Towards a Quantum Unified Field Theory: The Special Orthogonal Gauge Group $R(3)SO(3)$ in $\mathbb{R}^9$, and Quantised Topological Electromagnetic Solitons

Anton Lorenz Vrba

Ryde, UK

anton.vrba@neophysics.org

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Abstract: A mathematical framework is introduced in which gauge symmetries and field interactions are unified through a higher-dimensional geometric structure. The special orthogonal gauge group $R(3)\mathrm{SO}(3)$, embedded in $\mathbb{R}^9$, supports a formulation where Maxwell's equations, spatial quantisation, and interaction fields arise naturally from a shared underlying principle.

At the core lies a system of field equations involving vector cross and dot operations that reproduce classical Maxwell behaviour while suggesting deeper topological and structural constraints. These field equations further imply a discrete geometry of space, potentially addressing questions of quantisation and field stability from first principles.

This approach offers a mathematically smooth, singularity-free alternative to conventional Lie-algebra-based gauge theories, embedding known field laws in a structure that supports both unification and quantisation. In this setting, gravitational and weak interactions may be seen as emergent from symmetry breaking, while solitonic structures offer insight into particle structure, mass, and charge.

The framework proposes a generalisation of gauge theory that invites further examination—both as a conceptual unification and as a constructive model for fundamental physical interactions.

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3. *! Definition load and glue charge changed to load, fervour and vigour charge.*
4. *! Structure of electron and proton changed to take three charge types into account.*
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For minor changes and errata visit: https://r3so3.com/errata/

A New Perspective on Gauge Symmetry

The connection between physical forces and underlying symmetries is a cornerstone of modern theoretical physics. Traditionally, this connection has been encoded through gauge theory frameworks based on Lie algebras and infinitesimal generators. While highly successful, these tools leave certain foundational questions open—particularly those concerning the discrete structure of space, field quantisation, and the role of singularities in physical models.

This work explores a complementary formulation, built upon full vectorial operations within a higher-dimensional rotational structure. It seeks to retain the geometric intuition behind gauge symmetry while allowing for a more direct embedding of field equations, potentially free of singularities and with naturally emergent quantised solutions.

The special orthogonal group $R(3)\mathrm{SO}(3) \subset \mathrm{SO}(3\times3)$, realised in a nine-dimensional manifold, forms the mathematical basis for this construction. The core field system introduced here leads directly to the vacuum Maxwell equations, while also permitting new interpretations of field aggregation, soliton formation, and energy transport.

This formulation begins from a geometric principle in which symmetry is expressed through full vectorial rotations across higher-dimensional spaces. The resulting field equations reproduce the structure of classical electromagnetism in vacuum, but do so as part of a broader system in which topological stability, quantised structure, and field coherence arise naturally from the underlying geometry.

The intent here is not to assert replacement, but to offer a shift in perspective—a foundational lens through which familiar physical laws may be reinterpreted and extended. In that sense, this work proceeds as an invitation: to explore the geometric and physical consequences of an extraordinary orthogonal symmetry, and to follow where its structure leads.

The structure of this manuscript reflects a gradual build-up from basic algebraic definitions toward applications in field theory, soliton modelling, and ultimately, reinterpretations of known forces. To guide the reader through this layered development, a detailed table of contents follows.

Contents

PART I

Guiding Philosophy

Maxwell's field equations are well understood, yet they do not provide a satisfactory explanation for the wave-particle duality of light. In the 1930s, polarisation was considered a superposition of linear states. However, it is now known that photons can acquire geometric phase (Berry phase), carry orbital angular momentum in addition to intrinsic spin, and undergo spontaneous parametric down-conversion (SPDC) into entangled pairs. These findings challenge the sufficiency of superposition as a complete description, suggesting that a deeper structure underlies photonic behaviour.

If we model the photon—the particle, not the wave—as a quantised topological electromagnetic vortex, then its intrinsic structure may be expressed as

$$\begin{bmatrix} \nu \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_s t & \sin\omega_s t \\ 0 & -\sin\omega_s t & \cos\omega_s t \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix},$$

where ν denotes the propagation vector, ϕ the magnetic flux, and ψ the electric flux within a quantised unit of space. This describes a soliton supported by its own fields—an entity that bridges the wave and particle descriptions.

We may generalise this by introducing complex or imaginary-axis rotations to describe polarisation:

$$\begin{bmatrix} \nu \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\theta}\cos\omega_s t & \sin\omega_s t \\ 0 & -\sin\omega_s t & e^{i\theta}\cos\omega_s t \end{bmatrix} \begin{bmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{bmatrix},$$

with ellipticity $\varepsilon = \cos\theta$, or by rotating the velocity vector itself:

$$\begin{bmatrix} \nu \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} e^{i\theta} & 0 & 0 \\ 0 & \cos\omega_s t & \sin\omega_s t \\ 0 & -\sin\omega_s t & \cos\omega_s t \end{bmatrix} \begin{bmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{bmatrix}.$$

Such forms suggest how light's propagation may respond to the medium while preserving the effective causal speed—an interplay that invites deeper examination of the underlying field structure.

This led to the identification of a closed vector system:

$$\mathcal{M}(\nu,\phi,\psi) := \left\{ \nu = \frac{\phi\times\psi}{\phi\cdot\phi}, \quad \phi = \frac{\psi\times\nu}{\nu\cdot\nu}, \quad \psi = \nu\times\phi \right\},$$

which encodes a mutually reinforcing relationship among three orthogonal field vectors. This system does not arise from imposed constraints or external potentials, but from an intrinsic symmetry in the vector algebra itself.

Rather than employing imaginary rotations, the framework introduces a nine-dimensional real vector space, mathematically constructed to support transformations that extend conventional rotational symmetries. The orthogonal gauge groups $SO(3\times 3)$ and $R(3)SO(3)$, embedded in $\mathbb{R}^9$, offer a natural structure for field interactions that respect this extended symmetry.

Within this framework, topologically structured solitons—including spherular or multi-axial vortices—can be described by more general rotation matrices. For example:

$$\begin{bmatrix} \nu \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} c_\varnothing c_w & s_\varnothing c_w & -s_w \\ -c_s s_\varnothing + s_s c_\varnothing s_w & c_s c_\varnothing + s_s s_\varnothing s_w & s_s c_w \\ s_s s_\varnothing + c_s c_\varnothing s_w & -s_s c_\varnothing + c_s s_\varnothing s_w & c_s c_w \end{bmatrix} \begin{bmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{bmatrix}.$$

This description generalises the rotation to three angles, accommodating richer solitonic behaviour. Its full implications unfold in the subsequent sections, where the geometry of these rotations begins to illuminate questions of field layering, structure, and interaction.

1 $R(3)SO(3)$: Special Orthogonal Gauge Group

1.1 The Ansatz

AXIOM 1.1: *Cross Product Structure in $SO(3)$.*
For any rotation matrix $R \in SO(3)$, the third row is given by the cross product of the first two rows:

$$R_3 = R_1 \times R_2.$$

This follows from the definition of $SO(3)$, which requires that the rows of R form an orthonormal right-handed basis in $\mathbb{R}^3$, ensuring that R_3 is uniquely determined by R_1 and R_2.

DEFINITION 1.1: *Cyclicity of Unit Vector Cross Products.* A set of unit vectors $\{\vec{a}, \vec{b}, \vec{c}\}$ in $SO(3)$ satisfies *unit cyclicity* if it obeys the transformation:

$$\vec{a}_m = \vec{b}_n \times \vec{c}_n, \quad \vec{b}_m = \vec{c}_n \times \vec{a}_m, \quad \vec{c}_m = \vec{a}_m \times \vec{b}_m,$$

with $m = n+1$, such that the sequence remains periodic and satisfies $\vec{a}_m = \vec{a}_0$, $\vec{b}_m = \vec{b}_0$, and $\vec{c}_m = \vec{c}_0$.

This condition, unit cyclicity, ensures that the vectors $\vec{a}, \vec{b}, \vec{c}$ maintain their mutual orthogonality and unit length under successive transformations, stabilising the rotational sequence without convergence or divergence.

