundamental foundation for defining the speed of light, or is there another underlying explanation for the velocity c?

For instance, in the context of sound waves propagating through a material, the speed of sound is contingent upon the material's properties. In fluids, it is expressed as $c^2 = K_s/\rho$, where K_s represents the coefficient of stiffness, and ρ denotes the fluid's density. Alternatively, it can also be formulated as $c^2 = \partial P/\partial\rho$, where P represents pressure. Notably, none of these parameters—K_s, ρ, or P—are defined in terms of the speed of sound within the medium.

The proof establishing the derived values of ϵ_0 and μ_0 in Theorem 2 suggests that space possesses additional characteristics that lead to the concept of 'transportivity' denoted as $\mathcal{T} = ?_\mathrm{A}/?_\mathrm{B} = c^2$. Analogous to fluids, the transportivity could be regarded as a ratio of two properties that are independent of the speed of light. What precisely are these two properties denoted as $?_\mathrm{A}$ and $?_\mathrm{B}$, and how does one ascertain or define them?

2 Describing solitons as solutions of $\mathcal{M}$

Any solution to the three simultaneous equations in the set $\mathcal{M}(\mathbf{u}, \boldsymbol{\phi}_o, \boldsymbol{\Upsilon}_o)$ represents an EM-soliton. To express these solutions efficiently, we introduce a new mathematical syntax, utilising a row-by-row matrix product operator $\diamond$, defined as follows:

$$\begin{pmatrix} Pa_{1,1} & Pa_{1,2} \\ Qa_{2,1} & Qa_{2,2} \end{pmatrix} = \begin{pmatrix} P \\ Q \end{pmatrix} \diamond \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

A wave or soliton ξ, that is a solution of $\mathcal{M}$ is precisely defined by the three vectors $\mathbf{u}$, $\boldsymbol{\phi}_o$, and $\boldsymbol{\Upsilon}_o$, expressed in matrix form as

$$\xi \xrightarrow[\text{by}]{\text{def}} \begin{pmatrix} \mathbf{u} \\ \boldsymbol{\phi}_o \\ \boldsymbol{\Upsilon}_o \end{pmatrix} = \begin{pmatrix} c \\ \phi_o \\ c\phi_o \end{pmatrix} \diamond \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}$$

This expression can be further simplified by considering only the parameters of interest:

$$\xi \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} c \\ \phi_o \\ \Upsilon_o \end{pmatrix} \diamond \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}$$

Electromagnetic solitons of interest are described generically:

$$\left.\begin{array}{l} \xi_1 \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} c \\ \phi_o \\ \Upsilon_o \end{pmatrix} \diamond \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta\sin\vartheta & \cos\theta\sin\vartheta & \cos\vartheta \\ \sin\theta\cos\vartheta & -\cos\theta\cos\vartheta & \sin\vartheta \end{pmatrix} \\ \text{or} \\ \xi_2 \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} c \\ \phi_o \\ \Upsilon_o \end{pmatrix} \diamond \begin{pmatrix} \sin\theta\cos\vartheta & -\cos\theta\cos\vartheta & \sin\vartheta \\ \cos\theta & \sin\theta & 0 \\ -\sin\theta\sin\vartheta & \cos\theta\sin\vartheta & \cos\vartheta \end{pmatrix} \\ \text{or} \\ \xi_3 \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} c \\ \phi_o \\ \Upsilon_o \end{pmatrix} \diamond \begin{pmatrix} -\sin\theta\sin\vartheta & \cos\theta\sin\vartheta & \cos\vartheta \\ \sin\theta\cos\vartheta & -\cos\theta\cos\vartheta & \sin\vartheta \\ \cos\theta & \sin\theta & 0 \end{pmatrix} \end{array}\right\} \text{ where } \begin{cases} \theta = sn\omega_o t \\ \vartheta = r_z m\omega_o t \\ s_a \in \{1/2, 1, 3/2, \ldots\} \\ r_a \in \{-1, 0, 1\} \\ s = s_a r_a \\ r_z \in \{-1, 0, 1\} \\ n \in \mathbb{Q} \geq 0 \\ m \in \mathbb{Q} \geq 0 \end{cases} \tag{11}$$

In this context, the angles θ and ϑ may or may not be time-dependent. Rotations are characterised by two orientations: r_a, representing azimuthal rotation around the z-axis, and r_z, indicating rotation toward the zenith along the $x\cos\theta + y\sin\theta$ axis. The azimuthal rotation direction r_a, provides the sign for spin $s = r_a s_a$. We are working with rotating field vectors. For instance, $\boldsymbol{\phi}_o$ represents a rotating vector, which we define as the source of a north-pointing elementary magnetic emflux, denoted as $\phi_o = l_o^2 \phi_o$. Consequently, $-\boldsymbol{\phi}_o$ still acts as a source of a north-pointing emflux but in the opposite direction. We are now required to introduce $\overline{\boldsymbol{\phi}}_o$ as the magnetic field vector that absorbs a north-pointing emflux. This implies that $\boldsymbol{\phi}_o + \overline{\boldsymbol{\phi}}_o \equiv 0$, and $\boldsymbol{\phi}_o - \overline{\boldsymbol{\phi}}_o \equiv 2\boldsymbol{\phi}_o$ if and only if $\boldsymbol{\phi}_o = \hat{p}\phi_o$ and $\overline{\boldsymbol{\phi}}_o = \hat{p}\bar{\phi}_o$, where $\hat{p}$ represents any unit vector. Below is a visual representation of this concept where the symbol Ⓢ signifies the source or the sink:

$$\begin{array}{lll} \boldsymbol{\phi}_o \mapsto & \text{S}\textcircled{S}\longrightarrow\text{N} & \text{and} \quad -\boldsymbol{\phi}_o \mapsto \text{N}\longleftarrow\textcircled{S}\text{S} \\ \overline{\boldsymbol{\phi}}_o \mapsto & \text{N}\textcircled{S}\longleftarrow\text{S} & \text{and} \quad -\overline{\boldsymbol{\phi}}_o \mapsto \text{S}\longrightarrow\textcircled{S}\text{N} \end{array}$$

A solution to the equation (11) can take the following form:

$$\xi \xrightarrow[\text{by}]{\text{def}} \begin{pmatrix} c \\ \phi_o \\ n\Upsilon_o \end{pmatrix} \diamond \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega t & \sin\omega t \\ 0 & -\sin\omega t & \cos\omega t \end{pmatrix}$$

This solution represents a single rotating disturbance, akin to a propeller, propagating in a direction parallel to the **x**-axis. However, it's worth noting that the extent of the emflux $\boldsymbol{\phi}_o$ is precisely defined by the equation (**??**). According to Theorem 2, an elementary EM-soliton encloses a volume of l_o^3, forming a cylinder with a length of l_o and a radius of $r_o = l_o/\sqrt{2\pi}$. When the center of rotation is at zero potential, then the end of the $\boldsymbol{\phi}_o$ vector exhibits a raised magnetic potential, while the end of the $\boldsymbol{\Upsilon}_o$ vector has an elevated electrical potential.

3 The Family of Elementary Emtrons

An elementary *emtron* m_o—or more precisely, the set $\{m_o, \bar{m}_o, \dot{m}_o, \dot{\bar{m}}_o\}$—is defined as an EM-soliton by Theorem 2. It serves as a solution to the equations $\boldsymbol{\mathcal{M}}(\mathbf{u}, \boldsymbol{\phi}_o, \boldsymbol{\Upsilon}_o)$ and $\boldsymbol{\mathcal{M}}(\mathbf{u}, \overline{\boldsymbol{\phi}}_o, \overline{\boldsymbol{\Upsilon}}_o)$, and represents the least energetic configuration with an action h. The elementary emtron manifests in three distinct topological structures, or flavours:

1. A soliton moving in a straight line at velocity c, exemplified by a photon; this is a special case of the first solution $\boldsymbol{\xi}_1$ of (11) when $\lim \omega_o \to 0$.
2. A soliton in circular motion at velocity c around a stationary point, represented by $\boldsymbol{\xi}_1$ in (11).
3. A soliton in spherular motion at velocity c around a stationary point, described by either $\boldsymbol{\xi}_2$ or $\boldsymbol{\xi}_3$ in (11). Spherular motion follows a geometric path encompassing a sphere and exhibits rotational symmetry.

Experimental evidence, such as electron transitions or electron-positron annihilations, suggests that an emtron can transition between these flavours under certain atomic interactions.

3.1 Elementary Emtrons in Linear Motion: Spin=0

The set of elementary emtrons $\{m_o, \bar{m}_o, \dot{m}_o, \dot{\bar{m}}_o\}$ which serve as the least energetic configuration within the solutions of $\boldsymbol{\mathcal{M}}(\mathbf{u}, \boldsymbol{\phi}_o, \boldsymbol{\Upsilon}_o)$ and $\boldsymbol{\mathcal{M}}(\mathbf{u}, \overline{\boldsymbol{\phi}}_o, \overline{\boldsymbol{\Upsilon}}_o)$ are mathematically characterised as follows:

$$m_o \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} c \\ \phi_o \\ \Upsilon_o \end{pmatrix} \diamond \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \bar{m}_o \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} c \\ \bar{\phi}_o \\ \bar{\Upsilon}_o \end{pmatrix} \diamond \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\dot{m}_o \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} c \\ \phi_o \\ \Upsilon_o \end{pmatrix} \diamond \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{and} \quad \dot{\bar{m}}_o \xrightarrow[\text{by}]{\text{par}} \begin{pmatrix} c \\ \bar{\phi}_o \\ \bar{\Upsilon}_o \end{pmatrix} \diamond \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

In this context, the overaccented bar represents the concept of 'anti' (where the source becomes an absorber), while the underaccented dot signifies 'contra' (indicating a 180-degree rotation). One may further comprehend this family through the following relations:

			$\mathcal{E}$	p	A_o	φ_o
a)	$m_o + \bar{m}_o + \dot{m}_o + \dot{\bar{m}}_o$	$\Rightarrow$	4	0	0	0
b)	$m_o + \bar{m}_o$	$\Rightarrow$	2	0	0	2
c)	$m_o + \dot{\bar{m}}_o$	$\Rightarrow$	2	0	2	0
d)	$m_o + \dot{m}_o$	$\Rightarrow$	2	2	0	0
e)	$\bar{m}_o + \dot{\bar{m}}_o$	$\Rightarrow$	2	−2	0	0

Here, the energy $\mathcal{E} = ht_o$ and the momentum $p = h/c$ are specified, along with magnetic A_o and electric φ_o potentials. These are the quantities carried away by the emtrons resulting from the interactions that produced them.

3.2 Elementary Emtrons in Circular Self-Orbits: Spin=0

The notable finding is that the solutions to $\mathcal{M}$ permit circular self-orbits. As proven by Theorem 1, $\mathcal{M}$ leads to the Maxwell equations. Thus, circular and spherular self-orbits are intrinsic features of electromagnetic phenomena. Circular self-orbits are mathematically described by the following equations:

$$m_{\mathrm{o}}^{\odot} \xrightarrow[\mathsf{by}]{\mathsf{def}} \begin{pmatrix} c \\ \phi_{\mathrm{o}} \\ \Upsilon_{\mathrm{o}} \end{pmatrix} \diamond \begin{pmatrix} \cos\omega_{\mathrm{o}}t & \sin\omega_{\mathrm{o}}t & 0 \\ -\sin\omega_{\mathrm{o}}t & \cos\omega_{\mathrm{o}}t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{pmatrix} \tag{12a}$$

$$\bar{m}_{\mathrm{o}}^{\odot} \xrightarrow[\mathsf{by}]{\mathsf{def}} \begin{pmatrix} c \\ \bar{\phi}_{\mathrm{o}} \\ \bar{\Upsilon}_{\mathrm{o}} \end{pmatrix} \diamond \begin{pmatrix} -\sin\omega_{\mathrm{o}}t & \cos\omega_{\mathrm{o}}t & 0 \\ \cos\omega_{\mathrm{o}}t & \sin\omega_{\mathrm{o}}t & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{pmatrix}$$

$$\dot{m}_{\mathrm{o}}^{\odot} \xrightarrow[\mathsf{by}]{\mathsf{def}} \begin{pmatrix} c \\ \phi_{\mathrm{o}} \\ \Upsilon_{\mathrm{o}} \end{pmatrix} \diamond \begin{pmatrix} \cos\omega_{\mathrm{o}}t & \sin\omega_{\mathrm{o}}t & 0 \\ 0 & 0 & 1 \\ \sin\omega_{\mathrm{o}}t & -\cos\omega_{\mathrm{o}}t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{pmatrix}$$

$$\dot{\bar{m}}_{\mathrm{o}}^{\odot} \xrightarrow[\mathsf{by}]{\mathsf{def}} \begin{pmatrix} c \\ \bar{\phi}_{\mathrm{o}} \\ \bar{\Upsilon}_{\mathrm{o}} \end{pmatrix} \diamond \begin{pmatrix} \sin\omega_{\mathrm{o}}t & -\cos\omega_{\mathrm{o}}t & 0 \\ 0 & 0 & -1 \\ \cos\omega_{\mathrm{o}}t & \sin\omega_{\mathrm{o}}t & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{pmatrix}$$

These emtrons are classified as spin-zero because either $\boldsymbol{\phi}_{\mathrm{o}}$ or $\boldsymbol{\Upsilon}_{\mathrm{o}}$ remains static. To interpret these equations, we introduce an elementary electric emflux vector $\boldsymbol{\Upsilon}_{\mathrm{o}} = c\boldsymbol{\phi}_{\mathrm{o}}$, where $\boldsymbol{\phi}_{\mathrm{o}} = \boldsymbol{\phi}_{\mathrm{o}} l_{\mathrm{o}}^2$. For the emtron pairs specified in (12), we observe that the pair $(m^{\odot}, \bar{m}^{\odot})$ generates an electric emflux $\boldsymbol{\Upsilon}_{\mathrm{o}}$, while the pair $(\dot{m}^{\odot}, \dot{\bar{m}}^{\odot})$ establishes a magnetic emflux $\boldsymbol{\phi}_{\mathrm{o}}$, both aligned along the z-axis. The path of self-orbit lies in the xy-plane, defined by the unit position vector $\mathbf{p} = \int \hat{\mathrm{u}}\,\mathrm{d}t$. For (12a), it evaluates to $\mathbf{p} = \big(\hat{\mathrm{x}}\sin\omega_{\mathrm{o}}t - \hat{\mathrm{y}}\cos\omega_{\mathrm{o}}t\big)/\omega_{\mathrm{o}}$, yielding a circle with a radius of $l_{\mathrm{o}}/2\pi$.

We further note that the unit position vector $\hat{\mathrm{p}}(t)$ aligns with the field vectors for both $m^{\odot}$ and $\dot{\bar{m}}^{\odot}$; specifically, $\hat{\mathrm{p}}(t) = -\hat{\phi}(t)$ and $\hat{\mathrm{p}}(t) = \hat{\Upsilon}(t)$, indicating radial alignment of the respective emfluxes.

The energy of an elementary emtron in a circular self-orbit remains invariant at $\mathcal{E}_{\mathrm{o}} = ht_{\mathrm{o}}$. Equations (12) predict that such emtrons in circular self-orbits within the xy-plane project electric and magnetic emfluxes in the z direction. These emfluxes potentially elucidate the mechanisms behind charge accumulation in capacitors and the nature of permanent magnets as they provide the means for generating electric and magnetic motive forces.

3.3 Emtrons with Rotation and Linear Motion; Spin=1

The action of an emtron with rotation, denoted by m^{γ}, is governed by its rotational frequency. Emtrons in linear motion with rotating fields which are solutions of $\mathcal{M}$ are mathematically expressed as

$$m^{\gamma} \xrightarrow[\mathsf{by}]{\mathsf{def}} \begin{pmatrix} c \\ \phi_{\mathrm{o}} \\ n\Upsilon_{\mathrm{o}} \end{pmatrix} \diamond \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\mathrm{r_a}n\omega_{\mathrm{o}}t & \sin\mathrm{r_a}n\omega_{\mathrm{o}}t \\ 0 & -\sin\mathrm{r_a}n\omega_{\mathrm{o}}t & \cos\mathrm{r_a}n\omega_{\mathrm{o}}t \end{pmatrix}$$

$$\bar{m}^{\gamma} \xrightarrow[\mathsf{by}]{\mathsf{def}} \begin{pmatrix} c \\ \bar{\phi}_{\mathrm{o}} \\ n\bar{\Upsilon}_{\mathrm{o}} \end{pmatrix} \diamond \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos\mathrm{r_a}n\omega_{\mathrm{o}}t & \sin\mathrm{r_a}n\omega_{\mathrm{o}}t \\ 0 & \sin\mathrm{r_a}n\omega_{\mathrm{o}}t & -\cos\mathrm{r_a}n\omega_{\mathrm{o}}t \end{pmatrix}$$

$$\dot{m}^{\gamma} \xrightarrow[\mathsf{by}]{\mathsf{def}} \begin{pmatrix} c \\ \phi_{\mathrm{o}} \\ n\Upsilon_{\mathrm{o}} \end{pmatrix} \diamond \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\cos\mathrm{r_a}n\omega_{\mathrm{o}}t & -\sin\mathrm{r_a}n\omega_{\mathrm{o}}t \\ 0 & \sin\mathrm{r_a}n\omega_{\mathrm{o}}t & -\cos\mathrm{r_a}n\omega_{\mathrm{o}}t \end{pmatrix}$$

$$\dot{\bar{m}}^{\gamma} \xrightarrow[\text{by}]{\text{def}} \begin{pmatrix} c \\ \bar{\phi}_{\text{o}} \\ n\bar{\Upsilon}_{\text{o}} \end{pmatrix} \diamond \begin{pmatrix} -1 & 0 & 0 \\ 0 & -\cos \text{r}_{\text{a}} n\omega_{\text{o}} t & -\sin \text{r}_{\text{a}} n\omega_{\text{o}} t \\ 0 & -\sin \text{r}_{\text{a}} n\omega_{\text{o}} t & +\cos \text{r}_{\text{a}} n\omega_{\text{o}} t \end{pmatrix}$$

These equations describe emtrons where the fields exhibit a rotational velocity $n\omega_{\text{o}}$ while propagating, as illustrated in Figure 2. According to Theorem 2, an elementary emtron carries an elementary electromotive charge e and possesses an action h. Earlier, we introduced Equation (**??**) to describe the action of the EM-wave, restated here as:

$$h_e = \varrho h = \kappa \ell c l_{\text{o}} = \kappa \ell l_{\text{o}}^2 / t_{\text{o}} \tag{14}$$

However, Equation (13) describes EM-disturbances that, in addition to linear propagation, also possess rotational momentum.