Consider a solution matrix $S \in SO(3)$, where the rows are unit vectors:

$$\vec{a} := S_1, \quad \vec{b} := S_2, \quad \vec{c} := S_3.$$

Cross product cyclicity guarantees the following properties:

1. The vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ retain their orthogonal orientation, meaning a reference frame undergoing rotation preserves its mutually perpendicular directions.
2. If cyclicity = 1, then the vectors remain unit vectors throughout repeated transformations.
3. If cyclicity < 1, then under repeated cross-product transformations, $\vec{a}$, $\vec{b}$, and $\vec{c}$ converge to zero.
4. If cyclicity > 1, then under repeated cross-product transformations, $\vec{a}$, $\vec{b}$, and $\vec{c}$ diverge to infinity.
5. While unit cyclicity heuristically implies $\det S = \pm 1$, the reverse is strictly true: $\det S = 1$ ensures unit cyclicity, but unit cyclicity may still allow for $\det S = -1$ in specific cases.

The condition $\det S = 1$ ensures proper rotations (i.e., no reflections or change in handiness) and preserves inner products, guaranteeing that unit vectors remain mutually orthogonal.

Now, if S contains complex-valued coefficients of unit modulus, then in general, $\det S \neq 1$, which has traditionally been interpreted as a loss of orthogonality. However, by designing a cross product which normalises itself:

$$\vec{a}_m = \frac{\vec{b}_n \times \vec{c}_n}{\vec{b}_n \cdot \vec{b}_n}, \quad \vec{b}_m = \frac{\vec{c}_n \times \vec{a}_m}{\vec{a}_m \cdot \vec{a}_m}, \quad \vec{c}_m = \vec{a}_m \times \vec{b}_m,$$

unit cyclicity is restored, ensuring that the cross-product sequence remains structurally valid. However, for these transformations to represent proper rotations, additional constraints must be imposed to enforce $\det S$, a condition that will be addressed in detail in Section 1.2 (pp. 8).

DEFINITION 1.2: *Field Equation System in $R(3)SO(3)$.* The *field equation system* $\mathcal{M}$ in the $R(3)SO(3)$ framework is defined as:

$$\mathcal{M}(\nu,\phi,\psi) := \left\{ \nu = \frac{\phi\times\psi}{\phi\cdot\phi}, \quad \phi = \frac{\psi\times\nu}{\nu\cdot\nu}, \quad \psi = \nu\times\phi \right\},$$

where ν is the unit velocity vector, ϕ is the magnetic field vector, and ψ is the electric field vector.

This self-consistent vector system will be shown to recover Maxwell's equations in vacuum, and also support structured, quantised solutions beyond classical electromagnetism, we readily identify in the system following:

Poynting's energy flow: $\nu = \dfrac{\phi\times\psi}{\phi\cdot\phi}$,

Ampère-Maxwell Law (vacuum): $\phi = \dfrac{\psi\times\nu}{\nu\cdot\nu}$,

Faraday's Law of Induction: $\psi = \nu\times\phi$.

If $\{\nu,\phi,\psi\}$ are time-dependent, then $\mathcal{M}$ represents a wave equation system describing solitons. The trajectory of the wave is given by

$$\vec{s} = \int \nu \, \mathrm{d}t,$$

which can be open (photon), closed circular, or closed spherular paths. While conventional wave theory in vacuum imposes linear propagation along geodesics, the framework developed here permits coherent, curved propagation paths—including circular and spherular forms—emerging from internal field geometry. The construction of such spherular paths is formalised later in Lemma 5.2 (pp. 30), and visualised in Figure 1 (pp. 9).

1.2 Triaxial Rotations in $SO(3)$: A Restricted Rotational Framework

This framework adopts a specific subset of the full $SO(3)$ rotation group, referred to as *triaxial rotations*. These are parameterised by a triple of angles $(\alpha_s, \alpha_\varnothing, \alpha_w)$, corresponding to sequential rotations about the axes x, y, and z, where $\mathsf{z} = \mathsf{x}\times\mathsf{y}$.

The restriction to triaxial rotations serves a twofold purpose: it facilitates extension to higher-dimensional rotational structures, such as $R(3)SO(3)$, and preserves consistency when generalising from translational to angular quantities. This shift—replacing spatial displacement vectors (in metres) with angular displacement vectors (in radians), but retaining the same orthonormal basis—establishes a principle we refer to as *frame duality*.

DEFINITION 1.3: *Angle Vector and Frame Duality.* We define the *angle vector* as

$$\vec{\omega} = \hat{\mathsf{x}}\,\alpha_s + \hat{\mathsf{y}}\,\alpha_\varnothing + \hat{\mathsf{z}}\,\alpha_w,$$

where the angles $\alpha_s, \alpha_\varnothing, \alpha_w$ are expressed in radians. The unit vectors $(\hat{\mathsf{x}}, \hat{\mathsf{y}}, \hat{\mathsf{z}})$ maintain their usual spatial interpretation, but now also represent the axes of angular displacement.

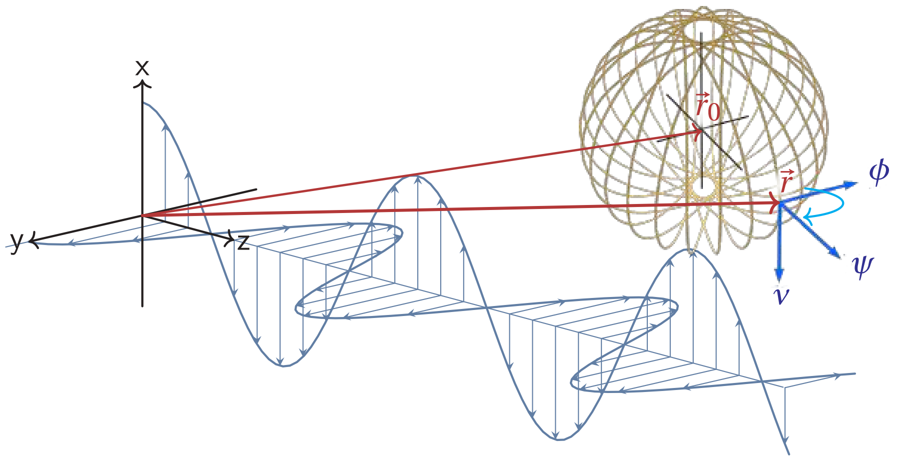
This dual usage of the frame allows for a unified treatment of translational and rotational quantities, forming the basis of frame duality: a conceptual symmetry in which directionality is preserved across both spatial and angular domains.

The angle vector $\vec{\varpi}$ defines a sequential rotation matrix $I^{(\vec{\varpi})}$, constructed by applying three component rotations on the identity matrix I—first about z by α_w, then about y by $\alpha_ø$, and finally about x by α_s. This follows the triaxial rotation convention introduced by Bryan and Tait.