Action is traditionally defined as momentum multiplied by distance. However, our system exhibits both linear and rotational motion, necessitating a comprehensive definition of action that includes both components. I introduce the concept of angular-action, defined as the product of angular momentum $L_{\text{o}} = I_{\text{o}}\omega_{\text{o}}$ times the angle θ subtended. Guided by (14) we define the elementary EM-moment of inertia as $I_{\text{o}} = \kappa \ell l_{\text{o}}^2/(2\pi)^2$ (see Figure 2) to give

$$\begin{aligned} h_{\text{rot}} &= I_{\text{o}}\omega_{\text{o}}\theta = \kappa \ell l_{\text{o}}^2 \omega_{\text{o}} \theta/(2\pi)^2 \\ &= \kappa \ell l_{\text{o}}^2 n f_{\text{o}} = n h_e \end{aligned}$$

Thus, an elementary emtron with spin has both linear recoil action h and rotational recoil action h_{rot}, giving it a total energy content of $\mathcal{E} = (h_{\text{rot}} + h)t_{\text{o}}$. This serves as a precise mathematical justification for Planck's energy-frequency equivalence $\mathcal{E} = h_{\text{rot}} f + h f_{\text{o}} = h_{\text{eff}} f_{\text{o}}$, where $h_{\text{eff}} = (n+1)h$.

Additionally, according to Equation (7), we find that $h \propto \left(\|\boldsymbol{\phi}_{\text{o}}\|, \|\boldsymbol{\Upsilon}_{\text{o}}\|\right)$. This implies that the electric field must scale with the factor n, *i.e.*, $\mathbf{E}_{\text{eff}} = (n+1)\boldsymbol{\Upsilon}_{\text{o}}$. Consequently, the effective action of the soliton can also be expressed as $h_{\text{eff}} \propto \left(\|\boldsymbol{\phi}_{\text{o}}\|, \|(n+1)\boldsymbol{\Upsilon}_{\text{o}}\|\right)$.

Remark 3. Instead of scaling by frequency, the energy of the emtron can also be scaled by increasing the magnetic emflux vector's displacement from the centre of rotation to $ml_{\text{o}}/2\pi$, which then gives $h_{\text{eff}} = nm^2 + 1$. Thus, three options for energy scaling are available: rotational frequency $n\omega_{\text{o}}$, radius $ml_{\text{o}}/2\pi$, and the combination of both.

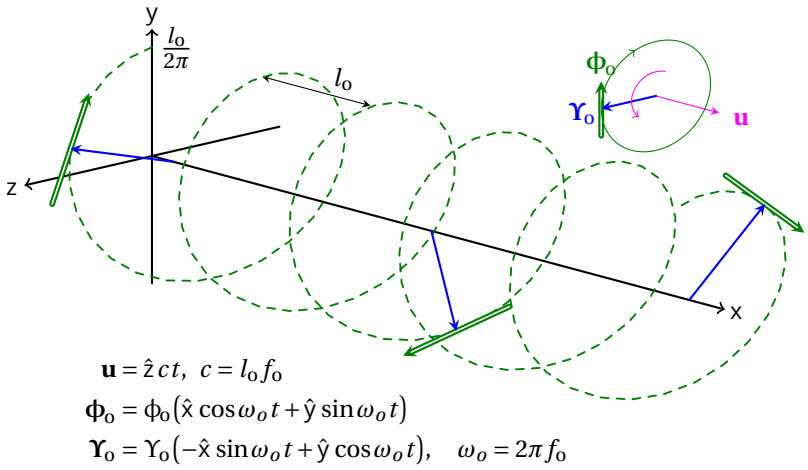

Figure 2: A depiction of a single emtron at three arbitrary positions at times $t = t_1$, t_2, and t_3. The cross product of two vectors does not require the vectors to originate at the same point. So that $I_{\text{o}} = \kappa e l_{\text{o}}^2/(2\pi)^2$ the vector $\boldsymbol{\phi}_{\text{o}}$ is displaced as illustrated

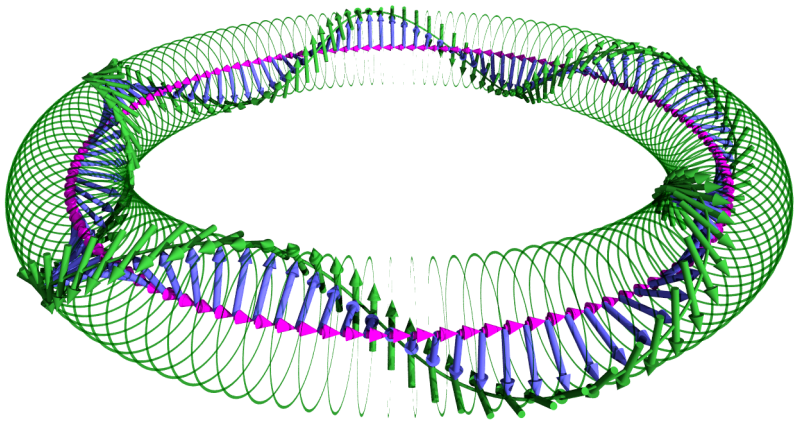

Figure 3: A toroidal eddy. The green represents the magnetic emflux, the emflux vectors $\boldsymbol{\phi}_o$ are tangential to the circles. The distance between each m is l_o.

3.4 Emtrons in Circular Self-Orbits: Spin=1

An elementary toroidal EM-eddy is described by

$$m^{\tau} \xrightarrow[\text{by}]{\text{def}} \begin{pmatrix} c \\ \phi_o \\ \Upsilon_o \end{pmatrix} \diamond \begin{pmatrix} \cos \omega_o t/n & \sin \omega_o t/n & 0 \\ -\cos \omega_o t/n \sin \omega_o t/n & \cos \omega_o t/n \cos \omega_o t/n & \sin \omega_o t/n \\ \sin \omega_o t/n \sin \omega_o t/n & -\sin \omega_o t/n \, \omega_o t/n & \cos \omega_o t/n \end{pmatrix}$$

and has elementary path length nl_o and traces a circle of radius $r_s = nl_o/2\pi$.

Alternatively on a larger scale, if m^{γ} is one charge of an electric current then m^{τ} is one loop of a transformer coil.

3.5 Emtrons in a Spherular Self-Orbit

In preceding sections, we discussed the possibility of spherular self-orbits. Mathematically, an EM-spherular soliton is described by one of the following two equations, or by one of their numerous variants:

$$m^{\oplus} \xrightarrow[\text{by}]{\text{def}} \begin{pmatrix} c \\ \phi_o \\ \Upsilon_o \end{pmatrix} \diamond \begin{pmatrix} \cos 2\omega_o t/m & -\sin 2\omega_o t/m \sin \omega_o t/mn & \sin 2\omega_o t/m \cos \omega_o t/mn \\ 0 & \cos \omega_o t/mn & \sin \omega_o t/mn \\ -\sin 2\omega_o t/m & -\cos 2\omega_o t/m \sin \omega_o t/mn & \cos 2\omega_o t/m \cos \omega_o t/mn \end{pmatrix} \tag{15a}$$

or

$$\dot{m}^{\oplus} \xrightarrow[\text{by}]{\text{def}} \begin{pmatrix} c \\ \phi_o \\ \Upsilon_o \end{pmatrix} \diamond \begin{pmatrix} -\sin 2\omega_o t/m & -\cos 2\omega_o t/m \sin \omega_o t/mn & \cos 2\omega_o t/m \cos \omega_o t/mn \\ \cos 2\omega_o t/m & -\sin 2\omega_o t/m \sin \omega_o t/mn & \sin 2\omega_o t/m \cos \omega_o t/mn \\ 0 & \cos \omega_o t/n & \sin \omega_o t/n \end{pmatrix}$$

where $n \in \{2,3,5,\ldots \text{prime}\}$, and m an integer scaling value. The integral $\mathbf{p} = \int \mathbf{u}\,\mathrm{d}t$ determines the path shape which has a length $2mnl_o$ and encloses a sphere of radius $r_s = ml_o/(2\pi)$.

Because $r_s = l_o 4\pi$, and referencing equation (14), we note that the length of the magnetic emflux vector for an $m^{\oplus}$ is half of that used for calculating the energy content of the m^{γ}. Therefore, the energy content is one-fourth. If the spherule $m^{\oplus}$ is to have the same energetic impact as m^{γ}, then we require four *half*-emtrons evenly distributed along the path.

For $n = x$, $x \in \{2,3,5,\ldots \text{prime}\}$ the spherule can be packed with one to x sets of four *half*-emtrons. This is made clear in Figure-4 that illustrates the paths calculated from $\mathbf{p} = \int \mathbf{u}\,\mathrm{d}t$.

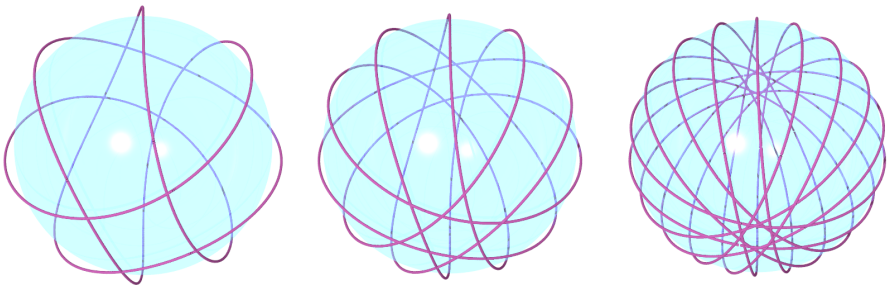

Figure 4: A graphic representation of the paths of the s, p, and d-shells. The visual representation reinforces the number of half-emtrons each shell can support.

4 Space has complex dimensionality

The spherule, as defined by Eq. (15a), is stationary in space because the cross-product equations do not permit linear motion superimposed on the spherular motion. One solution is to consider that the space xyz is complex, denoted by $\mathbb{C}^3$, as already suggested by Theorem 1. Consequently, the vectors in $\boldsymbol{\mathcal{M}}(\mathbf{u}, \boldsymbol{\phi}_o, \boldsymbol{\Upsilon}_o)$ can be complex, necessitating that the charge ℓ also be complex, and it can be shown that the constants ϵ_0 and μ_0 remain real. Extending from our earlier definitions, we generalise the relationship between the quantities as follows:

$$\ell \mapsto \begin{cases} \ell e^{i\alpha} \text{ thus } \phi_o \mapsto \phi_o e^{-i\alpha} \text{ and } \Upsilon_o \mapsto \begin{cases} \Upsilon_o e^{i\alpha}, & \text{if } c \mapsto c e^{i2\alpha} \\ \Upsilon_o e^{-i3\alpha}, & \text{if } c \mapsto c e^{-i2\alpha} \\ \Upsilon_o e^{-i\alpha}, & \text{if } c \mapsto c \end{cases} \\ \qquad\qquad \text{or} \\ \ell e^{-i\alpha} \text{ thus } \phi_o \mapsto \phi_o e^{i\alpha} \text{ and } \Upsilon_o \mapsto \begin{cases} \Upsilon_o e^{i3\alpha}, & \text{if } c \mapsto c e^{i2\alpha} \\ \Upsilon_o e^{-i\alpha}, & \text{if } c \mapsto c e^{-i2\alpha} \\ \Upsilon_o e^{i\alpha}, & \text{if } c \mapsto c \end{cases} \end{cases}$$

To validate this, consider $\ell \mapsto \ell e^{i\alpha}$ and $\phi_o \mapsto \phi_o e^{-i\alpha}$. Then $\Upsilon_o \mapsto \Upsilon_o e^{i\alpha}$ and the action $\epsilon_0 \boldsymbol{\phi}_o \times \boldsymbol{\Upsilon}_o = h$ remains unchanged. Specifically, if $\alpha = \pi/4$, the velocity $u = ic$.

As an illustrative example, we can define a spherular EM-soliton as follows:

$$m^{\oplus} \xrightarrow[\text{by}]{\text{def}} \begin{pmatrix} ic \\ e^{-i\pi/4}\phi_o \\ e^{i\pi/4}\Upsilon_o \end{pmatrix} \diamond \begin{pmatrix} \cos 2\omega_o t & -\sin 2\omega_o t \sin \omega_o t/2n & \sin 2\omega_o t \cos \omega_o t/2n \\ 0 & \cos \omega_o t/2n & \sin \omega_o t/2n \\ -\sin 2\omega_o t & -\cos 2\omega_o t \sin \omega_o t/2n & \cos 2\omega_o t \cos \omega_o t/2n \end{pmatrix}$$

It has real energy and an imaginary velocity. Alternatively, we can define

$$m^{\gamma} \xrightarrow[\text{by}]{\text{def}} \begin{pmatrix} c \\ e^{i\pi/4}\phi_o \\ e^{i\pi/4}\Upsilon_o \end{pmatrix} \diamond \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \omega_o t/2n & \sin \omega_o t/2n \\ 0 & -\sin \omega_o t/2n & \cos \omega_o t/2n \end{pmatrix}$$

In this case, the energy is imaginary or reactive, while the velocity is real. Working towards a profound result, we combine these through superposition and scaling, to derive:

$$m_{\mathrm{a}} \xrightarrow[\mathrm{by}]{\mathrm{def}} \begin{cases} m_{\mathrm{a}}^{\oplus} \xrightarrow[\mathrm{by}]{\mathrm{def}} \begin{pmatrix} i\cos\theta c \\ \sqrt{\sec\theta}e^{-i\pi/4}\phi_{\mathrm{o}} \\ \cos\theta\sqrt{\sec\theta}e^{i\pi/4}\Upsilon_{\mathrm{o}} \end{pmatrix} \diamond \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \\ \text{in superposition with} \\ m_{\mathrm{a}}^{\gamma} \xrightarrow[\mathrm{by}]{\mathrm{def}} \begin{pmatrix} \sin\theta c \\ \sqrt{\sec\theta}e^{i\pi/4}\phi_{\mathrm{o}} \\ \sin\theta\sqrt{\sec\theta}e^{i\pi/4}\Upsilon_{\mathrm{o}} \end{pmatrix} \diamond \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \end{cases}$$

From our definitions we have $h = \epsilon_0 \boldsymbol{\phi}_{\mathrm{o}} \times \boldsymbol{\Upsilon}_{\mathrm{o}}$ implying that the energy is proportional to the product of the magnetic and electric emflux magnitudes. For the superposition defined above, we evaluate the energy as:

$$\mathcal{E}_\tau = \mathcal{E}_{\mathrm{o}}\left(\sqrt{\sec\theta}e^{-i\pi/4} \times \cos\theta\sqrt{\sec\theta}e^{i\pi/4}\right)$$
$$\mathcal{E}_\gamma = \mathcal{E}_{\mathrm{o}}\left(\sqrt{\sec\theta}e^{i\pi/4} \times \sin\theta\sqrt{\sec\theta}e^{i\pi/4}\right)$$
$$\vec{\mathcal{E}}_{\mathrm{a}} = \mathcal{E}_\tau + \mathcal{E}_\gamma = \mathcal{E}_{\mathrm{o}}\left(1 + i\frac{\sin\theta}{\cos\theta}\right)$$

Here the over-accented arrow $\vec{\mathcal{E}}_{\mathrm{a}}$ indicates structural energy Since $u_\gamma = c\sin\theta$ and $u_\oplus = ic\cos\theta$, we can calculate the perceived energy:

$$\mathcal{E}_{\mathrm{A}} = \left|\vec{\mathcal{E}}_{\mathrm{a}}\right| = \mathcal{E}_\tau\sqrt{\frac{c^2}{c^2 - u_\gamma^2}} \tag{16}$$

Proceeding with this established energy $\mathcal{E}_{\mathrm{A}}$, if we increment the real velocity u_γ by $\mathrm{d}u_\gamma$ then we obtain:

$$\mathcal{E}_{\mathrm{A}} + \mathrm{d}\mathcal{E}_{\mathrm{A}} = \mathcal{E}_\tau\sqrt{1 + \frac{(u_\gamma + \mathrm{d}u_\gamma)^2}{c^2 - (u_\gamma + \mathrm{d}u_\gamma)^2}}$$

therefore

$$\mathrm{d}\mathcal{E}_{\mathrm{A}} = \mathcal{E}_\tau\sqrt{1 + \frac{(u_\gamma + \mathrm{d}u_\gamma)^2}{c^2 - (u_\gamma + \mathrm{d}u_\gamma)^2}} - \mathcal{E}_\tau\sqrt{1 + \frac{u_\gamma^2}{c^2 - u_\gamma^2}}$$

and performing a series expansion on $\mathrm{d}\mathcal{E}_{\mathrm{A}}$ gives

$$\mathrm{d}\mathcal{E}_{\mathrm{A}} = \mathcal{E}_\tau\frac{c\,u_\gamma\,\mathrm{d}u_\gamma}{(c^2 - u_\gamma^2)^{3/2}} + \mathcal{O}[\mathrm{d}u_\gamma^2]$$

Energy = force × distance and force is defined by Newton's second law of motion, hence we also have

$$\mathrm{d}\mathcal{E}_{\mathrm{Newton}} = m_{\mathrm{i}}\frac{\mathrm{d}u_\gamma}{\mathrm{d}t}\,u_\gamma\mathrm{d}t$$

where m_{i} is the inertial mass. Equating $\mathrm{d}\mathcal{E}_{\mathrm{Newton}} = \mathrm{d}\mathcal{E}_{\mathrm{A}}$ we obtain after cancelling common terms

$$m_{\mathrm{i}} = \mathcal{E}_\tau\frac{c}{(c^2 - u_\gamma^2)^{3/2}}$$

and if $u_\gamma = 0$ the above reduces to

$$\mathcal{E}_\tau = mc^2 \tag{17}$$

and using (16) it follows trivially that

$$\mathcal{E}_{\mathrm{A}} = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

The above discussion is not complete without noting that $E_\gamma + \mathcal{E}_\tau > \sqrt{E_\gamma^2 + \mathcal{E}_\tau^2}$, meaning that when m_{a} undergoes further acceleration, energy is released in some form or other, such as radiation.

Having successfully demonstrated that mass is an emergent property of the electromagnetic phenomenon—made possible only by the transformation $\ell \mapsto \ell e^{i\pi/4}$—we establish that a complex charge is a physical reality. We proceed to define the *complexification constants* $X_{\mathrm{o}} = e^{i\pi/4}$ and $\overline{X}_{\mathrm{o}} = -X_{\mathrm{o}} = e^{i5\pi/4}$ as physical variables. It is worth noting that although the electromagnetic phenomenon itself is not dependent on this value, its manifestations undoubtedly are.

5 Efficacy, Presence and Distant Interactions

It is now clear that a distinction must be made between motive EM-fields and static EM-fields. These two categories represent different facets of the electromagnetic phenomenon. A motive EM-field (for example, an electric field established between two plates of a capacitor) serves as an accelerating field. This field facilitates the transfer of energy between itself and an emtron with which it interacts. Conversely, a static EM-field alters the EM-structure of an emtron without affecting the structural energy of the emtron.

Contrary to the discussion in the previous section, consider the superposition described by:

$$m_{\mathrm{b}} \xrightarrow[\text{by}]{\text{def}} \begin{cases} m_{\mathrm{b}}^{\oplus} \xrightarrow[\text{by}]{\text{def}} \begin{pmatrix} i\cos\theta c \\ e^{-i\pi/4}\phi_{\mathrm{o}} \\ \cos\theta e^{i\pi/4}\Upsilon_{\mathrm{o}} \end{pmatrix} \diamond \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \\ \text{in superposition with} \\ m_{\mathrm{b}}^{\gamma} \xrightarrow[\text{by}]{\text{def}} \begin{pmatrix} \sin\theta c \\ e^{i\pi/4}\phi_{\mathrm{o}} \\ \sin\theta e^{i\pi/4}\Upsilon_{\mathrm{o}} \end{pmatrix} \diamond \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \end{cases} \tag{18}$$

This superposition yields

$$\vec{\mathcal{E}}_{\mathrm{b}} = \mathcal{E}_\tau + \mathcal{E}_\gamma = \mathcal{E}_{\mathrm{o}}(\cos\theta + i\sin\theta)$$

Interestingly, the perceived energy remains $\mathcal{E}_{\mathrm{B}} = \mathcal{E}_{\mathrm{o}}$, even when the emtron exhibits varying real velocity. While this result contradicts our limited experiential understanding, it persists and provides, I believe, a classical interpretation for the Aharonov-Bohm effect [1]. The significance of (18) is that it represents outcomes of interactions with static fields that do not change the energy content.

In this section, two new terms are introduced: *efficacy*, denoted by $\mathcal{F}$, and *presence*, denoted by $\mathcal{P}$. Until this point, the paper has focused primarily on the efficacy of the EM-phenomenon, that is collectively everything regarding the emtron's energetic structure. The emtron signals its state to the rest of the universe via its presence. That means that the presence of an emtron alters the nature of a vacuum.

Let us consider that the universe is composed of

$$\text{Universe} = \sum_{i=1}^{n} (m_i + \bar{m}_i + \dot{m}_i + \dot{\bar{m}}_i)$$

This corresponds to an energy $\mathcal{E}_U = 4n\mathcal{E}_\mathrm{o}$, which exhibits no net recoil upon creation; in other words, $\mathcal{E}_U$ as a whole is at rest, with neither movement nor rotations. Importantly, this energy $\mathcal{E}_U$ remains invariant, regardless of any interactions that may occur between emtrons.

In summary, the interaction outcome between two directly interacting emtrons is determined by both their efficacies and presences. Moreover, their presences also simultaneously govern the states of all other emtrons. The implications of this are profound: each entity is entangled with every other entity, thereby ensuring that the universe as a whole remains at rest.

We now formalise the EM-presence: An emtron efficacy, defined by $\boldsymbol{\mathcal{M}}(\mathbf{u}, \boldsymbol{\phi}_\mathrm{o}, \boldsymbol{\Upsilon}_\mathrm{o})$, is invariably accompanied by $\boldsymbol{\mathcal{M}}(\mathbf{u}, \boldsymbol{\Lambda}_\mathrm{o}, \boldsymbol{\Xi}_\mathrm{o})$, where $\boldsymbol{\Lambda}_\mathrm{o}$ and $\boldsymbol{\Xi}_\mathrm{o}$ are the magnetic and electric vector latencies, respectively. Given that the emflux vectors $\boldsymbol{\phi}_\mathrm{o}$ and $\boldsymbol{\Upsilon}_\mathrm{o}$ are defined from zero to a length l_o, these vector latencies extend the range from l_o to infinity.

Here, I introduce a new physical unit termed latentness, symbolised by $\mathcal{U}$ (pronounced inverse-h) with units expressed in changelings and dimensional characteristics $\mathrm{J}^{-1}\,\mathrm{s}^{-1}$. The numerical value of latentness is set to be the reciprocal of the Planck constant.

In a system involving two emtrons, the latentness of emtron-A influences the structural energy of emtron-B, and vice versa. Specifically, if the structural energy is denoted as $\vec{\mathcal{E}}_\mathrm{b} = \mathcal{E}_\mathrm{b} e^{i\beta}$, then latentness serves to modify the value of β.

We define the magnetic and electric latencies as follows:

$$\Lambda_\mathrm{o} = \frac{l_\mathrm{o}\mathcal{U}}{rh}\phi_\mathrm{o} = \frac{l_\mathrm{o}\mathcal{U}}{r\kappa\ell} \quad \text{and} \quad \Xi_\mathrm{o} = \frac{l_\mathrm{o}\mathcal{U}}{rh}\Upsilon_\mathrm{o} = \frac{cl_\mathrm{o}\mathcal{U}}{r\kappa\ell}$$

where $r = \rho l_\mathrm{o}$ represents a distance expressed in multiples ρ of the elementary length l_o. The definition of latencies necessitates the definition of latent permittivity and latent permeability, given by:

$$\tilde{\epsilon}_0 = \frac{\kappa^2\ell^2}{\mathcal{U}c} \quad \text{and} \quad \tilde{\mu}_0 = \frac{\mathcal{U}}{c\kappa^2\ell^2}$$

Here, the over-accented tilde indicates latency. In a previous section, we demonstrated that $h = \epsilon_0\left|\boldsymbol{\phi}_\mathrm{o} \times \boldsymbol{\Upsilon}_\mathrm{o}\right|$, having treated $\boldsymbol{\phi}_\mathrm{o}$ and $\boldsymbol{\Upsilon}_\mathrm{o}$ as impulses. The analogous calculation for the vector latencies involves evaluating the integral

$$\mathcal{U} = \int_\infty^{l_\mathrm{o}} \tilde{\epsilon}_0 |\boldsymbol{\Lambda}_\mathrm{o} \times \boldsymbol{\Xi}_\mathrm{o}|\,\mathrm{d}r$$

Latentness is responsible for the entanglement phenomenon, famously derided as "spooky action at a distance" [2]. Latentness causes changes in the wave structure of the emtrons subjected to it.

We are now in a position to define the presence $\mathcal{P}_\mathrm{o}$ as a function of location $r = \rho l_\mathrm{o}$. This is expressed in terms of the electromagnetic emfluxes and latencies, as follows:

$$\mathcal{P}_\mathrm{o}(r) = \mathcal{E}_\mathrm{o}\left(\epsilon_0\left|\boldsymbol{\phi}_\mathrm{o} \times \boldsymbol{\Upsilon}_\mathrm{o}\right|\right)\int_\infty^{\rho l_\mathrm{o}} \left(\tilde{\epsilon}_0 |\boldsymbol{\Lambda}_\mathrm{o} \times \boldsymbol{\Xi}_\mathrm{o}|\right)\mathrm{d}r = \frac{1}{\rho}$$

This evaluates to a complex scalar quantity that specifies the strength of the latentness at that particular point. In Question 1, I proposed that space possesses an additional property known as transportivity, which is defined as $\mathcal{T} = c^2$ in a vacuum. The presence modifies the transportivity according to the equation:

$$\mathcal{T} = c^2(1 - \mathcal{P}_\mathrm{o}(r)) = c^2(1 - \frac{1}{\rho}) \tag{19}$$

This is interpreted to mean that the speed of light at ρ is $c_\rho = ce^{i\beta}$, where $\sin\beta = \pm\sqrt{1/\rho}$. The sign of the angle β, or the sign of the imaginary component of c_ρ, is determined by the type of emtron—either positive for m or negative for $\bar{m}$

This concludes the foundational work required to analyse action at a distance. Let us consider two emtrons m_A and m_B separated by a distance $r = \rho l_\mathrm{o}$. Their energies are $\mathcal{E}_\mathrm{A} = n_\mathrm{a}\mathcal{E}_\mathrm{o}$ and $\mathcal{E}_\mathrm{B} = n_\mathrm{b}\mathcal{E}_\mathrm{o}$, respectively. The respective presences are scaled in the same proportions, thus $\mathcal{P}_\mathrm{A} = n_\mathrm{a}\mathcal{P}_\mathrm{o}(r)$ and $\mathcal{P}_\mathrm{B} = n_\mathrm{b}\mathcal{P}_\mathrm{o}(r)$.

Earlier, equation (17) established that energy is proportional to the square of the speed of light. Coupled with equation (19), this allows for the following manipulation:

$$\vec{\mathcal{E}}_\mathrm{A} = \mathcal{E}_\mathrm{A}\big(\cos\beta_\mathrm{b} + i\sin\beta_\mathrm{b}\big)$$

where $\sin\beta_\mathrm{b} = \sqrt{n_\mathrm{b}/\rho}$. This yields the perceived energy as:

$$\mathcal{E}_\mathrm{A} = \mathcal{E}_\mathrm{A}\big(\cos^2\beta_\mathrm{b} + \sin^2\beta_\mathrm{b}\big)$$

We now define $\mathcal{E}'_\mathrm{A} = \mathcal{E}_\mathrm{A}\cos^2\beta_\mathrm{b}$ and, after rearrangement, obtain:

$$\mathcal{E}'_\mathrm{A} = n_\mathrm{a}\mathcal{E}_\mathrm{o}\left(1 - \frac{n_\mathrm{b}}{\rho}\right)$$

$$\mathcal{E}'_\mathrm{B} = n_\mathrm{b}\mathcal{E}_\mathrm{o}\left(1 - \frac{n_\mathrm{a}}{\rho}\right)$$

Force is the product of energy and distance. Differentiating with respect to ρ yields:

$$F_\mathrm{A} = F_\mathrm{B} = \frac{n_\mathrm{a}n_\mathrm{b}}{\rho^2}$$

This is a well-known result; we shall determine its direction next.

With Theorem 1 and equation (**??**), we established that the electrical action is $h_e = \varrho h = \kappa\ell l_\mathrm{o} c$. This implies that the energy of an emtron is also proportional to its charge. Therefore, it follows that

$$\begin{aligned} F_\mathrm{A} &= k_e \frac{c_\mathrm{a}}{|c_\mathrm{a}|}\frac{n_\mathrm{a}\ell X_\mathrm{a} n_\mathrm{b}\ell X_\mathrm{b}}{\rho^2}, \\ F_\mathrm{B} &= k_e \frac{c_\mathrm{b}}{|c_\mathrm{b}|}\frac{n_\mathrm{a}\ell X_\mathrm{a} n_\mathrm{b}\ell X_\mathrm{b}}{\rho^2}, \end{aligned} \tag{20}$$

The factors $c_\mathrm{a}/|c_\mathrm{a}|$ and $c_\mathrm{b}/|c_\mathrm{b}|$ are necessary to define the forces in a manner that accommodates complex velocities.

For the stationary emtron $m^{\oplus}_\mathrm{A}$, we find that $c_\mathrm{a}/|c_\mathrm{a}| = i$. To ascertain the direction of the forces, we examine the products $iX_\mathrm{a}X_\mathrm{b}$. Recalling the definitions for the *complexification constants* $X_\mathrm{o} = e^{i\pi/4}$ and $\overline{X}_\mathrm{o} = -X_\mathrm{o} = e^{i5\pi/4}$, we have

$$iX_\mathrm{o}X_\mathrm{o} = -1, \quad iX_\mathrm{o}\overline{X}_\mathrm{o} = 1, \quad i\overline{X}_\mathrm{o}\overline{X}_\mathrm{o} = -1,$$

which demonstrates that like charges repel, while oppositely charged emtrons attract each other.

We have now recovered Coulomb's law, which serves as a special case of equation (20) when the charges are stationary. The inclusion of the Coulomb constant k_e was made retrospectively.

Let us consider two identical emtrons $m^{\oplus}_\mathrm{A} = m^{\oplus}_\mathrm{B} = m^{\oplus} + \bar{m}^{\oplus}$. From our previous discussions, it is evident that the sum of all forces on these emtrons will be zero. However, if we break the charge symmetry such that $X_\mathrm{o} = e^{i\pi/4+\delta_\mathrm{x}}$ and $\overline{X}_\mathrm{o} = e^{i5\pi/4-\delta_\mathrm{x}}$, the ensuing equation is:

$$i\big(X_\mathrm{o}X_\mathrm{o} + 2X_\mathrm{o}\overline{X}_\mathrm{o} + \overline{X}_\mathrm{o}\overline{X}_\mathrm{o}\big) = 4\delta_\mathrm{x}^2 \qquad \text{for } \delta_\mathrm{x} \ll 1,$$

This reveals not only the unification of the gravitational force with the electric force, but also introduces another universal constant, δ_x. In Appendix B, I offer a classical explanation for the precession of planetary orbits using a method derived from what was developed in this section.

6 Proton and Electron Orbits

Convention, as an example, symbolises a sodium isotope as ${}^{23}_{11}\mathrm{Na}$ indicating an atomic mass number of 23 and a charge of 11. This is interpreted that the nucleus is a bundle of 11 protons and 12 neutrons bound together by the strong force, but that is far from the truth. Specifically, there is no clear understanding of how to describe an atom as a comprehensive model where both the nucleus and the electron shells are described as a unified *onion*-shell model.

In this work, I propose the onion-shell model for atomic nuclei. Appendix C provides a partial list—for the isotopes of elements from hydrogen through to cadmium—and the rules for the progression of filling these nuclei shells. Each shell is numbered 1 through 8 with sub-shells labelled $\{s, p, d, f, g\}$. A well-defined algorithmic method is established, demonstrating the progression of the atomic mass number for each element and its isotopes. The electron shell packing is well understood using the same terminology but limited to the $\{s, p, d, f\}$ sub-shells.

Guided by Section 3.5, we define the sub-shells and their capacities as

$$s, p, d, f, g = \{1, 3, 5, 7, 11\}$$

which are populated by sets of four half-emtrons in the following sequence:

$$\begin{aligned}
&\big(1s + 2(s,p) + 3(s,p,d) + 4(s,p,d,f) + 5(s,p,d,f,g)\big) \times 4 \times 1/2m \\
&+\big(6(s,p,d,f,g)\big) \times 8 \times 1/2m \\
&+\big(7(s,p,d,f,g)\big) \times 16 \times 1/2m
\end{aligned}$$

Each half-emtron has an action $h_{[m/2]} = h/4$ but carries an equivalent charge of $\ell/2$. The p-shell is populated with tuples $\{3,1\}$, $\{2,2\}$, $\{1,3\}$ of $\{m, \bar{m}\}$, resulting in a charge of $\{-1, 0, +1\}$. As an example, we denote this as ${}^{3\ulcorner}_{1\llcorner}3p^{7}_{5}$, indicating a $3p$ sub-shell in the third onion shell of an atom's nucleus and its contribution to the atomic mass number is $3 = (7+5)/4$ and has a charge of $1 = (7-5)/2$. The left super- and subscript indicate the contributions to the atomic mass number and charge (atomic number), while the right super- and subscript specify the ratio of positive and negative half-charges.