The structure gives rise to three canonical forms of the rotation matrix, which respectively describe:

- a planar photonic mode ($\vec{\varpi} = \hat{x}\,\alpha_s$),
- a toroidal mode ($\vec{\varpi} = \hat{x}\,\alpha_s + \hat{y}\,\alpha_ø$),
- and a fully spherular mode ($\vec{\varpi} = \hat{x}\,\alpha_s + \hat{y}\,\alpha_ø + \hat{z}\,\alpha_w$).

$$\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_s & s_s \\ 0 & -s_s & c_s \end{bmatrix}}^{\vec{\varpi}=\hat{x}\,\alpha_s} \quad \overbrace{\begin{bmatrix} c_ø & 0 & -s_ø \\ s_s s_ø & c_s & s_s c_ø \\ c_s s_ø & -s_s & c_s c_ø \end{bmatrix}}^{\vec{\varpi}=\hat{x}\,\alpha_s+\hat{y}\,\alpha_ø} \quad \overbrace{\begin{bmatrix} c_ø c_w & s_ø c_w & -s_w \\ -c_s s_ø + s_s c_ø s_w & c_s c_ø + s_s s_ø s_w & s_s c_w \\ s_s s_ø + c_s c_ø s_w & -s_s c_ø + c_s s_ø s_w & c_s c_w \end{bmatrix}}^{\vec{\varpi}=\hat{x}\,\alpha_s+\hat{y}\,\alpha_ø+\hat{z}\,\alpha_w}$$



$$\nu = \frac{(\phi\times\psi)}{\phi\times\phi},\quad \phi = \frac{(\phi\times\nu)}{\nu\times\nu},\quad \psi = \nu\times\phi,\quad \vec{r} = \vec{r}_0 + \int \nu\,\mathrm{d}t$$

Figure 1: Here a travelling plane wave, with phase velocity c, is juxtaposed to a soliton described by the below solution of $\mathcal{M}$, both satisfying the Maxwell equations in vacuum. It is a gyration (spinning vortex) propagating with velocity $\|\nu\| = c$ along a spherular path defined by $\vec{r}$.

$$\begin{bmatrix} \nu \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} c_ø c_w & s_ø c_w & -s_w \\ -c_s s_ø + s_s c_ø s_w & c_s c_ø + s_s s_ø s_w & s_s c_w \\ s_s s_ø + c_s c_ø s_w & -s_s c_ø + c_s s_ø s_w & c_s c_w \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \quad \text{where} \quad \begin{aligned} c_s &:= \cos\omega_s t \\ s_ø &:= \sin\omega_ø t \\ &\textit{etc.} \end{aligned}$$

DEFINITION 1.4: *Spherular Motion.* A *spherular* rotation is defined as a trigonometric three-dimensional motion generated by triaxial angles $(\alpha_s, \alpha_\emptyset, \alpha_w)$, enclosing a sphere symmetrically without reaching its poles. The term captures a structure that is neither strictly spherical nor spheroidal, but maintains equatorial and axial symmetry across all three axes of rotation.

This triaxial formulation restricts the degrees of rotational freedom in a way that is both geometrically structured and algebraically closed. As we extend beyond conventional three-dimensional rotations, this constraint provides a stable foundation upon which higher-order structures may be defined.

In the next section, we introduce the algebraic generalisation of these triaxial angles, transitioning from classical $SO(3)$ to the broader symmetry space of $R(3)SO(3)$, where multiple triaxial frames interact within a unified topological algebra.

1.3 Expanding to $R(3)SO(3)$

Classical rotations in SO(3) describe spatial transformations adequately at macroscopic scales. However, they fall short in accounting for the structured, multi-layered rotational behaviour observed in quantum systems, such as spin–orbit coupling or generation symmetry. To address these limitations, we explore a natural extension into a structured higher-dimensional space.

During the development of this framework, it became evident that the pseudo-dimension introduced by the complex operator validates the field equation system $\mathcal{M}$. However, to fully accommodate quantum properties—such as the three-family structure of fundamental particles or the photon's orbital angular momentum, among others—a higher-dimensional space is required. This necessitates an expansion of the framework to $\mathbb{R}^9$, leading to the construction of $R(3)SO(3)$, an extended $SO(3)$ group characterised by *ternary rotations* in higher dimensions.

The guiding structural insight of this framework is that physical spaces should be treated as structured mathematical entities which preserve rotational symmetry while incorporating additional degrees of freedom. Conventional three-dimensional spaces, defined by $SO(3)$ rotations, describe ordinary spatial transformations, but they are insufficient for capturing the subtleties of quantum structure.

1.3.1 Nine Dimensions and Three 3D-Spaces: Yellow, Cyan and Magenta

To address the limitations inherent in SO(3), a hierarchical nine-dimensional spatial structure is introduced by "joining" three spaces—*yellow*, *cyan*, and *magenta*—each governed by SO(3) symmetries:

$$\{Y, C, M\} = \{S \in \mathbb{R}^{3\times 3} \mid SS^{\mathrm{T}} = I, \det(S) = 1, \text{ and } S \in T\} \subset SO(3)$$

Hence, $\{Y, C, M\} \subset SO(3)$, where T is the triaxial rotation matrix defined in Definition 1.5.

However, each of these component spaces must ultimately be structured under the symmetries of $R(3)SO(3)$. This is achieved by fixing their mutual orientation and embedding the yellow, cyan, and magenta spaces within a superordinate SO(3×3) group, thereby ensuring that all transformations preserve the required rotational invariances. Only under this construction can the orthogonal group $R(3)SO(3)$ be defined consistently.

A *radial reduction framework* is introduced to *join and orient* the three spaces. This is accomplished by defining a set of unit spatial vectors that encode their geometric relationships:

$$\{\hat{m} = \hat{y} \times \hat{c}\} \in \{\mathbb{R}^3 \times \mathbb{R}^3\}.$$

These unit vectors represent the radially reduced spaces, where, for example,

$$a\hat{m} = \hat{m}\sqrt{a_{m_x}^2 + a_{m_y}^2 + a_{m_z}^2},$$

so that $a\hat{m}$ is interpreted as a vector of magnitude a, oriented within the magenta space M.

DEFINITION 1.5: *Angle Vector Defines Triaxial Rotation Matrix.* The *hyper-angle vector* encodes rotational hyper-displacements:

$$\vec{w} = \hat{y}\lambda_s + \hat{c}\lambda_\varnothing + \hat{m}\lambda_w.$$

The corresponding hyper-triaxial rotation matrix $\mathcal{T}$ is defined as:

$$\mathcal{T} = I^{(\vec{w})},$$

where I is the identity matrix, and $I^{(\vec{w})}$ represents the rotation matrix obtained by sequentially rotating I by λ_w about the initial m axis, then by $\lambda_\varnothing$ about the initial y axis, and finally by λ_s about the initial m axis.

DEFINITION 1.6: *The Special Orthogonal Group $SO(3\times 3)$.*

$$\mathcal{W} := \{M = Y \times C\} = \left\{\mathcal{S} \in (\mathbb{R}^3)^{3\times 3} \mid \mathcal{S}\mathcal{S}^{\mathrm{T}} = I,\ \det\left(\mathcal{S}\right) = 1, \text{ and } \mathcal{S} \in \mathcal{T}\right\} \subset SO(3\times 3)$$

Therefore, $\mathcal{W} \subset SO(3\times 3)$.

This construction maps the hyper-space $\mathcal{W} := \{M = Y \times C\}$ to the group $SO(3\times 3)$. However, to describe oriented rotations from within Y toward C, the more refined group $R(3)SO(3)$ is required.

In this context, $SO(3\times 3)$ ensures rotational integrity for the hyper-space defined by the three axes $\{\hat{y}, \hat{c}, \hat{m}\}$, each of which represents a radially reduced $SO(3)$ space. The group $SO(3\times 3)$ introduces two additional degrees of rotational freedom to each of the spaces $\{Y, C, M\}$, while preserving their three-dimensional character. These additional degrees of rotational freedom are encapsulated by the group $R(3)SO(3)$.

Having established the extended rotational framework of $R(3)SO(3)$, we now return to the field equation system introduced earlier, and re-anchor our attention to the structured field dynamics that arise within the three-dimensional spaces defined by this framework.

The matrix $I^{\alpha_s,\alpha_\varnothing,\alpha_w}$ provides all rotation matrices S of interest, with each S residing in one of the spaces

$$\{Y,\ C,\ M\}.$$

Each such matrix S yields a solution to the field equation system

$$\mathcal{M}(\nu,\phi,\psi) := \left\{\nu = \frac{\phi\times\psi}{\phi\cdot\phi}, \quad \phi = \frac{\psi\times\nu}{\nu\cdot\nu}, \quad \psi = \nu\times\phi\right\},$$

where the unit velocity vector ν is given by S_1, the first row of S; the magnetic field vector ϕ by S_2; and the electric field vector ψ either by S_3, or equivalently, by computing $\psi = \nu\times\phi$.

The orientation of the three colour spaces is defined by the relation $M = Y \times C$. One of these spaces represents our physical reality; we choose Y as that space—the space in which *you* and I interact. The spaces C and M are orthogonal to Y and, in this framework, may be interpreted mathematically as "imaginary" dimensions.