The shell packing configurations for Hydrogen through to Carbon are excerpted from Appendix C:

$$\begin{aligned}
{}^{1}_{1}\mathrm{H} &= {}^{1\ulcorner}_{1\llcorner}1s^{3}_{1} \\
{}^{2}_{1}\mathrm{H} &= {}^{1\ulcorner}_{0\llcorner}1s^{2}_{2} + {}^{1\ulcorner}_{1\llcorner}2s^{3}_{1} \\
{}^{3}_{1}\mathrm{H} &= {}^{1\ulcorner}_{1\llcorner}1s^{3}_{1} + {}^{1\ulcorner}_{-1\llcorner}2s^{1}_{3} + {}^{1\ulcorner}_{1\llcorner}2p^{3}_{1} \\
{}^{4}_{2}\mathrm{He} &= {}^{1\ulcorner}_{1\llcorner}1s^{3}_{1} + {}^{1\ulcorner}_{0\llcorner}2s^{2}_{2} + {}^{2\ulcorner}_{1\llcorner}2p^{5}_{3} \\
{}^{5}_{2}\mathrm{He} &= {}^{1\ulcorner}_{1\llcorner}1s^{3}_{1} + {}^{1\ulcorner}_{0\llcorner}2s^{2}_{2} + {}^{3\ulcorner}_{1\llcorner}2p^{7}_{5} \\
{}^{6}_{3}\mathrm{Li} &= {}^{1\ulcorner}_{1\llcorner}1s^{3}_{1} + {}^{1\ulcorner}_{1\llcorner}2s^{3}_{1} + {}^{3\ulcorner}_{1\llcorner}2p^{7}_{5} + {}^{1\ulcorner}_{0\llcorner}3s^{2}_{2} + {}^{0\ulcorner}_{0\llcorner}3p \\
{}^{7}_{3}\mathrm{Li} &= {}^{1\ulcorner}_{1\llcorner}1s^{3}_{1} + {}^{1\ulcorner}_{1\llcorner}2s^{3}_{1} + {}^{2\ulcorner}_{1\llcorner}2p^{5}_{3} + {}^{1\ulcorner}_{-1\llcorner}3s^{1}_{3} + {}^{2\ulcorner}_{1\llcorner}3p^{5}_{3}
\end{aligned}$$

$$^{9}_{4}\mathrm{Be} = {}^{5}_{2}\mathrm{He} + {}^{1\lceil}_{1\lfloor}3s^{3}_{1} + {}^{3\lceil}_{1\lfloor}3p^{7}_{5}$$

$$^{10}_{5}\mathrm{B} = {}^{5}_{2}\mathrm{He} + {}^{1\lceil}_{1\lfloor}3s^{3}_{1} + {}^{2\lceil}_{1\lfloor}3p^{5}_{3} + {}^{2\lceil}_{1\lfloor}3d^{5}_{3}$$

$$^{12}_{6}\mathrm{C} = {}^{1\lceil}_{1\lfloor}1s^{3}_{1} + {}^{1\lceil}_{1\lfloor}2s^{3}_{1} + {}^{3\lceil}_{1\lfloor}2p^{7}_{5} + {}^{1\lceil}_{1\lfloor}3s^{3}_{1} + {}^{3\lceil}_{1\lfloor}3p^{7}_{5} + {}^{3\lceil}_{1\lfloor}3d^{7}_{5}$$

$$^{13}_{6}\mathrm{C} = {}^{1\lceil}_{1\lfloor}1s^{3}_{1} + {}^{1\lceil}_{1\lfloor}2s^{3}_{1} + {}^{3\lceil}_{1\lfloor}2p^{7}_{5} + {}^{1\lceil}_{1\lfloor}3s^{3}_{1} + {}^{3\lceil}_{1\lfloor}3d^{7}_{5} + {}^{5\lceil}_{1\lfloor}3d^{7}_{5}$$

$$^{14}_{6}\mathrm{C} = {}^{1\lceil}_{1\lfloor}1s^{3}_{1} + {}^{1\lceil}_{1\lfloor}2s^{3}_{1} + {}^{3\lceil}_{1\lfloor}2p^{7}_{5} + {}^{1\lceil}_{1\lfloor}3s^{3}_{1} + {}^{3\lceil}_{1\lfloor}3p^{7}_{5} + {}^{5\lceil}_{1\lfloor}3d^{11}_{9}$$

It is evident that the shell sequence { s, p, d, f, g } is fundamental and when the first five sets are filled in the sequence $1s+2(s,p)+3(s,p,d)+4(s,p,d,f)+5(s,p,d,f,g)$ they define either the isotope $^{57}_{26}\mathrm{Fe}$ or $^{57}_{25}\mathrm{Mn}$.

The next shell $6(s,p,d,f,g)$ is then filled with groups of eight half-emtrons instead of the sets of four of the earlier, beginning with Cobalt $^{59}_{27}\mathrm{Co} = {}^{57}_{25}\mathrm{Mn} + {}^{2\lceil}_{2\lfloor}6s^{6}_{2}$. The seventh shell is filled in groups of 16 half-emtrons establishing that the shell packings for gold and bismuth are

$$\begin{aligned}^{197}_{79}\mathrm{Au} = {}^{57}_{25}\mathrm{Mn} &+ {}^{2\lceil}_{2\lfloor}6s^{6}_{2} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{22\lceil}_{10\lfloor}6g^{54}_{34}\\ &+ {}^{4\lceil}_{4\lfloor}7s^{12}_{4} + {}^{12\lceil}_{4\lfloor}7p^{28}_{20} + {}^{20\lceil}_{4\lfloor}7d^{44}_{36} + {}^{28\lceil}_{4\lfloor}7f^{60}_{52} + {}^{22\lceil}_{10\lfloor}7g^{54}_{34}\end{aligned}$$

$$\begin{aligned}^{209}_{83}\mathrm{Bi} = {}^{57}_{25}\mathrm{Mn} &+ {}^{2\lceil}_{2\lfloor}6s^{6}_{2} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{22\lceil}_{10\lfloor}6g^{54}_{34}\\ &+ {}^{4\lceil}_{4\lfloor}7s^{12}_{4} + {}^{12\lceil}_{4\lfloor}7p^{28}_{20} + {}^{20\lceil}_{4\lfloor}7d^{44}_{36} + {}^{28\lceil}_{4\lfloor}7f^{60}_{52} + {}^{34\lceil}_{14\lfloor}7g^{82}_{54}\end{aligned}$$

Completing the 7^{th} shell would result in an atomic mass number of 219. I have not enough data to guide the shell configurations for elements with atomic mass higher than that of bismuth, such as $^{238}_{92}\mathrm{U}$.

The shell packing model is a convenient visualisation aid, but reality is different. Recalling $h = \epsilon_0 \left|\boldsymbol{\Phi}_\mathrm{o} \times \boldsymbol{\Upsilon}_\mathrm{o}\right| = \epsilon_0 c \left|\boldsymbol{\Phi}_\mathrm{o}\right|^2$ which we now express as

$$h = \int_0^{xl_\mathrm{o}} \epsilon_0 c \frac{\left|\boldsymbol{\Phi}_\mathrm{o}\right|^2}{x}, \mathrm{d}r$$

Therefore, a single $m^{\oplus}$ can inflate its volume and keep its energy content invariant. But one can also do the calculation for n emtrons in superposition as

$$nh = \int_0^{nxl_\mathrm{o}} \epsilon_0 c \frac{\left|\boldsymbol{\Phi}_\mathrm{o}\right|^2}{x}, \mathrm{d}r$$

therefore the superposition of two s-orbitals gives $^{1\lceil}_{1\lfloor}1s^{3}_{1} + {}^{1\lceil}_{1\lfloor}1s^{3}_{1} \rightarrow {}^{2\lceil}_{2\lfloor}\ddot{s}^{6}_{2}$ which is a fusion of the two. This explains the strong force. Once the initial repulsive energies are overcome, the fused result is extremely hard if not impossible to fission. However, the superposition of $\Leftarrow {}^{1\lceil}_{1\lfloor}1s^{3}_{1} + {}^{1\lceil}_{0\lfloor}1s^{2}_{2} \rightarrow {}^{2\lceil}_{1\lfloor}\ddot{s}^{5}_{3}$ would require a minimal force to fuse (no repulsive charge forces here) and possibly a weaker force to fission, as this configuration may prove to be unstable.

Having successfully described the atomic nuclei, we now shift our focus to the electron. Unlike the atomic nuclei, the electron is generally considered as a point particle but assumed to have a finite radius. Drawing upon insights from previous sections, I propose that an electron behaves as a photon-like particle that propagates in an imaginary direction when observed in a state of rest. This perspective immediately accounts for the three generations observed in the Standard Model. Earlier, in Section 5 I demonstrated that particles can be considered as solitons travelling with an imaginary speed of light when observed at rest.

We can represent the electron by m^{γ}, the muon by $m^{\odot}$, and the tau by $m^{\oplus}$, each propagating with an imaginary speed of light when in a state of rest.

The packing orders for the $\{s, p, d, f\}$ electron orbits are well-established. Given that each electron can be described as a pair of half-emtrons, it becomes clear why the s-orbit contains two electrons and the p-orbit can accommodate six, and so forth. The kinetic and potential energy of the electron modulate the radius of its orbit in order to satisfy the Bohr conditions.

In conclusion, both the atomic nucleus and the electron shell can be understood as a superposition of synchronous three-dimensional wave structures. These structures share a common center, and a common emflux vector $\boldsymbol{\phi}_o$. In effect forming a spherical onion-shell structure.

If the proton is described by ${}^{1\lceil}_{1\lfloor}1s^3_1$ then we can describe a neutron as ${}^{1+e\lceil}_{0\lfloor}1p^3_{1+2(1/2e)}$. The temporary $1p$ shell requires extra potential, so when the neutron decays to a proton, the elevated potential is released in kinetic energy of the ejected electron requiring a recoil, namely the neutrino. A p-shell is required because a proton plus an electron constitute six half charges which cannot fit in an s-shell. This explanation is consistent with the onion shell proton packing.

7 General Discussion

Where should I begin? The mathematical evidence is compelling in its own right!

This paper conclusively demonstrates that electrostatic and electromotive fields are two distinct phenomena. The immediate implication is that Quantum Electrodynamics (QED) is incomplete. Calculations in QED, guided by Feynman diagrams, must be expanded to include electromagnetic potential energy. It is inconceivable to assume that the outcomes of atomic interactions are invariant of electromagnetic potential energy. In my opinion, any efforts to achieve sustainable and controllable fusion must address this issue, particularly the need to neutralise self-generated electromotive effects—should they exist in the plasma—in the effort to maintain plasma stability.

The latencies that generate the presence of a particle provide an explanation for the phenomenon of entanglement. Two entangled photons are bathed by the other's presence. As each photon experiences different environmental conditions, the degree of their entanglement diminishes over time. This process of de-entanglement explains why outcomes that initially are causal later manifest as probabilistic upon observation. In essence, this gives free will, which a purely deterministic interpretation would negate. In my opinion, developers of quantum computers will inevitably encounter a signal-to-noise ratio threshold, as it is impossible to shield against the ever-varying presence field emanating from a constantly, and randomly, changing environment.

I shall diplomatically refrain from expressing any further opinions at this juncture. From a generalised philosophical perspective, it suffices to state that this work provides an explanation, using classic methods, for all major discoveries that have shaped our current understanding of nature. Furthermore, it establishes the framework for a comprehensive theory of unified forces and perhaps even paves the way for the formulation of a grand theory of everything, should such a theory ever be attainable.

8 Energy production by isotope transmutation in electro-weak reactions

Transmutation of the elements are widely reported. Vysotskii [3] studied transmutations in biological systems, Cardone [4] subjected iron bars to sonic excitation and observed neutron emissions and he reports the presence of transmuted elements. Transmutation in low energy nuclear reactions using deuterium are extensively reported, a recent summary

is provided by Srinivasan [5]; common to all reactions is that deuterium is used as a donor catalyst; that is deuterium transmutes to hydrogen releasing a 'neutron'. In reactions using palladium and deuterium the isotope composition of the palladium was altered; as a rule, the ratio of ${}^{106}_{46}\mathrm{Pd}$ to ${}^{108}_{46}\mathrm{Pd}$ was decreased. This is explained by the reaction ${}^{106}_{46}\mathrm{Pd} + 2\times{}^{2}_{1}\mathrm{H} \Rightarrow {}^{108}_{46}\mathrm{Pd} + 2\times{}^{1}_{1}\mathrm{H} + 11.3\mathrm{MeV}$, the deuterium donated two 'neutrons' and the palladium captured them.

Here I propose to exploit an isotope transmutation within the element's crystal structure were an isotope acts as the donor and a neighbouring isotope of the same element captures it. This phenomenon if controlled is exploitable for sustainable energy production. As an example, the charge neutral isotope transmutation in zinc $2\times{}^{67}_{30}\mathrm{Zn} \Rightarrow {}^{66}_{30}\mathrm{Zn} + {}^{68}_{30}\mathrm{Zn}$ releases 3.15MeV, that is about one eighth of the energy produced by the fusion of hydrogen to helium reaction, *i. e.* $4\times{}^{2}_{1}\mathrm{H} \Rightarrow {}^{4}_{2}\mathrm{He} + 2e + 2\nu + 26.7\mathrm{MeV}$. The same reaction in germanium releases 3.41MeV, and in zirconium 1.44MeV. These elements are abundantly available. Furthermore, because isotope transmutations are charge neutral means that a low temperature electroweak reaction is possible.

The above proposed isotope transmutation reaction, I believe, was unknowingly demonstrated at a conference in 2017 by the SAFIRE project team [6]. Childs reported that 180 watts of power in a plasma reactor melted, repeatedly, a tungsten Langmuir probe. The reaction here is $2\times{}^{183}_{74}\mathrm{W} \Rightarrow {}^{182}_{74}\mathrm{W} + {}^{184}_{74}\mathrm{W} + 1.22\mathrm{MeV}$. This abundance of energy released within the tungsten's crystal matrix was responsible for the destruction of the Langmuir probe. Childs also reported visible flaring—documented in the video recordings—and he showed electron microscopy photographs which revealed changes in the tungsten crystal structure. I assert that this is evidence to the isotope transmutation reaction but unfortunately analysis for this was not done at the time.

The process is explained as follows: Using zinc as an example the atomic weights and the nuclei compositions are:

Atomic weight	Shell packing
65.9260334	${}^{66}_{30}\mathrm{Zn} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{2\lceil}_{2\lfloor}6d^{6}_{2}$
72.9234590	${}^{67}_{30}\mathrm{Zn} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{-1\lfloor}6s^{3}_{5} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{2\lceil}_{2\lfloor}6d^{6}_{2}$
66.9271273	${}^{68}_{30}\mathrm{Zn} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{4\lceil}_{2\lfloor}6d^{10}_{6}$

Our attention is on ${}^{67}_{30}\mathrm{Zn}$; the $7s$ shell is a superposition of two solitons, that is ${}^{2\lceil}_{1\lfloor}7s^{5}_{3} = {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{1\lceil}_{0\lfloor}7s^{2}_{2}$. The ${}^{1\lceil}_{0\lfloor}7s^{2}_{2}$ soliton is a ${}^{1}_{0}\Pi$ nucleoid without charge. In an energised environment with the correct conditions, a ${}^{67}_{30}\mathrm{Zn}$ decays to ${}^{66}_{30}\mathrm{Zn}$ isotope by releasing a ${}^{1}_{0}\Pi$ nucleoid. The freed ${}^{1}_{0}\Pi$ nucleoid is captured by the second ${}^{67}_{30}\mathrm{Zn}$ isotope producing a ${}^{68}_{30}\mathrm{Zn}$; summarised as follows:

$$\left.\begin{array}{l} {}^{67}_{30}\mathrm{Zn} - {}^{1}_{0}\Pi \Rightarrow {}^{206}_{82}\mathrm{Pb} \\ {}^{67}_{30}\mathrm{Zn} + {}^{1}_{0}\Pi \Rightarrow {}^{208}_{82}\mathrm{Pb} \end{array}\right\} \equiv 2\times{}^{67}_{30}\mathrm{Zn} \Rightarrow {}^{206}_{82}\mathrm{Pb} + {}^{208}_{82}\mathrm{Pb} + 6.30\mathrm{MeV}$$

That is 11.8% of the energy that a hydrogen to helium fusion yields.

The ${}^{1}_{0}\Pi$ nucleoid—lets refer to it as a pitron—differs from the neutron as it will decay into a positron, an electron, and a pair of pi-neutrinos (or an electron- and a positron neutrino) that carry away any excess energies.

I plead for the immediate funding of institutions capable of research in and developing this technology, because environmentally friendly and sustainable energy is producible by this method. Furthermore, the process is not weaponisable allowing these isotope transmu-

tation reactors to be erected, without concern, in all parts of the World. We must note that these isotope transmutation reactions do not produce radioactive byproducts, but rather the spent reaction material is returned to industry for its usual industrial usages.

9 Conclusion

It was only the fortuitous discovery of the soliton equation set

$$\mathcal{M}(\mathbf{u},\mathbf{a},\mathbf{r}) \xrightarrow{\text{defines}} \left\{ \mathbf{u} = \frac{1}{\mathbf{a}\cdot\mathbf{a}^*}\mathbf{a}\times\mathbf{r}, \quad \mathbf{a} = \frac{1}{\mathbf{u}\cdot\mathbf{u}^*}\mathbf{r}\times\mathbf{u}, \quad \mathbf{r} = \mathbf{u}\times\mathbf{a} \right\}$$

which allowed me to advance our understanding of electromagnetic theory in ways that had previously eluded comprehension.

Since the time when Maxwell formulated his field equations, what has largely been overlooked is the formal separation of the electromotive and the electrostatic fields. I have not conducted a study of Maxwell's original work to ascertain whether he made this separation or not. However, it is evident that Heaviside's vector algebraic interpretation of Maxwell's work, treats the electrostatic and electromotive fields as identical.

In conclusion, I contend that this paper comprehensively addresses the Clay Mathematics Institute's Millennium Prize Problem concerning "Yang-Mills & The Mass Gap." The Institute acknowledges in its problem description that: *Progress in establishing the existence of the Yang-Mills theory and a mass gap will require the introduction of fundamental new ideas both in physics and in mathematics.*

The introduction of this paper began with an excerpt from Aharonov and Bohm; fittingly, I will conclude with another, courtesy of Poincaré [7]:

"If we were to admit the postulate of relativity, we would find the same number in the law of gravitation and the laws of electromagnetism—the speed of light—and we would find it again in all other forces of any origin whatsoever. This state of affairs may be explained in one of two ways: either everything in the universe would be of electromagnetic origin, or this aspect—shared, as it were, by all physical phenomena—would be a mere epiphenomenon, something due to our methods of measurement. How do we go about measuring? The first response will be: we transport solid objects considered to be rigid, one on top of the other. But that is no longer true in the current theory if we admit the Lorentzian contraction. In this theory, two lengths are equal, by definition, if they are traversed by light in equal times.

Perhaps if we were to abandon this definition, Lorentz's theory would be as fully overthrown as was Ptolemy's system by Copernicus's intervention. Should that happen someday, it would not prove that Lorentz's efforts were in vain, because regardless of what one may think, Ptolemy was useful to Copernicus."

APPENDIX

A Electric current is not a drift of electrons

In a normal conductor, Drude's [8] 1900 theory visualises electric current as a fluid of free electrons flowing through the atomic lattice. In 1927 Sommerfeld [9] combined the classical Drude model with the Fermi–Dirac statistics providing a quantum mechanical description for the behaviour of charge carriers in a metallic solid. However, this theory can be easily refuted:

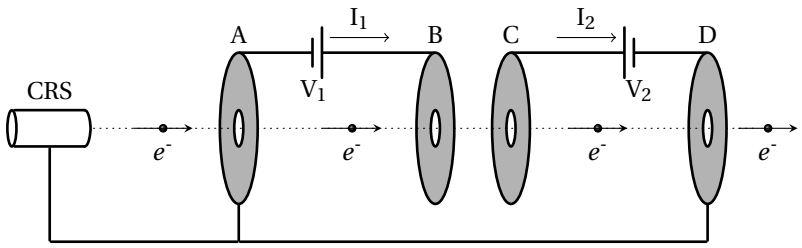

Figure 5: Apparatus to test Drude's theory.

PROOF. Now let's devise a thought experiment to test Drude's electron drift theory. Figure-5 sketches the apparatus, an adaptation of a particle accelerator. The cathode ray source (CRS) emits electrons, at ground potential, and with a defined kinetic energy. The electrons do not gain, nor lose, any kinetic energy on the path CRS–A, because A is at the same potential as CRS. Along the path from A to B, the electrons gain energy; this is due to the electromotive field between points A and B. The battery V_1 supplies the power to accelerate the electrons and subsequently discharges, causing an electric current I_1 to flow from A to B. Anything else would violate energy conservation laws

The path from B to C is electrically neutral; it features no potential difference and serves to electrically isolate circuits A–B and C–D. On the path from C to D, the electrons decelerate, returning their gained energy and charging battery V_2 through an electric current I_2.

Electrons exit the apparatus with their original kinetic energy. The sum of energies stored in batteries V_1 and V_2 remains unchanged; one battery discharges while the other charges. The electron beam can be sustained indefinitely, implying that the electric currents I_1 and I_2 are also maintained indefinitely. We must then question the validity of Drude's electron drift theory. Specifically, the question "In electrically isolated circuits, where do the infinitely many charge carriers (i.e., electrons) come from, which are required to sustain the electric currents I_1 and I_2 that discharge and charge the two batteries respectively?" remains unanswered in this experimental setup. □

B Precession of Planetary Orbits

Transportivity $\mathcal{T}$ is defined in Section 5 by Equation (19). This appendix aims to demonstrate how to apply the concept of transportivity in classical mechanics. Specifically, we analyse a two-body system in a circular orbit around their common barycentre, which serves as the origin for this analysis. We define $x = m_\mathrm{b}/m_\mathrm{a}$, $r_\mathrm{a} = x\rho$ and $r_\mathrm{b} = \rho$ with the aim that m_a and m_b are separated by $r = (1+x)\rho$. The transportivity $\mathcal{T}$ at positions A and B is given by

$$\mathcal{T}_\mathrm{A} = c_\mathrm{a}^2 = c^2 - \frac{Gm_\mathrm{b}}{r} \quad \text{and} \quad \mathcal{T}_\mathrm{B} = c_\mathrm{b}^2 = c^2 - \frac{Gm_\mathrm{a}}{r} \tag{B.1}$$

For the case $m_\mathrm{b} < m_\mathrm{a}$ and assuming $Gm_\mathrm{a}/r \ll c^2$, we relate the kinetic energy of the orbiting bodies m_a and m_b to a fraction of the available potential energy $(Gm_\mathrm{a}m_\mathrm{b})/r$ as follows:

$$\frac{m_\mathrm{b}v_\mathrm{b}^2}{2} = \frac{m_\mathrm{b}c^2 - m_\mathrm{b}c_\mathrm{b}^2}{2(1+x)^2} = \frac{Gm_\mathrm{a}m_\mathrm{b}}{2(1+x)^2\rho} \tag{B.2}$$

$$\frac{m_\mathrm{a}v_\mathrm{a}^2}{2} = \frac{m_\mathrm{a}c^2 - m_\mathrm{a}c_\mathrm{a}^2}{2\frac{(1+x)^2}{x}} = \frac{Gm_\mathrm{a}m_\mathrm{b}}{2\frac{(1+x)^2}{x}\rho} \tag{B.3}$$

The orbiting velocities v are thus found to be:

$$v_\mathrm{b} = \sqrt{\frac{Gm_\mathrm{a}}{\rho(1+x)^2}} \quad \text{and} \quad v_\mathrm{a} = \sqrt{\frac{Gm_\mathrm{b}x}{\rho(1+x)^2}} \tag{B.4}$$

Recalling the relation $m_\mathrm{b} = xm_\mathrm{a}$ we obtain

$$v_\mathrm{a} = \sqrt{\frac{Gm_\mathrm{a}x^2}{\rho(1+x)^2}} = xv_\mathrm{b}$$

confirming the division of the available potential energy which led to equations (B.2) and (B.3)

In Section 5, Equation (18) describes the outcome of a body in free fall interacting with a static field. The perceived energy of the body remains constant, but its inertial mass changes. The inertial mass is directly proportional to the ratio of the transportivity $\mathcal{T}$ to c^2. Consequently, the centrifugal forces acting on both m_a and m_b are also modified as follows:

$$F_\mathrm{b}^\mathrm{centri} = \frac{c_\mathrm{b}^2}{c^2}\frac{m_\mathrm{b}v_\mathrm{b}^2}{\rho} \quad \text{and} \quad F_\mathrm{a}^\mathrm{centri} = \frac{c_\mathrm{a}^2}{c^2}\frac{m_\mathrm{a}v_\mathrm{a}^2}{x\rho}$$

The gravitational force acting on m_b and m_a is determined by the derivatives of $m_\mathrm{a}c_\mathrm{a}^2$ and $m_\mathrm{b}c_\mathrm{b}^2$ with respect to r, yielding the expected

$$F_\mathrm{b}^\mathrm{gravi} = \frac{Gm_\mathrm{a}m_\mathrm{b}}{r^2} \quad \text{and} \quad F_\mathrm{a}^\mathrm{gravi} = \frac{Gm_\mathrm{a}m_\mathrm{b}}{r^2}$$

Using Newtonian mechanics, the effective potential of the orbits for m_b and m_a are formulated as

$$V_\mathrm{b}(\rho) = \frac{c_\mathrm{b}^2}{c^2}\frac{L_\mathrm{b}^2}{2m_\mathrm{b}\rho^2} - \frac{Gm_\mathrm{a}m_\mathrm{b}}{\rho(1+x)} \tag{B.5}$$

$$V_\mathrm{a}(x\rho) = \frac{c_\mathrm{a}^2}{c^2}\frac{L_\mathrm{a}^2}{2m_\mathrm{a}x^2\rho^2} - \frac{Gm_\mathrm{a}m_\mathrm{b}}{\rho(1+x)} \tag{B.6}$$

where $L = rmv$. We continue with (B.5)—the treatment for the second equation (B.6) proceeds analogously—and multiply and divide by the second term c^2, giving

$$V_\mathrm{b}(\rho) = \frac{c_\mathrm{b}^2}{c^2}\frac{L_\mathrm{b}^2}{2m_\mathrm{b}\rho^2} - \frac{Gm_\mathrm{a}}{c^2\rho(1+x)}m_\mathrm{b}c^2 \tag{B.7}$$

Next, we expand $m_\mathrm{b}c^2$ in terms of its constituent components. Utilising Equation (B.1) and recalling that $r = (1+x)\rho$, we obtain

$$m_\mathrm{b}c^2 = m_\mathrm{b}c_\mathrm{b}^2 + \frac{Gm_\mathrm{a}m_\mathrm{b}}{\rho(1+x)}$$

Rewriting the last term using Equation (B.4), we get

$$m_\mathrm{b}c^2 = m_\mathrm{b}c_\mathrm{b}^2 + (1+x)m_\mathrm{b}v_\mathrm{b}^2$$

Incorporating $L_\mathrm{b} = m_\mathrm{b}v_\mathrm{b}\rho$ into the second term yields

$$m_\mathrm{b}c^2 = m_\mathrm{b}c_\mathrm{b}^2 + (1+x)\frac{L_\mathrm{b}^2}{m_\mathrm{b}\rho^2} \tag{B.8}$$

Substituting $m_\mathrm{b}c^2$ from Equation (B.8) into Equation (B.7), we arrive at

$$V_\mathrm{b}(\rho) = \frac{c_\mathrm{b}^2}{c^2}\left(\frac{L_\mathrm{b}^2}{2m_\mathrm{b}\rho^2} - \frac{Gm_\mathrm{a}m_\mathrm{b}}{(1+x)\rho}\right) - \frac{Gm_\mathrm{a}L_\mathrm{b}^2}{m_\mathrm{b}c^2\rho^3} \tag{B.9}$$

Similarly, we have

$$V_{\mathrm{a}}(x\rho) = \frac{c_{\mathrm{a}}^2}{c^2}\left(\frac{L_{\mathrm{a}}^2}{2m_{\mathrm{a}}x^2\rho^2} - \frac{Gm_{\mathrm{a}}m_{\mathrm{b}}}{(1+x)\rho}\right) - \frac{Gm_{\mathrm{b}}L_{\mathrm{a}}^2}{m_{\mathrm{a}}c^2x^3\rho^3} \tag{B.10}$$

This result is familiar; upon simplifying Equation (B.9) by setting $M = m_{\mathrm{a}}$, $m = m_{\mathrm{b}}$, $c = c_{\mathrm{a}} = c_{\mathrm{b}}$, and considering the limit $x \to 0$, the variables ρ and $m_{\mathrm{b}}\rho$ transform into r and μr, respectively, where μ represents the reduced mass. We obtain:

$$V_{\mathrm{b}} \approx \frac{L^2}{2\mu r^2} - \frac{GMm}{r} - \frac{GML^2}{\mu c^2 r^3}, \tag{B.11}$$

This equation was first derived following Schwarzschild's discovery of the first exact solution to Einstein's field equations. The first two terms of Equation (B.11) are well-known classical energies: the second term represents the attractive Newtonian gravitational potential energy, while the first corresponds to the repulsive 'centrifugal' potential energy. The additional third term is an attractive energy term. As documented in numerous textbooks, this inverse-cubic energy results in a gradual precession of elliptical orbits by an angle $\delta\varphi$ per revolution,

$$\delta\varphi \approx \frac{6\pi G(M+m)}{c^2 A(1-e^2)}$$

where A is the semi-major axis and e is the eccentricity.

The result given in Equation (B.11) was initially thought to be unique to general relativity. It has now been derived through a classical approach by incorporating the transportivity $\mathcal{T}$, equation (B.1). Furthermore, the motion of both bodies is analysed simultaneously, eschewing approximations like reduced mass. Equations (B.9) and (B.10) describe the effective potential for the orbits of the two bodies around their common centre of mass.

C Proton Packing

Atomic shell packing is by half charges. A proton ${}^{1\lceil}_{1\lfloor}1s_1^3$ indicates 3 positive half charges and one negative half charge. In this case the atomic mass is $(3+1)/4 = 1$ and the charge $(3-1)/2 = 1$ and an anti-proton is indicated by ${}^{1\lceil}_{-1\lfloor}1s_3^1$. The shells are compound structures, *e. g.* a configuration of the $6p\ {}^{4\lceil}_{2\lfloor}6p_6^{10}$ gives an atomic mass of 4 and a positive charge 2. The shell packing follows the following rules:

$s \mapsto \langle 1,3\rangle,\ \langle 2,2\rangle,\ \langle 3,1\rangle$ gives charges -1, 0, +1

$p \mapsto \langle 3,1\rangle,\ \langle 5,3\rangle,\ \{\langle 7,5\rangle, \langle 9,3\rangle\}$ gives charges 1, 1, {1, 3}

$d \mapsto p,\ \{\langle 9,7\rangle, \langle 11,5\rangle\},\ \{\langle 11,9\rangle, \langle 13,7\rangle\}$

$f \mapsto d,\ \{\langle 13,11\rangle, \langle 15,9\rangle\},\ \{\langle 15,13\rangle, \langle 17,11\rangle\}$

$g \mapsto f,\ \{\langle 17,15\rangle, \langle 19,13\rangle\},\ \{\langle 19,17\rangle, \langle 21,15\rangle\},\ \{\langle 21,19\rangle, \langle 23,17\rangle\},\ \{\langle 23,21\rangle, \langle 25,19\rangle\}$

Note the maximum shell packing quantities are $\{s, p, d, f, g\} = 4 \times \{1, 3, 5, 7, 11\}$. The shell packing sequence is $1s, 2s, 2p, 3s \ldots 3d, 4s \ldots 4f, 5s \ldots 5g$. The completion of the first five shells results in either the iron isotope ${}^{57}_{26}\mathrm{Fe}$ or the cobalt isotope ${}^{57}_{27}\mathrm{Co}$

The sixth shell is filled as above, but instead of groups of four they are filled in groups of eight to give packing quantities for the 6^{th} shell $\{6s, 6p, 6d, 6f, 6g\}$ as $8 \times \{1, 3, 5, 7, 11\}$ with exceptions in the $6s$ which packs either in groups of four or eight. Completing the sixth shell gives the cadmium stable isotope ${}^{111}_{48}\mathrm{Cd}$. Similarly the 7^{th} shell packs in groups of sixteen.

We know that the nickel isotope ${}^{62}_{28}\mathrm{Ni} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s_3^1 + {}^{4\lceil}_{2\lfloor}6p_6^{10}$ has the least binding energy but this is not reflected in the packing, The charge difference between the $6s$ and $6p$ shell is $3e$. However, with a rearrangement of ${}^{62}_{28}\mathrm{Ni} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s_1^3 + {}^{4\lceil}_{0\lfloor}6p_{5+3}^{3+5}$ this charge difference is reduced to e which heuristically is a lower energy state. This indicates that a further rule needs to be applied.