To describe rotations into these imaginary spaces in a consistent and structured manner, we now introduce a ternary number system that preserves the three-dimensional character of each space Y, C, and M.

DEFINITION 1.7: *Ternary Operators and Ternary Numbers.* The cyclic cross-product relations

$$\hat{y} = \hat{c}\times\hat{m}, \quad \hat{c} = \hat{m}\times\hat{y}, \quad \hat{m} = \hat{y}\times\hat{c} \tag{1}$$

- define the orientation of the spaces Y, C, and M,
- introduce the ternary operators $\{y, c, m\}$,
- and establish the following ternary-complex equivalences:

$$\begin{aligned} y &:= -c^2 = cm = -mc = -y^{-1},\\ c &:= -m^2 = my = -ym = -c^{-1},\\ m &:= -y^2 = yc = -cy = -m^{-1}. \end{aligned}$$

A ternary number r is defined as a sequence of rotations:

$$\begin{aligned} r = y\,e^{(c\alpha+m\beta)} &:= y\,e^{c\alpha}e^{m\beta}\\ &= y\,e^{c\alpha}\cos\beta + m\left\|y\,e^{c\alpha}\right\|\sin\beta\\ &= (y\cos\alpha + c\sin\alpha)\cos\beta + m\sin\beta\\ &= y\cos\alpha\cos\beta + c\sin\alpha\cos\beta + m\sin\beta \end{aligned}$$

and always respects the right-hand rule, as defined by (1), yielding any of the following cyclically equivalent forms:

$$r_y = y\,e^{(c\alpha+m\beta)} \tag{2}$$

$$r_c = c\,e^{(m\alpha+y\beta)}$$

$$r_m = m\,e^{(y\alpha+c\beta)} \tag{3}$$

These expressions define ternary numbers in the near-field $\mathbb{T}\subset\mathbb{R}^3$, governed by non-distributive multiplication and a cyclic imaginary structure. The ordering of the rotations in (2)–(3) adheres to the right-hand rule implicit in the ternary algebra.

DEFINITION 1.8: *Unit Vectors on $R(3)SO(3)$.* In this work, we repurpose Euler's classical notation for rotations on the complex number plane to express a structurally meaningful representation of vector orientations within the $R(3)SO(3)$ framework. The expression

$$\vec{a} = \hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha+\mathsf{z}\beta)}$$

denotes a unit vector initially aligned with the basis direction x (i.e., $\vec{a}'' = \hat{\mathsf{x}}$), and successively rotated: first by an angle α about the z-axis, yielding $\vec{a}' = \hat{\mathsf{x}}\cos\alpha + \hat{\mathsf{y}}\sin\alpha$; and then by an angle β from the xy-plane toward the z-axis, resulting in

$$\vec{a} = \hat{\mathsf{x}}\cos\alpha\cos\beta + \hat{\mathsf{y}}\sin\alpha\cos\beta + \hat{\mathsf{z}}\sin\beta.$$

This construct mirrors the geometric philosophy underlying the construction of $R(3)SO(3)$.

This representation introduces a novel operation in vector algebra—a *rotation-product*, denoted by $\odot$—defined by additive composition of angular arguments:

$$\hat{e}_{\mathsf{x}}^{(\mathsf{y}(\alpha_1+\alpha_2)+\mathsf{z}(\beta_1+\beta_2))} := \hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha_1+\mathsf{z}\beta_1)} \odot \hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha_2+\mathsf{z}\beta_2)}.$$

This directional composition, which preserves both orientation and structural form, serves as a natural algebraic extension complementing the dot and cross products in the $R(3)SO(3)$ framework.

NOTATION (Vectors in $R(3)SO(3)$). In $SO(3)$, let the vector $\vec{a} = a_x\hat{\mathsf{x}} + a_y\hat{\mathsf{y}} + a_z\hat{\mathsf{z}}$. To represent this vector within one of the spaces $\{Y, C, M\}$, we use the spatial–imaginary operator to explicitly place

$$\vec{a} = \hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha+\mathsf{z}\beta)}$$

into a given space:

$$y\,\vec{a} = a_x\hat{y}_{\mathsf{x}} + a_y\hat{y}_{\mathsf{y}} + a_z\hat{y}_{\mathsf{z}} = y\,\hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha+\mathsf{z}\beta)}$$

$$c\,\vec{a} = a_x\hat{c}_{\mathsf{x}} + a_y\hat{c}_{\mathsf{y}} + a_z\hat{c}_{\mathsf{z}} = c\,\hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha+\mathsf{z}\beta)}$$

$$m\,\vec{a} = a_x\hat{m}_{\mathsf{x}} + a_y\hat{m}_{\mathsf{y}} + a_z\hat{m}_{\mathsf{z}} = m\,\hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha+\mathsf{z}\beta)}$$

With vectors now explicitly situated in their respective spaces via the spatial–imaginary operators, we next define how these objects interact through a structured algebraic operation: the ternary cross product. This operation preserves magnitude, respects orientation, and aligns with the geometric foundations established by the right-hand rule on $\{\hat{y}, \hat{c}, \hat{m}\}$.

DEFINITION 1.9: *Ternary Cross Product.* Let

$$y\,\hat{a} = y\,\hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha_a+\mathsf{z}\beta_a)} \qquad \text{and} \qquad c\,\hat{b} = c\,\hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha_b+\mathsf{z}\beta_b)}$$

be two vectors in Y and C, respectively. Then the *ternary cross product*—which preserves vector length—is defined as:

$$y\,\hat{a} \times c\,\hat{b} := y \times c\,\sin\theta + \cos\theta\left(y\,\hat{a}\cos\vartheta + c\,\hat{b}\sin\vartheta\right),$$

where $y \times c = m$, and

$$\cos\vartheta = \hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha_a+\mathsf{z}\beta_a)} \cdot \hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha_b+\mathsf{z}\beta_b)}.$$

For the special case where both vectors lie in the same subspace Y:

$$\mathcal{y}\,\hat{a} = \mathcal{y}\,\hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha_a + \mathsf{z}\beta_a)} \qquad \text{and} \qquad \mathcal{y}\,\hat{b} = \mathcal{y}\,\hat{e}_{\mathsf{x}}^{(\mathsf{y}\alpha_b + \mathsf{z}\beta_b)},$$

the ternary cross product simplifies to:

$$\mathcal{y}\,\hat{a} \times \mathcal{y}\,\hat{b} := \mathcal{y} \times \mathcal{y}\, \sin\theta + \cos\theta \left(\mathcal{y}\,\hat{a} \odot \hat{b}\right),$$

where $\mathcal{y} \times \mathcal{y} = -\mathcal{m}$, and where $\odot$ denotes the *rotation-product* defined by angle composition in the Euler-inspired vector representation.

The cross product between vectors situated in different $R(3)SO(3)$ subspaces preserves both geometric structure and rotational integrity. However, as ternary numbers constructed from different cyclic bases belong to algebraically distinct subspaces, direct multiplication between them is not defined within the non-distributive ternary number field.

To reconcile this, we now establish a structural bridge between the ternary algebra and the corresponding vector geometry—an operation that allows cross-subspace interactions via a consistent and geometrically grounded product.

DEFINITION 1.10: *Ternary Number–Vector Duality via Cross Product.* Ternary numbers constructed from different cyclic bases—such as

$$r_{\mathcal{y}} = \mathcal{y}\,\mathrm{e}^{(\mathcal{c}\alpha + \mathcal{m}\beta)},$$

$$r_{\mathcal{c}} = \mathcal{c}\,\mathrm{e}^{(\mathcal{m}\alpha + \mathcal{y}\beta)},$$

—belong to distinct multiplicative subspaces defined by their respective axis operators $\mathcal{y}$ and $\mathcal{c}$. Because the ternary number field lacks distributivity and does not support direct cross-cyclic multiplication, the expression $r_{\mathcal{y}} \cdot r_{\mathcal{c}}$ is undefined in the algebraic sense.