From the 6th shell onwards, after packing with above rules, the charge differences between the sub shells are minimised. Example: ${}^{86}_{36}\mathrm{Kr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{12\lceil}_{2\lfloor}6f^{26}_{22}$ The $6d$ $\langle 26, 14\rangle$ packing is according to the above rules but the charge difference between the $6s$ and $6d$ shell is $7e$. The rearrangement ${}^{1\lceil}_{-1\lfloor}6s^{1}_{3} \rightarrow {}^{1\lceil}_{1\lfloor}6s^{3}_{1}$ and ${}^{10\lceil}_{6\lfloor}6d^{26}_{14} \rightarrow {}^{10\lceil}_{4\lfloor}6d^{24}_{16}$ reduces the difference to $3e$, clearly a lower energy state, giving ${}^{86}_{36}\mathrm{Kr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{4\lfloor}6d^{24}_{16} + {}^{12\lceil}_{2\lfloor}6f^{26}_{22}$. The packing sequence listed in the table below does not consider charge difference minimisation, The list serves to confirm the basic shell packing rules.

$${}^{1}_{1}\mathrm{H} = \; + {}^{1\lceil}_{1\lfloor}1s^{3}_{1}$$

$${}^{2}_{1}\mathrm{D} = {}^{1}_{1}\mathrm{H} + {}^{1\lceil}_{0\lfloor}2s^{2}_{2}$$

$${}^{3}_{1}\mathrm{T} = {}^{1}_{1}\mathrm{H} + {}^{1\lceil}_{0\lfloor}2s^{2}_{2} + {}^{1\lceil}_{0\lfloor}2p^{2}_{2}$$

$${}^{3}_{2}\mathrm{He} = {}^{1}_{1}\mathrm{H} + {}^{1\lceil}_{0\lfloor}2s^{2}_{2} + {}^{2\lceil}_{1\lfloor}2p^{5}_{3}$$

$${}^{4}_{2}\mathrm{He} = {}^{1}_{1}\mathrm{H} + {}^{1\lceil}_{0\lfloor}2s^{2}_{2} + {}^{3\lceil}_{1\lfloor}2p^{7}_{5}$$

$${}^{5}_{3}\mathrm{LI} = {}^{1}_{1}\mathrm{H} + {}^{1\lceil}_{1\lfloor}2s^{3}_{1} + {}^{3\lceil}_{1\lfloor}2p^{7}_{5}$$

$${}^{6}_{3}\mathrm{Li} = {}^{5}_{3}\mathrm{LI} + {}^{1\lceil}_{0\lfloor}3s^{2}_{2}$$

$${}^{7}_{3}\mathrm{Li} = {}^{5}_{3}\mathrm{LI} + {}^{1\lceil}_{-1\lfloor}3s^{1}_{3} + {}^{1\lceil}_{1\lfloor}3p^{3}_{1}$$

$${}^{9}_{4}\mathrm{Be} = {}^{5}_{2}\mathrm{He} + {}^{1\lceil}_{1\lfloor}3s^{3}_{1} + {}^{3\lceil}_{1\lfloor}3p^{7}_{5}$$

$${}^{10}_{5}\mathrm{B} = {}^{5}_{2}\mathrm{He} + {}^{1\lceil}_{1\lfloor}3s^{3}_{1} + {}^{2\lceil}_{1\lfloor}3p^{5}_{3} + {}^{2\lceil}_{1\lfloor}3d^{5}_{3}$$

$${}^{11}_{5}\mathrm{B} = {}^{5}_{2}\mathrm{He} + {}^{1\lceil}_{1\lfloor}3s^{3}_{1} + {}^{3\lceil}_{1\lfloor}3p^{7}_{5} + {}^{2\lceil}_{1\lfloor}3d^{5}_{3}$$

$${}^{12}_{6}\mathrm{C} = {}^{5}_{3}\mathrm{LI} + {}^{1\lceil}_{1\lfloor}3s^{3}_{1} + {}^{3\lceil}_{1\lfloor}3p^{7}_{5} + {}^{3\lceil}_{1\lfloor}3d^{7}_{5}$$

$${}^{13}_{6}\mathrm{C} = {}^{5}_{3}\mathrm{LI} + {}^{1\lceil}_{1\lfloor}3s^{3}_{1} + {}^{3\lceil}_{1\lfloor}3p^{7}_{5} + {}^{4\lceil}_{1\lfloor}3d^{9}_{7}$$

$${}^{14}_{6}\mathrm{C} = {}^{5}_{3}\mathrm{LI} + {}^{1\lceil}_{1\lfloor}3s^{3}_{1} + {}^{3\lceil}_{1\lfloor}3p^{7}_{5} + {}^{5\lceil}_{1\lfloor}3d^{11}_{9}$$

$${}^{14}_{7}\mathrm{N} = {}^{13}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1}$$

$${}^{15}_{7}\mathrm{N} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1}$$

$${}^{16}_{8}\mathrm{O} = {}^{13}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1} + {}^{2\lceil}_{1\lfloor}4p^{5}_{3}$$

$${}^{17}_{8}\mathrm{O} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1} + {}^{2\lceil}_{1\lfloor}4p^{5}_{3}$$

$${}^{18}_{8}\mathrm{O} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1} + {}^{3\lceil}_{1\lfloor}4p^{7}_{5}$$

$${}^{19}_{9}\mathrm{F} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1} + {}^{2\lceil}_{1\lfloor}4p^{5}_{3} + {}^{2\lceil}_{1\lfloor}4d^{5}_{3}$$

$${}^{20}_{10}\mathrm{Ne} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{0\lfloor}4s^{2}_{2} + {}^{3\lceil}_{3\lfloor}4p^{9}_{3} + {}^{2\lceil}_{1\lfloor}4d^{5}_{3}$$

$${}^{21}_{10}\mathrm{Ne} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{0\lfloor}4s^{2}_{2} + {}^{3\lceil}_{3\lfloor}4p^{9}_{3} + {}^{3\lceil}_{1\lfloor}4d^{7}_{5}$$

$${}^{22}_{10}\mathrm{Ne} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{0\lfloor}4s^{2}_{2} + {}^{3\lceil}_{3\lfloor}4p^{9}_{3} + {}^{4\lceil}_{1\lfloor}4d^{9}_{7}$$

$${}^{23}_{11}\mathrm{Na} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1} + {}^{3\lceil}_{1\lfloor}4p^{7}_{5} + {}^{5\lceil}_{3\lfloor}4d^{13}_{7}$$

$$^{24}_{12}\mathrm{Mg} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1} + {}^{3\lceil}_{1\lfloor}4p^{7}_{5} + {}^{5\lceil}_{3\lfloor}4d^{13}_{7} + {}^{1\lceil}_{1\lfloor}4f^{3}_{1}$$

$$^{25}_{12}\mathrm{Mg} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1} + {}^{3\lceil}_{1\lfloor}4p^{7}_{5} + {}^{5\lceil}_{3\lfloor}4d^{13}_{7} + {}^{2\lceil}_{1\lfloor}4f^{5}_{3}$$

$$^{26}_{12}\mathrm{Mg} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1} + {}^{3\lceil}_{1\lfloor}4p^{7}_{5} + {}^{5\lceil}_{3\lfloor}4d^{13}_{7} + {}^{3\lceil}_{1\lfloor}4f^{7}_{5}$$

$$^{27}_{13}\mathrm{Al} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{0\lfloor}4s^{2}_{2} + {}^{3\lceil}_{1\lfloor}4p^{7}_{5} + {}^{5\lceil}_{3\lfloor}4d^{13}_{7} + {}^{4\lceil}_{3\lfloor}4f^{11}_{5}$$

$$^{28}_{14}\mathrm{Si} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1} + {}^{3\lceil}_{1\lfloor}4p^{7}_{5} + {}^{5\lceil}_{3\lfloor}4d^{13}_{7} + {}^{5\lceil}_{3\lfloor}4f^{13}_{7}$$

$$^{29}_{14}\mathrm{Si} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1} + {}^{3\lceil}_{1\lfloor}4p^{7}_{5} + {}^{5\lceil}_{3\lfloor}4d^{13}_{7} + {}^{6\lceil}_{3\lfloor}4f^{15}_{9}$$

$$^{30}_{14}\mathrm{Si} = {}^{14}_{6}\mathrm{C} + {}^{1\lceil}_{1\lfloor}4s^{3}_{1} + {}^{3\lceil}_{1\lfloor}4p^{7}_{5} + {}^{5\lceil}_{3\lfloor}4d^{13}_{7} + {}^{7\lceil}_{3\lfloor}4f^{17}_{11}$$

$$^{31}_{15}\mathrm{P} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1}$$

$$^{32}_{16}\mathrm{S} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{1\lceil}_{1\lfloor}5p^{3}_{1}$$

$$^{33}_{16}\mathrm{S} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{2\lceil}_{1\lfloor}5p^{5}_{3}$$

$$^{34}_{16}\mathrm{S} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5}$$

$$^{36}_{16}\mathrm{S} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{2\lceil}_{1\lfloor}5d^{5}_{3}$$

$$^{35}_{17}\mathrm{Cl} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{1\lceil}_{1\lfloor}5d^{3}_{1}$$

$$^{37}_{17}\mathrm{Cl} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{3\lceil}_{1\lfloor}5d^{7}_{5}$$

$$^{36}_{18}\mathrm{Ar} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{2\lceil}_{1\lfloor}5p^{5}_{3} + {}^{3\lceil}_{3\lfloor}5d^{9}_{3}$$

$$^{38}_{18}\mathrm{Ar} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{4\lceil}_{3\lfloor}5d^{11}_{5}$$

$$^{40}_{18}\mathrm{Ar} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{6\lceil}_{3\lfloor}5d^{15}_{9}$$

$$^{39}_{19}\mathrm{K} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7}$$

$$^{40}_{19}\mathrm{K} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{6\lceil}_{3\lfloor}5d^{15}_{9}$$

$$^{41}_{19}\mathrm{K} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{7\lceil}_{3\lfloor}5d^{17}_{11}$$

$$^{40}_{20}\mathrm{Ca} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{1\lceil}_{1\lfloor}5f^{3}_{1}$$

$$^{42}_{20}\mathrm{Ca} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{3\lceil}_{1\lfloor}5f^{7}_{5}$$

$$^{43}_{20}\mathrm{Ca} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{4\lceil}_{1\lfloor}5f^{9}_{7}$$

$$^{44}_{20}\mathrm{Ca} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{5\lceil}_{1\lfloor}5f^{11}_{9}$$

$$^{46}_{20}\mathrm{Ca} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{1\lfloor}5f^{15}_{13}$$

$$^{48}_{20}\mathrm{Ca} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{1\lfloor}5f^{15}_{13}$$

$$^{45}_{21}\mathrm{Sc} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{6\lceil}_{3\lfloor}5f^{15}_{9}$$

$$^{46}_{22}\mathrm{Ti} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11}$$

$$^{47}_{22}\mathrm{Ti} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{1\lceil}_{1\lfloor}5g^{3}_{1}$$

$$^{48}_{22}\mathrm{Ti} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{2\lceil}_{1\lfloor}5g^{5}_{3}$$

$$^{49}_{22}\mathrm{Ti} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{3\lceil}_{1\lfloor}5g^{7}_{5}$$

$$^{50}_{22}\mathrm{Ti} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{4\lceil}_{1\lfloor}5g^{9}_{7}$$

$$^{50}_{23}\mathrm{V} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{4\lceil}_{1\lfloor}5g^{9}_{7}$$

$$^{51}_{23}\mathrm{V} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{5\lceil}_{1\lfloor}5g^{11}_{9}$$

$$^{50}_{24}\mathrm{Cr} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{4\lceil}_{3\lfloor}5g^{11}_{5}$$

$$^{52}_{24}\mathrm{Cr} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{6\lceil}_{3\lfloor}5g^{15}_{9}$$

$$^{53}_{24}\mathrm{Cr} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{7\lceil}_{3\lfloor}5g^{17}_{11}$$

$$^{54}_{24}\mathrm{Cr} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{8\lceil}_{3\lfloor}5g^{19}_{13}$$

$$^{55}_{25}\mathrm{Mn} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{3\lfloor}5d^{13}_{7} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{9\lceil}_{3\lfloor}5g^{21}_{15}$$

$$^{54}_{26}\mathrm{Fe} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{5\lfloor}5d^{15}_{5} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{8\lceil}_{3\lfloor}5g^{19}_{13}$$

$$^{56}_{26}\mathrm{Fe} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{5\lfloor}5d^{15}_{5} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{10\lceil}_{3\lfloor}5g^{23}_{17}$$

$$^{57}_{26}\mathrm{Fe} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{0\lfloor}5s^{2}_{2} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{5\lfloor}5d^{15}_{5} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{11\lceil}_{3\lfloor}5g^{25}_{19}$$

$$^{57}_{27}\mathrm{Co} = {}^{30}_{14}\mathrm{Si} + {}^{1\lceil}_{1\lfloor}5s^{3}_{1} + {}^{3\lceil}_{1\lfloor}5p^{7}_{5} + {}^{5\lceil}_{5\lfloor}5d^{15}_{5} + {}^{7\lceil}_{3\lfloor}5f^{17}_{11} + {}^{11\lceil}_{3\lfloor}5g^{25}_{19}$$

$$^{58}_{26}\mathrm{Fe} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3}$$

$$^{59}_{27}\mathrm{Co} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4}$$

$$^{58}_{28}\mathrm{Ni} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1}$$

$$^{60}_{28}\mathrm{Ni} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{0\lfloor}6s^{2}_{2} + {}^{2\lceil}_{1\lfloor}6p^{5}_{3}$$

$$^{61}_{28}\mathrm{Ni} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4} + {}^{2\lceil}_{1\lfloor}6p^{5}_{3}$$

$$^{62}_{28}\mathrm{Ni} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{0\lfloor}6s^{2}_{2} + {}^{4\lceil}_{1\lfloor}6p^{9}_{7}$$

$$^{64}_{28}\mathrm{Ni} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10}$$

$$^{63}_{29}\mathrm{Cu} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4} + {}^{4\lceil}_{2\lfloor}6p^{10}_{6}$$

$$^{65}_{29}\mathrm{Cu} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10}$$

$$^{64}_{30}\mathrm{Zn} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10}$$

$$^{66}_{30}\mathrm{Zn} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{2\lceil}_{2\lfloor}6d^{6}_{2}$$

$$^{67}_{30}\mathrm{Zn} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{-1\lfloor}6s^{3}_{5} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{2\lceil}_{2\lfloor}6d^{6}_{2}$$

$$^{68}_{30}\mathrm{Zn} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{4\lceil}_{2\lfloor}6d^{10}_{6}$$

$$^{70}_{30}\mathrm{Zn} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{6\lceil}_{2\lfloor}6d^{14}_{10}$$

$$^{69}_{31}\mathrm{Ga} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{4\lceil}_{2\lfloor}6d^{10}_{6}$$

$$^{71}_{31}\mathrm{Ga} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{6\lceil}_{2\lfloor}6d^{14}_{10}$$

$$^{70}_{32}\mathrm{Ge} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{6\lceil}_{2\lfloor}6d^{14}_{10}$$

$$^{72}_{32}\mathrm{Ge} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{8\lceil}_{2\lfloor}6d^{18}_{14}$$

$$^{73}_{32}\mathrm{Ge} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{1\lfloor}6s^{5}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{8\lceil}_{2\lfloor}6d^{18}_{14}$$

$$^{74}_{32}\mathrm{Ge} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18}$$

$$^{76}_{32}\mathrm{Ge} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{12\lceil}_{2\lfloor}6d^{26}_{22}$$

$$^{75}_{33}\mathrm{As} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{2\lfloor}6s^{6}_{2} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18}$$

$$^{74}_{34}\mathrm{Se} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14}$$

$$^{76}_{34}\mathrm{Se} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{2\lceil}_{2\lfloor}6f^{6}_{2}$$

$$^{77}_{34}\mathrm{Se} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{1\lfloor}6s^{5}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{2\lceil}_{2\lfloor}6f^{6}_{2}$$

$$^{78}_{34}\mathrm{Se} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{4\lceil}_{2\lfloor}6f^{10}_{6}$$

$$^{80}_{34}\mathrm{Se} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{6\lceil}_{2\lfloor}6f^{14}_{10}$$

$$^{82}_{34}\mathrm{Se} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{8\lceil}_{2\lfloor}6f^{18}_{14}$$

$$^{79}_{35}\mathrm{Br} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{2\lfloor}6s^{6}_{2} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{4\lceil}_{2\lfloor}6f^{10}_{6}$$

$$^{81}_{35}\mathrm{Br} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{2\lfloor}6s^{6}_{2} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{6\lceil}_{2\lfloor}6f^{14}_{10}$$

$$^{78}_{36}\mathrm{Kr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{4\lceil}_{6\lfloor}6f^{14}_{2}$$

$$^{80}_{36}\mathrm{Kr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{6\lceil}_{6\lfloor}6f^{18}_{6}$$

$$^{82}_{36}\mathrm{Kr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{8\lceil}_{6\lfloor}6f^{22}_{10}$$

$$^{83}_{36}\mathrm{Kr} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{-1\lfloor}6s^{3}_{5} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{8\lceil}_{6\lfloor}6f^{22}_{10}$$

$$^{84}_{36}\mathrm{Kr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{10\lceil}_{6\lfloor}6f^{26}_{14}$$

$$^{86}_{36}\mathrm{Kr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{12\lceil}_{6\lfloor}6f^{30}_{18}$$

$$^{85}_{37}\mathrm{Rb} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{10\lceil}_{6\lfloor}6f^{26}_{14}$$

$$^{87}_{37}\mathrm{Rb} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{12\lceil}_{6\lfloor}6f^{30}_{18}$$

$$^{84}_{38}\mathrm{Sr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{10\lceil}_{6\lfloor}6f^{26}_{14}$$

$$^{86}_{38}\mathrm{Sr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{12\lceil}_{6\lfloor}6f^{30}_{18}$$

$$^{87}_{38}\mathrm{Sr} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{1\lfloor}6s^{5}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{12\lceil}_{6\lfloor}6f^{30}_{18}$$

$$^{88}_{38}\mathrm{Sr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22}$$

$$^{89}_{39}\mathrm{Y} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{2\lfloor}6s^{6}_{2} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22}$$

$$^{90}_{40}\mathrm{Zr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{2\lceil}_{2\lfloor}6g^{6}_{2}$$

$$^{91}_{40}\mathrm{Zr} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{1\lfloor}6s^{5}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{2\lceil}_{2\lfloor}6g^{6}_{2}$$

$$^{92}_{40}\mathrm{Zr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{4\lceil}_{2\lfloor}6g^{10}_{6}$$

$$^{94}_{40}\mathrm{Zr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{6\lceil}_{2\lfloor}6g^{14}_{10}$$

$$^{96}_{40}\mathrm{Zr} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{8\lceil}_{2\lfloor}6g^{18}_{14}$$

$$^{93}_{41}\mathrm{Nb} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{2\lfloor}6s^{6}_{2} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{4\lceil}_{2\lfloor}6g^{10}_{6}$$

$$^{92}_{42}\mathrm{Mo} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{4\lceil}_{6\lfloor}6g^{14}_{2}$$

$$^{94}_{42}\mathrm{Mo} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{6\lceil}_{6\lfloor}6g^{18}_{6}$$

$$^{95}_{42}\mathrm{Mo} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{-1\lfloor}6s^{3}_{5} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{6\lceil}_{6\lfloor}6g^{18}_{6}$$

$$^{96}_{42}\mathrm{Mo} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{8\lceil}_{6\lfloor}6g^{22}_{10}$$

$$^{97}_{42}\mathrm{Mo} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{-1\lfloor}6s^{3}_{5} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{8\lceil}_{6\lfloor}6g^{22}_{10}$$

$$^{98}_{42}\mathrm{Mo} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{10\lceil}_{6\lfloor}6g^{26}_{14}$$

$$^{100}_{42}\mathrm{Mo} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{12\lceil}_{6\lfloor}6g^{30}_{18}$$

$$^{97}_{43}\mathrm{Tc} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{8\lceil}_{6\lfloor}6g^{22}_{10}$$

$$^{98}_{43}\mathrm{Tc} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{0\lfloor}6s^{2}_{2} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{10\lceil}_{6\lfloor}6g^{26}_{14}$$

$$^{99}_{43}\mathrm{Tc} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{10\lceil}_{6\lfloor}6g^{26}_{14}$$

$$^{96}_{44}\mathrm{Ru} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{8\lceil}_{6\lfloor}6g^{22}_{10}$$

$$^{98}_{44}\mathrm{Ru} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{10\lceil}_{6\lfloor}6g^{26}_{14}$$

$$^{99}_{44}\mathrm{Ru} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{1\lfloor}6s^{5}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{10\lceil}_{6\lfloor}6g^{26}_{14}$$

$$^{100}_{44}\mathrm{Ru} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{12\lceil}_{6\lfloor}6g^{30}_{18}$$

$$^{101}_{44}\mathrm{Ru} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{1\lfloor}6s^{5}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{12\lceil}_{6\lfloor}6g^{30}_{18}$$

$$^{102}_{44}\mathrm{Ru} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{14\lceil}_{6\lfloor}6g^{34}_{22}$$

$$^{104}_{44}\mathrm{Ru} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{16\lceil}_{6\lfloor}6g^{38}_{26}$$

$$^{103}_{45}\mathrm{Rh} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{2\lfloor}6s^{6}_{2} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{2\lfloor}6d^{22}_{18} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{14\lceil}_{6\lfloor}6g^{34}_{22}$$

$$^{102}_{46}\mathrm{Pd} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{14\lceil}_{6\lfloor}6g^{34}_{22}$$

$$^{104}_{46}\mathrm{Pd} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{16\lceil}_{6\lfloor}6g^{38}_{26}$$

$$^{105}_{46}\mathrm{Pd} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{-1\lfloor}6s^{3}_{5} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{16\lceil}_{6\lfloor}6g^{38}_{26}$$

$$^{106}_{46}\mathrm{Pd} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{18\lceil}_{6\lfloor}6g^{42}_{30}$$

$$^{108}_{46}\mathrm{Pd} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{20\lceil}_{6\lfloor}6g^{46}_{34}$$

$$^{110}_{46}\mathrm{Pd} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{22\lceil}_{6\lfloor}6g^{50}_{38}$$

$$^{111}_{46}\mathrm{Pd} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{-1\lfloor}6s^{3}_{5} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{22\lceil}_{6\lfloor}6g^{50}_{38}$$

$$^{107}_{47}\mathrm{Ag} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{18\lceil}_{6\lfloor}6g^{42}_{30}$$

$$^{109}_{47}\mathrm{Ag} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{20\lceil}_{6\lfloor}6g^{46}_{34}$$

$$^{111}_{47}\mathrm{Ag} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{0\lfloor}6s^{4}_{4} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{22\lceil}_{6\lfloor}6g^{50}_{38}$$

$$^{106}_{48}\mathrm{Cd} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{18\lceil}_{6\lfloor}6g^{42}_{30}$$

$$^{108}_{48}\mathrm{Cd} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{20\lceil}_{6\lfloor}6g^{46}_{34}$$

$$^{110}_{48}\mathrm{Cd} = {}^{57}_{27}\mathrm{Co} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{22\lceil}_{6\lfloor}6g^{50}_{38}$$

$$^{111}_{48}\mathrm{Cd} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{1\lfloor}6s^{5}_{3} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{22\lceil}_{6\lfloor}6g^{50}_{38}$$

$$^{112}_{48}\mathrm{Cd} = {}^{111}_{47}\mathrm{Ag} + {}^{1\lceil}_{1\lfloor}6s^{3}_{1}$$

$$^{113}_{48}\mathrm{Cd} = {}^{111}_{47}\mathrm{Ag} + {}^{2\lceil}_{1\lfloor}6s^{5}_{3}$$

$$^{114}_{48}\mathrm{Cd} = {}^{111}_{47}\mathrm{Ag} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{2\lceil}_{2\lfloor}6p^{6}_{2}$$

$$^{116}_{48}\mathrm{Cd} = {}^{111}_{47}\mathrm{Ag} + {}^{1\lceil}_{-1\lfloor}6s^{1}_{3} + {}^{4\lceil}_{2\lfloor}6p^{10}_{6}$$

$$^{111}_{49}\mathrm{In} = {}^{57}_{27}\mathrm{Co} + {}^{2\lceil}_{2\lfloor}6s^{6}_{2} + {}^{6\lceil}_{2\lfloor}6p^{14}_{10} + {}^{10\lceil}_{6\lfloor}6d^{26}_{14} + {}^{14\lceil}_{6\lfloor}6f^{34}_{22} + {}^{22\lceil}_{6\lfloor}6g^{50}_{38}$$

$$^{113}_{49}\mathrm{In} = {}^{111}_{47}\mathrm{Ag} + {}^{2\lceil}_{2\lfloor}7s^{6}_{2}$$

$$^{115}_{49}\mathrm{In} = {}^{111}_{47}\mathrm{Ag} + {}^{4\lceil}_{2\lfloor}7s^{10}_{6}$$

$$^{112}_{50}\mathrm{Sn} = {}^{110}_{48}\mathrm{Cd} + {}^{2\lceil}_{2\lfloor}7s^{6}_{2}$$

$$^{114}_{50}\mathrm{Sn} = {}^{110}_{48}\mathrm{Cd} + {}^{4\lceil}_{2\lfloor}7s^{10}_{6}$$

$$^{115}_{50}\mathrm{Sn} = {}^{111}_{48}\mathrm{Cd} + {}^{2\lceil}_{0\lfloor}7s^{4}_{4} + {}^{2\lceil}_{2\lfloor}7p^{6}_{2}$$

$$^{116}_{50}\mathrm{Sn} = {}^{111}_{48}\mathrm{Cd} + {}^{1\lceil}_{0\lfloor}7s^{2}_{2} + {}^{4\lceil}_{2\lfloor}7p^{10}_{6}$$

$$^{117}_{50}\mathrm{Sn} = {}^{111}_{48}\mathrm{Cd} + {}^{2\lceil}_{0\lfloor}7s^{4}_{4} + {}^{4\lceil}_{2\lfloor}7p^{10}_{6}$$

$$^{118}_{50}\mathrm{Sn} = {}^{111}_{48}\mathrm{Cd} + {}^{3\lceil}_{0\lfloor}7s^{6}_{6} + {}^{4\lceil}_{2\lfloor}7p^{10}_{6}$$

$$^{119}_{50}\mathrm{Sn} = {}^{111}_{48}\mathrm{Cd} + {}^{4\lceil}_{0\lfloor}7s^{8}_{8} + {}^{4\lceil}_{2\lfloor}7p^{10}_{6}$$

$$^{120}_{50}\mathrm{Sn} = {}^{111}_{48}\mathrm{Cd} + {}^{1\lceil}_{0\lfloor}7s^{2}_{2} + {}^{8\lceil}_{2\lfloor}7p^{18}_{14}$$

$$^{122}_{50}\mathrm{Sn} = {}^{111}_{48}\mathrm{Cd} + {}^{3\lceil}_{0\lfloor}7s^{6}_{6} + {}^{8\lceil}_{2\lfloor}7p^{18}_{14}$$

$$^{124}_{50}\mathrm{Sn} = {}^{111}_{48}\mathrm{Cd} + {}^{1\lceil}_{0\lfloor}7s^{2}_{2} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22}$$

$$^{121}_{51}\mathrm{Sb} = {}^{111}_{48}\mathrm{Cd} + {}^{2\lceil}_{1\lfloor}7s^{5}_{3} + {}^{8\lceil}_{2\lfloor}7p^{18}_{14}$$

$$^{123}_{51}\mathrm{Sb} = {}^{111}_{48}\mathrm{Cd} + {}^{4\lceil}_{1\lfloor}7s^{9}_{7} + {}^{8\lceil}_{2\lfloor}7p^{18}_{14}$$

$$^{120}_{52}\mathrm{Te} = {}^{110}_{48}\mathrm{Cd} + {}^{2\lceil}_{2\lfloor}7s^{6}_{2} + {}^{8\lceil}_{2\lfloor}7p^{18}_{14}$$

$$^{122}_{52}\mathrm{Te} = {}^{110}_{48}\mathrm{Cd} + {}^{4\lceil}_{2\lfloor}7s^{10}_{6} + {}^{8\lceil}_{2\lfloor}7p^{18}_{14}$$

$$^{123}_{52}\mathrm{Te} = {}^{110}_{48}\mathrm{Cd} + {}^{1\lceil}_{0\lfloor}7s^{2}_{2} + {}^{8\lceil}_{2\lfloor}7p^{18}_{14}$$

$$^{124}_{52}\mathrm{Te} = {}^{110}_{48}\mathrm{Cd} + {}^{2\lceil}_{0\lfloor}7s^{4}_{4} + {}^{8\lceil}_{2\lfloor}7p^{18}_{14}$$

$$^{125}_{52}\mathrm{Te} = {}^{110}_{48}\mathrm{Cd} + {}^{3\lceil}_{0\lfloor}7s^{6}_{6} + {}^{8\lceil}_{2\lfloor}7p^{18}_{14}$$

$$^{126}_{52}\mathrm{Te} = {}^{110}_{48}\mathrm{Cd} + {}^{4\lceil}_{0\lfloor}7s^{8}_{8} + {}^{8\lceil}_{2\lfloor}7p^{18}_{14}$$

$$^{128}_{52}\mathrm{Te} = {}^{110}_{48}\mathrm{Cd} + {}^{2\lceil}_{0\lfloor}7s^{4}_{4} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{4\lceil}_{2\lfloor}7d^{10}_{6}$$

$$^{130}_{52}\mathrm{Te} = {}^{110}_{48}\mathrm{Cd} + {}^{4\lceil}_{0\lfloor}7s^{8}_{8} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{4\lceil}_{2\lfloor}7d^{10}_{6}$$

$$^{127}_{53}\mathrm{I} = {}^{110}_{48}\mathrm{Cd} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{4\lceil}_{2\lfloor}7d^{10}_{6}$$

$$^{124}_{54}\mathrm{Xe} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{8\lceil}_{2\lfloor}7p^{18}_{14} + {}^{4\lceil}_{2\lfloor}7d^{10}_{6}$$

$$^{126}_{54}\mathrm{Xe} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{8\lceil}_{2\lfloor}7p^{18}_{14} + {}^{4\lceil}_{2\lfloor}7d^{10}_{6}$$

$$^{128}_{54}\mathrm{Xe} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{4\lceil}_{2\lfloor}7d^{10}_{6}$$

$$^{129}_{54}\mathrm{Xe} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{1\lfloor}7s^{5}_{3} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{4\lceil}_{2\lfloor}7d^{10}_{6}$$

$$^{130}_{54}\mathrm{Xe} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{4\lceil}_{2\lfloor}7d^{10}_{6}$$

$$^{131}_{54}\mathrm{Xe} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{1\lfloor}7s^{9}_{7} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{4\lceil}_{2\lfloor}7d^{10}_{6}$$

$$^{132}_{54}\mathrm{Xe} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{8\lceil}_{2\lfloor}7d^{18}_{14}$$

$$^{134}_{54}\mathrm{Xe} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{8\lceil}_{2\lfloor}7d^{18}_{14}$$

$$^{136}_{54}\mathrm{Xe} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{12\lceil}_{2\lfloor}7d^{26}_{22}$$

$$^{133}_{55}\mathrm{Cs} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{2\lfloor}7s^{6}_{2} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{8\lceil}_{2\lfloor}7d^{18}_{14}$$

$$^{130}_{56}\mathrm{Ba} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{4\lceil}_{6\lfloor}7d^{14}_{2}$$

$$^{132}_{56}\mathrm{Ba} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{8\lceil}_{6\lfloor}7d^{22}_{10}$$

$$^{134}_{56}\mathrm{Ba} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{8\lceil}_{6\lfloor}7d^{22}_{10}$$

$$^{135}_{56}\mathrm{Ba} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{-1\lfloor}7s^{7}_{9} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{8\lceil}_{6\lfloor}7d^{22}_{10}$$

$$^{136}_{56}\mathrm{Ba} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{12\lceil}_{6\lfloor}7d^{30}_{18}$$

$$^{137}_{56}\mathrm{Ba} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{-1\lfloor}7s^{3}_{5} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{12\lceil}_{6\lfloor}7d^{30}_{18}$$

$$^{138}_{56}\mathrm{Ba} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{12\lceil}_{6\lfloor}7d^{30}_{18}$$

$$^{138}_{57}\mathrm{La} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{0\lfloor}7s^{6}_{6} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{12\lceil}_{6\lfloor}7d^{30}_{18}$$

$$^{139}_{57}\mathrm{La} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{0\lfloor}7s^{8}_{8} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{12\lceil}_{6\lfloor}7d^{30}_{18}$$

$$^{136}_{58}\mathrm{Ce} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{12\lceil}_{6\lfloor}7d^{30}_{18}$$

$$^{138}_{58}\mathrm{Ce} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{12\lceil}_{6\lfloor}7d^{30}_{18}$$

$$^{140}_{58}\mathrm{Ce} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{16\lceil}_{6\lfloor}7d^{38}_{26}$$

$$^{142}_{58}\mathrm{Ce} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{16\lceil}_{6\lfloor}7d^{38}_{26}$$

$$^{141}_{59}\mathrm{Pr} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{2\lfloor}7s^{6}_{2} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{16\lceil}_{6\lfloor}7d^{38}_{26}$$

$$^{142}_{60}\mathrm{Nd} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{16\lceil}_{6\lfloor}7d^{38}_{26} + {}^{2\lceil}_{2\lfloor}7f^{6}_{2}$$

$$^{143}_{60}\mathrm{Nd} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{1\lfloor}7s^{5}_{3} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{16\lceil}_{6\lfloor}7d^{38}_{26} + {}^{2\lceil}_{2\lfloor}7f^{6}_{2}$$

$$^{144}_{60}\mathrm{Nd} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{16\lceil}_{6\lfloor}7d^{38}_{26} + {}^{2\lceil}_{2\lfloor}7f^{6}_{2}$$

$$^{145}_{60}\mathrm{Nd} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{1\lfloor}7s^{9}_{7} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{16\lceil}_{6\lfloor}7d^{38}_{26} + {}^{2\lceil}_{2\lfloor}7f^{6}_{2}$$

$$^{146}_{60}\mathrm{Nd} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{2\lceil}_{2\lfloor}7f^{6}_{2}$$

$$^{148}_{60}\mathrm{Nd} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{2\lceil}_{2\lfloor}7f^{6}_{2}$$

$$^{150}_{60}\mathrm{Nd} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{4\lceil}_{2\lfloor}7f^{10}_{6}$$

$$^{145}_{61}\mathrm{Pm} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{2\lfloor}7s^{6}_{2} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{16\lceil}_{6\lfloor}7d^{38}_{26} + {}^{4\lceil}_{2\lfloor}7f^{10}_{6}$$

$$^{147}_{61}\mathrm{Pm} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{2\lfloor}7s^{6}_{2} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{2\lceil}_{2\lfloor}7f^{6}_{2}$$

$$^{144}_{62}\mathrm{Sm} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{16\lceil}_{6\lfloor}7d^{38}_{26} + {}^{4\lceil}_{6\lfloor}7f^{14}_{2}$$