To resolve this, we define a duality between ternary numbers and ternary vectors: the *cross-basis product* of two ternary numbers is defined using the ternary vector cross product. This product is denoted by the symbol $\otimes$, and is evaluated via:

$$r_{\mathcal{y}} \otimes r_{\mathcal{c}} \overset{\text{def}}{:=} \vec{r}_y \times \vec{r}_c,$$

where

$$\vec{r}_y := \mathcal{y}\,\hat{e}^{(\mathcal{c}\alpha + \mathcal{m}\beta)} \qquad \text{and} \qquad \vec{r}_c := \mathcal{c}\,\hat{e}^{(\mathcal{m}\alpha + \mathcal{y}\beta)}$$

are the corresponding vector realisations of $r_{\mathcal{y}}$ and $r_{\mathcal{c}}$.

This cross-basis product satisfies:

- *norm preservation*: $\|r_{\mathcal{y}} \otimes r_{\mathcal{c}}\| = \|r_{\mathcal{y}}\|\,\|r_{\mathcal{c}}\|$,
- *directional closure*: alignment with the ternary cyclic ordering $\mathcal{y} \times \mathcal{c} = \mathcal{m}$,
- *geometric realisability*: the product remains in $\mathbb{R}^3$ with well-defined orientation,
- *algebraic compatibility*: it extends the ternary number system into a closed geometric structure under cross-cyclic composition.

This establishes a duality between the algebra of ternary numbers and the geometry of ternary vectors, wherein the ternary cross product provides a consistent mechanism for inter-subspace composition.

Having established the algebraic–geometric duality that underlies inter-subspace structure, we now turn to the foundational constraint that ensures consistency across all such representations: the condition of unit cyclicity.

DEFINITION 1.11: *Ternary Unit Cyclicity in $R(3)SO(3)$.* The $SO(3)$ condition that $\det(S) = 1$ ensures that the matrix S represents a rotation that maintains the handedness (orientation) of the coordinate system. In the special orthogonal group $R(3)SO(3)$, this is generalised: the unit determinant condition is replaced by unit cyclicity $\operatorname{cyc}(S) = 1$. Cyclicity is defined as:

$$\operatorname{cyc} S := \frac{\|S_1\|^2 \|S_2\|^2}{(S_1 \cdot S_1)(S_2 \cdot S_2)} \det S,$$

where S_1 and S_2 denote the first and second rows of S, respectively. Here, $S_1 \cdot S_1$ is the standard (non-Hermitian) inner product, and $\|S_1\|^2$ is the sum of the squares of all coefficients in S_1.

The condition of unit cyclicity generalises the determinant constraint of $SO(3)$, ensuring that all structural and algebraic relations established in the previous sections are preserved under transformation.

All necessary structural components—vectorial rotation, ternary algebra, unit cyclicity, and geometric duality—have now been established. These elements converge in the formal construction of a new symmetry group: $R(3)SO(3)$. This group generalises $SO(3)$ by incorporating ternary structure and extending the field dynamics into a unified algebraic and geometric framework.

1.4 The Special Orthogonal Gauge Group $R(3)SO(3)$

DEFINITION 1.12 The group $R(3)SO(3)$ is defined by the hierarchical structure:

$$\{Y,\, C,\, M\} = \{S \in \mathbb{R}^{3\times 3} \mid SS^{\mathrm{T}} = I, \det(S) = 1, \text{ and } S \in T\} \subset SO(3),$$

$$\mathscr{W} := \{M = Y \times C\} = \{\mathcal{S} \in (\mathbb{R}^3)^{3\times 3} \mid \mathcal{S}\mathcal{S}^{\mathrm{T}} = I, \det\big(\mathcal{S}\big) = 1, \text{ and } \mathcal{S} \in \mathscr{T}\} \subset SO(3\times 3),$$

$$\{Y,\, C,\, M\} = \{S \in (\mathbb{R}^3)^{3\times 3} \mid SS^{\mathrm{T}} = I, \operatorname{cyc}(S) = 1, \text{ and } S \in \mathscr{T}\} \subset T(3)SO(3).$$

The three spaces $\{Y, C, M\}$ form a subgroup of $SO(3)$, defined by matrices T constructed through specific triaxial rotations (see Definition 1.5 (pp. 11)). These spaces are embedded in a structured nine-dimensional space $\mathscr{W}$, forming a hyper-$SO(3\times 3)$ framework. This space is defined by the unit hypervectors $\{\hat{y}, \hat{c}, \hat{m}\}$, which correspond to the radially reduced spaces $\{Y, C, M\}$, respectively.

These unit vectors yield the ternary operators $\{y, c, m\}$ (see Definition 1.7 (pp. 12)), which define a near-field of ternary numbers $\mathbb{T} \subset \mathbb{R}^3$, notable for lacking distributivity. Just as complex numbers may be viewed as rotations of the real number line, ternary numbers describe structured rotations across the three spaces $\{Y, C, M\} \subset SO(3)$, extending these to $\{Y, C, M\} \subset T(3)SO(3)$. This extension defines the extraordinary orthogonal group $R(3)SO(3)$, formally equivalent to $SO(3, \mathbb{T})$.

The group $R(3)SO(3)$ satisfies the following properties:

1. *Unit Cyclicity Condition:* Any matrix $S \in R(3)SO(3)$ satisfies $\operatorname{cyc} S = 1$, enabling:

2. *Field Equation System:* For unit vectors ν, ϕ, ψ defined as the rows of S, the field equation system is:

$$\mathcal{M}(\nu,\phi,\psi) := \left\{ \nu = \frac{\phi\times\psi}{\phi\cdot\phi}, \quad \phi = \frac{\psi\times\nu}{\nu\cdot\nu}, \quad \psi = \nu\times\phi \right\}. \tag{4}$$

   These equations are valid if and only if $\operatorname{cyc} S = 1$, and they define the foundational field system within $R(3)SO(3)$.

3. *Smooth Transformations:* All transformations in $R(3)SO(3)$ are continuously differentiable and integrable, ensuring analytical robustness.

4. *Closure and Group Properties:*

   a. *Closure:* $S_1, S_2 \in R(3)SO(3) \Rightarrow S_1 S_2 \in R(3)SO(3)$.

   b. *Identity:* There exists an identity element $I \in R(3)SO(3)$ such that $SI = IS = S$.

   c. *Inverses:* Each $S \in R(3)SO(3)$ has an inverse $S^{-1} \in R(3)SO(3)$ with $SS^{-1} = S^{-1}S = I$.

   d. *Associativity:* $(S_1S_2)S_3 = S_1(S_2S_3)$ for all $S_1, S_2, S_3 \in R(3)SO(3)$.

5. *Invariant Symmetries:* As an extension of $SO(3)$, $R(3)SO(3)$ preserves all SO(3) invariants while introducing ternary-structured symmetry transformations.

6. *Compactness:* $R(3)SO(3)$ is a smooth and differentiable gauge group that preserves unit cyclicity. Its compactness is defined through the finiteness and boundedness of the associated field solutions.

7. *Gauge Conditions:* The field equation system (4) provides natural gauge conditions for $R(3)SO(3)$, extending Maxwell's equations in vacuum to a generalised setting. The ternary algebraic structure unifies strong and electric forces within a Maxwell-like framework, supporting quantised and topologically stable field interactions.

The gauge group $R(3)SO(3)$ emerges as a structurally complete and algebraically rich extension of the classical rotation group $SO(3)$. Through its ternary structure, unit cyclicity condition, and integrated vector–number duality, $R(3)SO(3)$ provides a coherent framework in which field configurations are geometrically structured and algebraically constrained.

Within this framework, the field equation system $\mathcal{M}(\nu,\phi,\psi)$ arises not as an imposed law, but as a natural consequence of the internal symmetries and structural properties of $R(3)SO(3)$. Later in this work we discover that it defines a zero-order gauge field description that preserves orientation, supports smooth transformations, and ensures topological stability across the extended field space.

We now turn to this field equation system, $\mathcal{M}(\nu,\phi,\psi)$, and demonstrate how Maxwell's equations in vacuum emerge directly from the $R(3)SO(3)$ structure.

2 $\mathcal{M}(\nu,\phi,\psi)$ are the zero-order Maxwell field equations in vacuum

We now demonstrate that the field equation system $\mathcal{M}(\nu,\phi,\psi)$, derived from the rotational structure of $R(3)SO(3)$, recovers the Maxwell equations in vacuum as a natural consequence.

THEOREM 2.1: *Soliton Equation System.*
In a vacuum, let ψ and ϕ be the electric and magnetic fields, respectively, defined at a point $\vec{s} = \vec{s}_0 + \int v \, \mathrm{d}t$, where v is the velocity vector. The system of simultaneous equations:

$$\mathcal{M}(v,\phi,\psi) := \left\{ v = \frac{\phi \times \psi}{\phi \cdot \phi}, \quad \phi = \frac{\psi \times v}{v \cdot v}, \quad \psi = v \times \phi \right\} \tag{5}$$

constitutes the linear form of Maxwell's equations in vacuum.

PROOF (Proof). Applying the curl operation to the second and third equations in the system (5) yields:

$$\nabla \times \phi = \frac{1}{v \cdot v} \nabla \times \left(\psi \times v\right), \qquad \nabla \times \psi = \nabla \times \left(v \times \phi\right). \tag{6}$$

To evaluate these vector triple products, we apply standard vector calculus identities:

$$\nabla \times \left(\psi \times v\right) = (v \cdot \nabla)\psi - (\psi \cdot \nabla)v + \psi(\nabla \cdot v) - v(\nabla \cdot \psi),$$
$$\nabla \times \left(v \times \phi\right) = (\phi \cdot \nabla)v - (v \cdot \nabla)\phi + v(\nabla \cdot \phi) - \phi(\nabla \cdot v).$$

Evaluating the terms of $\nabla \times \left(v \times \phi\right)$:

1. The term $(\phi \cdot \nabla)v = \phi_x \, \partial v / \partial x + \phi_y \, \partial v / \partial y + \phi_z \, \partial v / \partial z = 0$ since v is a function of time only, not of position.
2. To evaluate $(v \cdot \nabla)\phi$, let $v = v' + \delta v$, where v' is time-dependent and δv accounts for spatial inhomogeneity. For the homogeneous case where $\delta v = 0$, we have:

   $$v' = \hat{x} \frac{\partial x}{\partial t} + \hat{y} \frac{\partial y}{\partial t} + \hat{z} \frac{\partial z}{\partial t} \quad \Rightarrow \quad (v' \cdot \nabla) = \frac{\partial}{\partial t}.$$

   Hence,

   $$(v \cdot \nabla)\phi = \frac{\partial \phi}{\partial t}.$$
3. The term $v(\nabla \cdot \phi) = 0$, since ϕ is not a function of position.
4. Similarly, $\phi(\nabla \cdot v) = 0$. Even if multiplied by a constant k, we obtain:

   $$\phi \, (\nabla \cdot v) \, k = \phi \frac{\partial k}{\partial t} = 0.$$

Substituting these into equation (6), we find:

$$\nabla \times \phi = \frac{1}{u^2} \frac{\partial \psi}{\partial t}, \qquad \nabla \times \psi = -\frac{\partial \phi}{\partial t},$$

where $u^2 = v \cdot v$ is the squared magnitude of the propagation velocity. These are recognised as the curl forms of Maxwell's equations in vacuum. Including the divergence conditions:

$$\begin{cases} \nabla \times \phi = \dfrac{1}{u^2} \dfrac{\partial \psi}{\partial t}, & \nabla \times \psi = -\dfrac{\partial \phi}{\partial t}, \\ \nabla \cdot \phi = 0, & \nabla \cdot \psi = 0, \end{cases} \tag{7}$$

we recover the complete Maxwell system for free space.

□

Later, in Theorem 4.1 (pp. 20), we will demonstrate axiomatically that $u^2 = 1/\epsilon_0\mu_0$, confirming that the simultaneous equation set

$$\mathcal{M}(v,\phi,\psi) := \left\{ v = \frac{\phi\times\psi}{\phi\cdot\phi}, \quad \phi = \frac{\psi\times v}{v\cdot v}, \quad \psi = v\times\phi \right\}$$

is hierarchically superordinate to the Maxwell equations. This hierarchical structure implies that solutions to $\mathcal{M}$ not only satisfy classical electromagnetic theory, but also fulfil the d'Alembert wave equations for electric and magnetic fields.

The equations within the system $\mathcal{M}$ may be interpreted as follows:

1. $v = \phi\times\psi/_{\phi\cdot\phi}$: defines the direction of energy flow (Poynting vector) and characterises wave action.
2. $\phi = \psi\times v/_{v\cdot v}$: is consistent with the form of Maxwell's displacement current.
3. $\psi = v\times\phi$: encodes Faraday's law of electromagnetic induction.

3 Gyrations as Maxwellian Vortices

In a quantised context, "gyration" denotes a spinning quantum point of finite extent that generates a gyrating field. Unlike a lighthouse beam, this field exhibits gyration at every point in space. Consider a gridded disc, with orthogonal lines representing the electric and magnetic fields. An observer viewing a fixed window on this gyrating disc perceives a field rotating about the window's centre.

Theorems 2.1 (pp. 17) and 4.1 (pp. 20) establish the field equation system as an alternative form of Maxwell's equations. Introducing a gyration reference frame (expressed in radians) overlaid onto the laboratory frame (expressed in metres) yields the gyratory Maxwell equations. Consequently, a gyration propagates indefinitely as a wave, analogous to a photon.

NOTATION (Overset Circle for Angular Quantities). To distinguish between spatial and angular quantities, we adopt an overset circle (e.g., $\mathring{x}$) to denote variables expressed in radians rather than metres, without altering the identity or directional interpretation of the symbol. For example:

- $x, y, z \in \mathbb{R}$ denote spatial positions in the laboratory frame (measured in metres),
- $\mathring{x}, \mathring{y}, \mathring{z} \in \mathbb{R}$ denote angular positions in the internal gyration frame (measured in radians).

This notational convention allows us to define, for example, the *gyro-velocity* ω, which is the gyration rate of a vortex, and the *gyro-gradient* $\mathring{\nabla}$, analogously to their spatial counterparts v and ∇, while preserving the geometric roles of the symbols:

$$v = \hat{x}\frac{dx}{dt} + \hat{y}\frac{dy}{dt} + \hat{z}\frac{dz}{dt}, \qquad \omega = \hat{x}\frac{d\mathring{x}}{dt} + \hat{y}\frac{d\mathring{y}}{dt} + \hat{z}\frac{d\mathring{z}}{dt},$$

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}, \qquad \mathring{\nabla} = \hat{x}\frac{\partial}{\partial \mathring{x}} + \hat{y}\frac{\partial}{\partial \mathring{y}} + \hat{z}\frac{\partial}{\partial \mathring{z}}.$$

This change of units from metres to radians reflects the transition from translational motion to internal vortex structure—central to the formulation of gyrating solitons in the $R(3)SO(3)$ framework.

DEFINITION 3.1: *Gyro-Field Equation System.* The *Gyro-Field Equation System* in the $R(3)SO(3)$ framework describes the internal dynamics of a gyrating electromagnetic soliton in a quantised context. It is defined as:

$$\mathring{\mathcal{M}}(\omega, \phi, \mathring{\psi}) := \left\{ \omega = \frac{\phi \times \mathring{\psi}}{\phi \cdot \phi}, \quad \phi = \frac{\mathring{\psi} \times \omega}{\omega \cdot \omega}, \quad \mathring{\psi} = \omega \times \phi \right\},$$

where:

- ω is the gyro-velocity vector, defined in the internal (gyration) frame,
- ϕ is the magnetic field vector,
- $\mathring{\psi}$ is the electric field vector arising from gyration.