$$^{147}_{62}\mathrm{Sm} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{-1\lfloor}7s^{7}_{9} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{16\lceil}_{6\lfloor}7d^{38}_{26} + {}^{4\lceil}_{6\lfloor}7f^{14}_{2}$$

$$^{148}_{62}\mathrm{Sm} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{4\lceil}_{6\lfloor}7f^{14}_{2}$$

$$^{149}_{62}\mathrm{Sm} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{-1\lfloor}7s^{3}_{5} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{4\lceil}_{6\lfloor}7f^{14}_{2}$$

$$^{150}_{62}\mathrm{Sm} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{4\lceil}_{6\lfloor}7f^{14}_{2}$$

$$^{152}_{62}\mathrm{Sm} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{8\lceil}_{6\lfloor}7f^{22}_{10}$$

$$^{154}_{62}\mathrm{Sm} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{8\lceil}_{6\lfloor}7f^{22}_{10}$$

$$^{151}_{63}\mathrm{Eu} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{0\lfloor}7s^{8}_{8} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{4\lceil}_{6\lfloor}7f^{14}_{2}$$

$$^{153}_{63}\mathrm{Eu} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{0\lfloor}7s^{4}_{4} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{8\lceil}_{6\lfloor}7f^{22}_{10}$$

$$^{152}_{64}\mathrm{Gd} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{8\lceil}_{6\lfloor}7f^{22}_{10}$$

$$^{154}_{64}\mathrm{Gd} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{8\lceil}_{6\lfloor}7f^{22}_{10}$$

$$^{155}_{64}\mathrm{Gd} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{1\lfloor}7s^{9}_{7} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{8\lceil}_{6\lfloor}7f^{22}_{10}$$

$$^{156}_{64}\mathrm{Gd} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{12\lceil}_{6\lfloor}7f^{30}_{18}$$

$$^{157}_{64}\mathrm{Gd} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{1\lfloor}7s^{5}_{3} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{12\lceil}_{6\lfloor}7f^{30}_{18}$$

$$^{158}_{64}\mathrm{Gd} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{12\lceil}_{6\lfloor}7f^{30}_{18}$$

$$^{160}_{64}\mathrm{Gd} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{16\lceil}_{6\lfloor}7f^{38}_{26}$$

$$^{159}_{65}\mathrm{Tb} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{2\lfloor}7s^{10}_{6} + {}^{12\lceil}_{2\lfloor}7p^{26}_{22} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{12\lceil}_{6\lfloor}7f^{30}_{18}$$

$$^{156}_{66}\mathrm{Dy} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{12\lceil}_{6\lfloor}7f^{30}_{18}$$

$$^{158}_{66}\mathrm{Dy} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{12\lceil}_{6\lfloor}7f^{30}_{18}$$

$$^{160}_{66}\mathrm{Dy} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{16\lceil}_{6\lfloor}7f^{38}_{26}$$

$$^{161}_{66}\mathrm{Dy} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{-1\lfloor}7s^{3}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{16\lceil}_{6\lfloor}7f^{38}_{26}$$

$$^{162}_{66}\mathrm{Dy} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{16\lceil}_{6\lfloor}7f^{38}_{26}$$

$$^{163}_{66}\mathrm{Dy} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{-1\lfloor}7s^{7}_{9} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{16\lceil}_{6\lfloor}7f^{38}_{26}$$

$$^{164}_{66}\mathrm{Dy} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{20\lceil}_{6\lfloor}7f^{46}_{34}$$

$$^{165}_{67}\mathrm{Ho} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{0\lfloor}7s^{4}_{4} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{20\lceil}_{6\lfloor}7f^{46}_{34}$$

$$^{162}_{68}\mathrm{Er} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{16\lceil}_{6\lfloor}7f^{38}_{26}$$

$$^{164}_{68}\mathrm{Er} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{20\lceil}_{6\lfloor}7f^{46}_{34}$$

$$^{166}_{68}\mathrm{Er} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{20\lceil}_{6\lfloor}7f^{46}_{34}$$

$$^{167}_{68}\mathrm{Er} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{1\lfloor}7s^{9}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{20\lceil}_{6\lfloor}7f^{46}_{34}$$

$$^{168}_{68}\mathrm{Er} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{24\lceil}_{6\lfloor}7f^{54}_{42}$$

$$^{170}_{68}\mathrm{Er} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{24\lceil}_{6\lfloor}7f^{54}_{42}$$

$$^{169}_{69}\mathrm{Tm} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{2\lfloor}7s^{6}_{2} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{24\lceil}_{6\lfloor}7f^{54}_{42}$$

$$^{168}_{70}\mathrm{Yb} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{20\lceil}_{6\lfloor}7f^{46}_{34} + {}^{2\lceil}_{2\lfloor}7g^{6}_{2}$$

$$^{170}_{70}\mathrm{Yb} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{24\lceil}_{6\lfloor}7f^{54}_{42} + {}^{2\lceil}_{2\lfloor}7g^{6}_{2}$$

$$^{171}_{70}\mathrm{Yb} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{1\lfloor}7s^{5}_{3} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{24\lceil}_{6\lfloor}7f^{54}_{42} + {}^{2\lceil}_{2\lfloor}7g^{6}_{2}$$

$$^{172}_{70}\mathrm{Yb} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{24\lceil}_{6\lfloor}7f^{54}_{42} + {}^{2\lceil}_{2\lfloor}7g^{6}_{2}$$

$$^{173}_{70}\mathrm{Yb} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{1\lfloor}7s^{9}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{24\lceil}_{6\lfloor}7f^{54}_{42} + {}^{2\lceil}_{2\lfloor}7g^{6}_{2}$$

$$^{174}_{70}\mathrm{Yb} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{24\lceil}_{6\lfloor}7f^{54}_{42} + {}^{4\lceil}_{2\lfloor}7g^{10}_{6}$$

$$^{176}_{70}\mathrm{Yb} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{2\lceil}_{2\lfloor}7g^{6}_{2}$$

$$^{175}_{71}\mathrm{Lu} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{2\lfloor}7s^{6}_{2} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{2\lceil}_{2\lfloor}7g^{6}_{2}$$

$$^{176}_{71}\mathrm{Lu} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{2\lfloor}7s^{8}_{4} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{2\lceil}_{2\lfloor}7g^{6}_{2}$$

$$^{174}_{72}\mathrm{Hf} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{24\lceil}_{6\lfloor}7f^{54}_{42} + {}^{4\lceil}_{6\lfloor}7g^{14}_{2}$$

$$^{176}_{72}\mathrm{Hf} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{4\lceil}_{6\lfloor}7g^{14}_{2}$$

$$^{177}_{72}\mathrm{Hf} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{-1\lfloor}7s^{3}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{4\lceil}_{6\lfloor}7g^{14}_{2}$$

$$^{178}_{72}\mathrm{Hf} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{4\lceil}_{6\lfloor}7g^{14}_{2}$$

$$^{179}_{72}\mathrm{Hf} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{-1\lfloor}7s^{7}_{9} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{4\lceil}_{6\lfloor}7g^{14}_{2}$$

$$^{180}_{72}\mathrm{Hf} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{8\lceil}_{6\lfloor}7g^{22}_{10}$$

$$^{180}_{73}\mathrm{Ta} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{0\lfloor}7s^{2}_{2} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{8\lceil}_{6\lfloor}7g^{22}_{10}$$

$$^{181}_{73}\mathrm{Ta} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{0\lfloor}7s^{4}_{4} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{8\lceil}_{6\lfloor}7g^{22}_{10}$$

$$^{180}_{74}\mathrm{W} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{8\lceil}_{6\lfloor}7g^{22}_{10}$$

$$^{182}_{74}\mathrm{W} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{8\lceil}_{6\lfloor}7g^{22}_{10}$$

$$^{183}_{74}\mathrm{W} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{1\lfloor}7s^{9}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{8\lceil}_{6\lfloor}7g^{22}_{10}$$

$$^{184}_{74}\mathrm{W} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{12\lceil}_{6\lfloor}7g^{30}_{18}$$

$$^{186}_{74}\mathrm{W} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{12\lceil}_{6\lfloor}7g^{30}_{18}$$

$$^{185}_{75}\mathrm{Re} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{2\lfloor}7s^{6}_{2} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{12\lceil}_{6\lfloor}7g^{30}_{18}$$

$$^{187}_{75}\mathrm{Re} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{2\lfloor}7s^{10}_{6} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{12\lceil}_{6\lfloor}7g^{30}_{18}$$

$$^{184}_{76}\mathrm{Os} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{12\lceil}_{10\lfloor}7g^{34}_{14}$$

$$^{186}_{76}\mathrm{Os} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{12\lceil}_{10\lfloor}7g^{34}_{14}$$

$$^{187}_{76}\mathrm{Os} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{-1\lfloor}7s^{7}_{9} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{12\lceil}_{10\lfloor}7g^{34}_{14}$$

$$^{188}_{76}\mathrm{Os} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{16\lceil}_{10\lfloor}7g^{42}_{22}$$

$$^{189}_{76}\mathrm{Os} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{-1\lfloor}7s^{3}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{16\lceil}_{10\lfloor}7g^{42}_{22}$$

$$^{190}_{76}\mathrm{Os} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{16\lceil}_{10\lfloor}7g^{42}_{22}$$

$$^{192}_{76}\mathrm{Os} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{20\lceil}_{10\lfloor}7g^{50}_{30}$$

$$^{191}_{77}\mathrm{Ir} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{0\lfloor}7s^{8}_{8} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{16\lceil}_{10\lfloor}7g^{42}_{22}$$

$$^{193}_{77}\mathrm{Ir} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{0\lfloor}7s^{4}_{4} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{20\lceil}_{10\lfloor}7g^{50}_{30}$$

$$^{190}_{78}\mathrm{Pt} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{16\lceil}_{10\lfloor}7g^{42}_{22}$$

$$^{192}_{78}\mathrm{Pt} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{20\lceil}_{10\lfloor}7g^{50}_{30}$$

$$^{194}_{78}\mathrm{Pt} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{20\lceil}_{10\lfloor}7g^{50}_{30}$$

$$^{195}_{78}\mathrm{Pt} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{1\lfloor}7s^{9}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{20\lceil}_{10\lfloor}7g^{50}_{30}$$

$$^{196}_{78}\mathrm{Pt} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{24\lceil}_{10\lfloor}7g^{58}_{38}$$

$$^{198}_{78}\mathrm{Pt} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{24\lceil}_{10\lfloor}7g^{58}_{38}$$

$$^{197}_{79}\mathrm{Au} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{2\lfloor}7s^{6}_{2} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{6\lfloor}7f^{62}_{50} + {}^{24\lceil}_{10\lfloor}7g^{58}_{38}$$

$$^{196}_{80}\mathrm{Hg} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{24\lceil}_{10\lfloor}7g^{58}_{38}$$

$$^{198}_{80}\mathrm{Hg} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{24\lceil}_{10\lfloor}7g^{58}_{38}$$

$$^{199}_{80}\mathrm{Hg} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{-1\lfloor}7s^{7}_{9} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{24\lceil}_{10\lfloor}7g^{58}_{38}$$

$$^{200}_{80}\mathrm{Hg} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{28\lceil}_{10\lfloor}7g^{66}_{46}$$

$$^{201}_{80}\mathrm{Hg} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{-1\lfloor}7s^{3}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{28\lceil}_{10\lfloor}7g^{66}_{46}$$

$$^{202}_{80}\mathrm{Hg} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{28\lceil}_{10\lfloor}7g^{66}_{46}$$

$$^{204}_{80}\mathrm{Hg} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{-1\lfloor}7s^{1}_{3} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{32\lceil}_{10\lfloor}7g^{74}_{54}$$

$$^{203}_{81}\mathrm{Tl} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{0\lfloor}7s^{8}_{8} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{28\lceil}_{10\lfloor}7g^{66}_{46}$$

$$^{205}_{81}\mathrm{Tl} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{0\lfloor}7s^{4}_{4} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{32\lceil}_{10\lfloor}7g^{74}_{54}$$

$$^{204}_{82}\mathrm{Pb} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{32\lceil}_{10\lfloor}7g^{74}_{54}$$

$$^{206}_{82}\mathrm{Pb} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{1\lfloor}7s^{7}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{32\lceil}_{10\lfloor}7g^{74}_{54}$$

$$^{207}_{82}\mathrm{Pb} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{1\lfloor}7s^{9}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{32\lceil}_{10\lfloor}7g^{74}_{54}$$

$$^{208}_{82}\mathrm{Pb} = {}^{111}_{49}\mathrm{In} + {}^{1\lceil}_{1\lfloor}7s^{3}_{1} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{36\lceil}_{10\lfloor}7g^{82}_{62}$$

$$^{209}_{83}\mathrm{Bi} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{2\lfloor}7s^{6}_{2} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{6\lfloor}7d^{46}_{34} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{36\lceil}_{10\lfloor}7g^{82}_{62}$$

$$^{209}_{84}\mathrm{Po} = {}^{111}_{49}\mathrm{In} + {}^{2\lceil}_{-1\lfloor}7s^{3}_{5} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{10\lfloor}7d^{50}_{30} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{36\lceil}_{10\lfloor}7g^{82}_{62}$$

$$^{210}_{84}\mathrm{Po} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{-1\lfloor}7s^{5}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{10\lfloor}7d^{50}_{30} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{36\lceil}_{10\lfloor}7g^{82}_{62}$$

$$^{210}_{85}\mathrm{At} = {}^{111}_{49}\mathrm{In} + {}^{3\lceil}_{0\lfloor}7s^{6}_{6} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{10\lfloor}7d^{50}_{30} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{36\lceil}_{10\lfloor}7g^{82}_{62}$$

$$^{211}_{85}\mathrm{At} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{0\lfloor}7s^{8}_{8} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{10\lfloor}7d^{50}_{30} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{36\lceil}_{10\lfloor}7g^{82}_{62}$$

$$^{211}_{86}\mathrm{Rn} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{1\lfloor}7s^{9}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{10\lfloor}7d^{50}_{30} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{36\lceil}_{10\lfloor}7g^{82}_{62}$$

$$^{219}_{86}\mathrm{Rn} = {}^{111}_{49}\mathrm{In} + {}^{4\lceil}_{1\lfloor}7s^{9}_{7} + {}^{12\lceil}_{6\lfloor}7p^{30}_{18} + {}^{20\lceil}_{10\lfloor}7d^{50}_{30} + {}^{28\lceil}_{10\lfloor}7f^{66}_{46} + {}^{44\lceil}_{10\lfloor}7g^{98}_{78}$$

Hereafter, the eighth shell is packed.

D Nomenclature

In the table below the explanation mark denotes a new definition.

	$\mathbf{a}, \hat{\mathrm{a}}, a$	Generalised, each vector is expresses as the product of a scalar that carries dimensionality and a unit vector, *e. g.* $\mathbf{a} = a\hat{\mathrm{a}}$
	$\hat{\mathrm{x}}$, $\hat{\mathrm{y}}$, $\hat{\mathrm{z}}$	Unit vectors $\hat{\mathrm{z}} = \hat{\mathrm{x}} \times \hat{\mathrm{y}}$ defining an Euclidean space [XYZ] in $\mathbb{C}^3$.
	e	the Euler number to express a complex number $e^{i\theta}$ as a rotation.
	$\sqcup_{\mathrm{o}}$	An elementary quantity $\sqcup$, convention dictates the exceptions for e, ℓ, h, $\mathcal{U}$ and c (see below)
!	l_{o}	elementary length
!	t_{o}	elementary time
	e	elementary charge $e = 1.602176634 \times 10^{-19}$ coulombs
!	ℓ	elementary electromotive charge $\lvert\ell\rvert = e$ coulombs

	h	Planck's constant $h = 6.62607015 \times 10^{-34}$
!	$\mathcal{U}$	elementary latentness $\mathcal{U} = 1/h$ and has unit changelings
!	$\mathcal{T}$	The transportivity of space, $\mathcal{T} = c^2$
!	$\boldsymbol{\phi}_o, \phi_o$	Magnetic vector emflux, its magnitude
!	$\boldsymbol{\Upsilon}_o, \Upsilon_o$	Electromotive vector emflux, its magnitude
!	$\boldsymbol{\Lambda}_o, \Lambda_o$	Magnetic latency emflux vector, its magnitude
!	$\boldsymbol{\Xi}_o, \Xi_o$	Electromotive latency emflux vector, its magnitude
!	X_o	The complexification constant $e^{i(pi/4+\delta_x)}$
!	$\vec{\mathcal{E}}$	Energy as a complex quantity, $\vec{\mathcal{E}} = \mathcal{E}e^{i\theta}$
!	m	An emtron is a solitary wave that propagates on a path that is either straight, circular, or spherular.
!	$\bar{m}, \dot{m}$	The over-accented bar indicates an anti-emtron, and the under-accented dot the contra-emtron.

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Typographic note: This article is typeset with LaTeX and using a mathematical script font to define the symbols $\ell\mathcal{U}m$ as `\mathscr{elm}`, or $\mathcal{M}$ as `\mathbfcal{M}` by including in the preamble.

```
\PassOptionsToPackage{widespace}{fourier}
\RequirePackage{fourier}
\linespread{1.05} % a bit more
\PassOptionsToPackage{
scr=boondoxo, % gives \mathscr and \mathbfscr
scrscaled=1.10,
cal=stix, % gives \mathcal and \mathbfcal
}{mathalpha}
\RequirePackage{mathalpha}
```