The gyro-frame is expressed in radians rather than metres, and derivatives are taken with respect to angular coordinates. Analogous to the laboratory-frame velocity v and gradient ∇, the gyro-velocity and gyro-gradient are defined as:

$$v = \hat{x}\frac{dx}{dt} + \hat{y}\frac{dy}{dt} + \hat{z}\frac{dz}{dt}, \qquad \omega = \hat{x}\frac{d\mathring{x}}{dt} + \hat{y}\frac{d\mathring{y}}{dt} + \hat{z}\frac{d\mathring{z}}{dt},$$

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}, \qquad \mathring{\nabla} = \hat{x}\frac{\partial}{\partial \mathring{x}} + \hat{y}\frac{\partial}{\partial \mathring{y}} + \hat{z}\frac{\partial}{\partial \mathring{z}}.$$

The system $\mathring{\mathcal{M}}$ is structurally analogous to the field equation system $\mathcal{M}$ and gives rise to the *gyro-Maxwell equations*, which extend classical Maxwell theory to describe the rotational dynamics of internal fields. Together, $\mathcal{M}$ and $\mathring{\mathcal{M}}$ provide a unified framework for electromagnetic solitons exhibiting both translational and internal vortex structures.

NOTE. Since $\mathcal{M}$ theorematically (by Theorems 2.1 (pp. 17) and 4.1 (pp. 20)) yields Maxwell's equations in vacuum, $\mathring{\mathcal{M}}$ necessarily implies the gyro-Maxwell field equations, opening a novel domain in electromagnetic field theory. These gyro-field equations provide a theoretical basis for the Planck–Einstein relation $E = hf$, thereby validating the formulation of $\mathring{\mathcal{M}}$.

4 Constructing a Generalised and Quantised Field Theory

Theorem 2.1 (pp. 17), derived from $\mathcal{M}$, generalises Maxwell's equations in vacuum with a general velocity vector v. To avoid the hypothesis that $\|v\| = c$, we adopt the following five elementary quantities as axioms:

AXIOM 4.1: *Electromagnetic Action and Coupling Constants.*
We adopt the following physical definitions as foundational:

1. *Action is defined as momentum times distance. By mechanical analogy, electromagnetic momentum (EM-momentum) is charge times velocity:*

$$\mathfrak{p} := ec \qquad \left[\mathrm{C\,m\,s^{-1}}\right].$$

2. *Electromagnetic action (EM-action) is the product of EM-momentum and propagation distance. Let $l_o = ct_0$ be the elementary propagation length, then:*

$$\mathfrak{h} := ecl_o \qquad \left[\mathrm{Cm^2/s}\right].$$

3. *Relating EM-action to mechanical action yields:*

$$h = \varrho\kappa\mathfrak{h}, \qquad \text{with} \quad \varrho := 1\,\text{kg}\,\text{C}^{-1},$$

where κ is a dimensionless coupling factor that connects electromagnetic and inertial formulations of action.

We therefore adopt the following eight elementary quantities as axiomatic constants in the unified field framework, even though κ remains an undetermined coupling constant to be evaluated within the theory.

h *Planck's constant, or elementary action (joule second).*
e *Elementary charge (coulomb).*
c *Speed of light in vacuum (metres per second).*
$\dot{\omega}_0$ *Radial angular velocity, $\dot{\omega}_0 = 2\pi$ (radians per second).*
$|\mu_0|$ *Numeric value for magnetic permeability $4\pi \times 10^{-7}$, (Alternatively, we can use the fine structure constant alpha)*
$\mathfrak{h}$ *Electromagnetic action $\mathfrak{h} = ecl_o$ (coulomb metre squared per second).*
ϱ *Charge-to-mass ratio, here set to unity $\varrho = 1\,\text{kg}\,\text{C}^{-1}$.*
κ *Dimensionless coupling constant relating electromagnetic and Newtonian action.*

The aim of Theorem 4.1 (pp. 20) is to employ the field equations systems $\mathcal{M}$ and $\mathring{\mathcal{M}}$ to quantise Maxwell's equations in vacuum, embedding them fully within the structured geometry of $R(3)SO(3)$. This theorem accomplishes the following objectives:

1. Establish the magnetic quantum flux ϕ.
2. Derive the vacuum light-speed identity $c^2 = (\epsilon_0\mu_0)^{-1}$.
3. Derive the dimensionless coupling constant κ, thereby completing the relation $h = \varrho\kappa\mathfrak{h}$.
4. Define an elementary length scale l_o, fully quantising the electromagnetic field framework.
5. Provide a rigorous derivation of the Planck–Einstein relation $E = hf$ within the field structure.

To simplify notation, fields and potentials are represented as functions of quantised flux; for instance, the magnetic field $\underset{\cdot\cdot}{\mathcal{F}}(\phi)$ and magnetic potential $\underset{\cdot}{\mathcal{V}}(\phi)$, where each underset dot symbolically encodes spatial division— respectively by l_o^2 and l_o.

THEOREM 4.1: *General Field Quantisation Theorem.*
The electromagnetic phenomenon is fully quantifiable by adopting the following assertions:

1. *There exist an elementary length, l_o, and an elementary time, t_o, that are related to the speed of light in vacuum, c, by $l_o = ct_o$.*
2. *and an elementary angle, $\vartheta_o = \dot{\omega}_0 t_o$ rad.*

3. *There exists an elementary magnetic field quantum, $\underset{\cdot\cdot}{\phi}$, and electric field quanta, $\underset{\cdot\cdot}{\psi}$ and $\underset{\cdot\cdot}{\mathring{\psi}}$, that mutually reinforce each other by self induction manifesting in an elementary electromagnetic soliton, described by:*

$$\mathcal{M}(\nu,\underset{\cdot\cdot}{\phi},\underset{\cdot\cdot}{\psi}) := \left\{ \nu = \frac{\underset{\cdot\cdot}{\phi}\times\underset{\cdot\cdot}{\psi}}{\underset{\cdot\cdot}{\phi}\cdot\underset{\cdot\cdot}{\phi}}, \quad \underset{\cdot\cdot}{\phi} = \frac{\underset{\cdot\cdot}{\psi}\times\nu}{\nu\cdot\nu}, \quad \underset{\cdot\cdot}{\psi} = \nu\times\underset{\cdot\cdot}{\phi} \right\} \tag{8}$$

in union with

$$\mathring{\mathcal{M}}(\omega,\underset{\cdot\cdot}{\phi},\underset{\cdot\cdot}{\mathring{\psi}}) := \left\{ \omega = \frac{\underset{\cdot\cdot}{\phi}\times\underset{\cdot\cdot}{\mathring{\psi}}}{\underset{\cdot\cdot}{\phi}\cdot\underset{\cdot\cdot}{\phi}}, \quad \underset{\cdot\cdot}{\phi} = \frac{\underset{\cdot\cdot}{\mathring{\psi}}\times\omega}{\omega\cdot\omega}, \quad \underset{\cdot\cdot}{\mathring{\psi}} = \omega\times\underset{\cdot\cdot}{\phi} \right\} \tag{9}$$

4. *The elementary soliton described by the solution $\mathcal{M}\cup\mathring{\mathcal{M}}$ carries an elementary charge e (also refer to discussion on page 25.)*
5. *An elementary soliton $\mathcal{M}$ has action, $\mathcal{S} = h$ while propagating at light speed,*
6. *and $\mathring{\mathcal{M}}$ provides additional gyro-action, $\mathring{\mathcal{S}}^{\S} = n\hbar$, while gyrating with a gyration velocity $\omega = n\dot{\omega}_0$ radian per second (Plank's energy-frequency relation),*
7. *and occupies a volume l_o^3 in space.*
8. *The magnetic field quantum $\underset{\cdot\cdot}{\phi}$ in the elementary soliton $\mathcal{M}\cup\mathring{\mathcal{M}}$ also manifests itself in an elementary* electromagnetic *action $\mathcal{S}$ and $\mathring{\mathcal{S}}^{\S}$.*

PROOF. We begin by repeating the first equations of (8) and (9) to obtain

$$\nu = \frac{\underset{\cdot\cdot}{\phi}\times\underset{\cdot\cdot}{\psi}}{\underset{\cdot\cdot}{\phi}\cdot\underset{\cdot\cdot}{\phi}} \qquad \text{and} \qquad \omega = \frac{\underset{\cdot\cdot}{\phi}\times\underset{\cdot\cdot}{\mathring{\psi}}}{\underset{\cdot\cdot}{\phi}\cdot\underset{\cdot\cdot}{\phi}} \tag{10}$$

On the premise that $\underset{\cdot\cdot}{\phi}\times\underset{\cdot\cdot}{\psi}$ is indicative of the wave action, we multiply (10) by the quantised action h and $\hbar$, respectively (Planck constant). Evaluating the norms and using $\|\nu\| = c$ and $\|\omega\| = n\dot{\omega}_0$, gives :

$$hc = \frac{h}{\|\underset{\cdot\cdot}{\phi}\|^2}\|\underset{\cdot\cdot}{\phi}\|\,\|\underset{\cdot\cdot}{\psi}\| \qquad \text{and} \qquad \hbar n\dot{\omega}_0 = \frac{\hbar}{\|\underset{\cdot\cdot}{\phi}\|^2}\|\underset{\cdot\cdot}{\phi}\|\,\|\underset{\cdot\cdot}{\mathring{\psi}}\|$$

Here it is important to note that the Planck constant

- h has units $\mathrm{kg\,m^2\,s^{-1}}$, and
- $\hbar$ has units $\mathrm{kg\,rad^2\,s^{-1}}$

The magnetic flux $\|\phi\|$ is related to magnetic flux density $\|\underset{\cdot\cdot}{\phi}\|$ by $\|\phi\| = l_o^2\|\underset{\cdot\cdot}{\phi}\|$. Similarly, the electric fluxes $\|\psi\|$ and $\|\mathring{\psi}\|$ are related to the electric fields $\|\underset{\cdot\cdot}{\psi}\|$ and $\|\underset{\cdot\cdot}{\mathring{\psi}}\|$ by $\|\psi\| = l_o^2\|\underset{\cdot\cdot}{\psi}\|$ and $\|\mathring{\psi}\| = l_o^2\|\underset{\cdot\cdot}{\mathring{\psi}}\|$, respectively. Expressing the preceding equations in terms of fluxes, after dividing by c and $\dot{\omega}_0$, respectively, yields:

$$h = \left[\frac{h}{\|\phi\|^2 c}\right]\|\phi\|\,\|\psi\| \qquad \text{and} \qquad n\hbar = \left[\frac{\hbar}{\|\phi\|^2\dot{\omega}_0}\right]\|\phi\|\,\|\mathring{\psi}\| \tag{11}$$

Here the square brackets indicate the development of a physical constant, which we want to determine by eliminating $\|\phi\|$.

Assertion 4 of Theorem 4.1 (pp. 20) establishes that an electromagnetic wave transports a quantised charge e at the velocity c. From Axiom 4.1 (pp. 19), this implies that the mechanical wave action h relates to the electromagnetic action $\mathfrak{h}$ via

$$h = \varrho\kappa\,\mathfrak{h}, \qquad \text{where} \quad \varrho = 1\ \mathrm{kg\,C^{-1}}$$

with κ remaining a dimensionless coupling constant to be determined. (This satisfies Theorem 4.1 (pp. 20) Assertion-5.)

Similarly, gyro-electromagnetic angular momentum is $L_{\mathrm{EM}} = \mathring{\mathcal{I}} e\dot{\omega}_0$, where $I_{\mathrm{EM}} = \mathring{\mathcal{I}} e$ is gyro-EM-moment of inertia, and $\mathring{\mathcal{I}}$ is an inertial-scaling constant ($\mathrm{m^2\,rad^{-2}}$). Angular EM-momentum times angle ϑ_o yields gyro-EM-action. Elementary actions $\mathcal{S}$ and $\mathring{\mathcal{S}}^{\S}$, with units $\mathrm{kg\,m^2\,s^{-1}}$ and $\mathrm{kg\,rad^2\,s^{-1}}$, respectively, are now expressible.

$$\mathcal{S} = h = \varrho\kappa e c l_o \qquad \text{and} \qquad \mathring{\mathcal{S}}^{\S} = \hbar = \varrho\kappa\mathring{\mathcal{I}} e\dot{\omega}_0\vartheta_o \tag{12}$$

Let us think about the magnetic flux $\|\phi\|$ in the context of the elementary EM-wave and Assertion-8 of Theorem 4.1 (pp. 20): The magnetic flux $\|\phi\|$ of the EM-wave results from the transportation of an elementary charge e. Because the charge is carried by the elementary EM-wave we can postulate that by choosing the units and dimensioning of χ that $\chi\|\phi\| = ec$ *i.e.* electric momentum, and multiplying by l_o we obtain wave action. Similarly, the gyro EM-action is $\mathring{\chi}\|\phi\| = \mathring{\mathcal{I}} e\dot{\omega}_0$ times angle subtended, ϑ_o, gives:

$$\mathcal{S} = h = \varrho\,\chi\|\phi\|\,l_o \qquad \text{and} \qquad \mathring{\mathcal{S}}^{\S} = \hbar = \varrho\mathring{\chi}\|\phi\|\vartheta_o$$

and where χ and $\mathring{\chi}$ are constants with units and scaling to be determined. Combining the above with (12) gives

$$\|\phi\| = \frac{\kappa ec}{\chi} \qquad \text{and} \qquad \|\phi\| = \frac{\kappa\mathring{\mathcal{I}} e\dot{\omega}_0}{\mathring{\chi}}$$

and we substitute $\|\phi\|$ from the above into (11) to get

$$h = \left[\frac{h}{\|\phi\|^2 c}\right]\left(\frac{\kappa ec}{\chi}\|\psi\|\right) \qquad \text{and} \qquad n\hbar = \left[\frac{\hbar}{\|\phi\|^2\dot{\omega}_0}\right]\left(\frac{\kappa\mathring{\mathcal{I}} e\dot{\omega}_0}{\mathring{\chi}}\|\mathring{\psi}\|\right).$$

To eliminate $\|\psi\|$ and $\|\mathring{\psi}\|$, we substitute $\|\psi\| = c\|\phi\|$ and $\|\mathring{\psi}\| = n\dot{\omega}_0\|\phi\|$, obtained from the third equations of (8) and (9), respectively, which yields:

$$h = \left[\frac{h}{\|\phi\|^2 c}\right]\left[\frac{1}{\chi}\right]\kappa e c^2\|\phi\| \qquad \text{and} \qquad n\hbar = \left[\frac{\hbar}{\|\phi\|^2\dot{\omega}_0}\right]\left[\frac{\mathring{\mathcal{I}}}{\mathring{\chi}}\right]\kappa e n\dot{\omega}_0^2\|\phi\|$$

We can now express the magnetic flux $\|\phi\|$ as

$$\|\phi\| = \frac{h}{\kappa e} \qquad \text{and} \qquad \|\phi\| = \frac{h}{\kappa e} \tag{13}$$

contingent on

$$\left[\frac{h}{\|\phi\|^2 c}\right]\left[\frac{1}{\chi}\right]c^2 = 1 \qquad \text{and} \qquad \left[\frac{\hbar}{\|\phi\|^2\dot{\omega}_0}\right]\left[\frac{\mathring{\mathcal{I}}}{\mathring{\chi}}\right]n\dot{\omega}_0^2 = \frac{n}{2\pi}$$

and replacing $\|\phi\|$ using (13) gives

$$\left[\frac{\kappa^2 e^2}{hc}\right]\left[\frac{1}{\chi}\right]c^2 = 1 \qquad \text{and} \qquad \left[\frac{\kappa^2 e^2}{2\pi h\dot{\omega}_0}\right]\left[\frac{2\pi\mathring{\mathcal{I}}}{\mathring{\chi}}\right]\dot{\omega}_0^2 = 1$$

which requires $\frac{1}{\chi} = \frac{h}{\kappa^2 e^2 c}$ and $\frac{\mathring{\mathcal{I}}}{\mathring{\chi}} = \frac{h}{\kappa^2 e^2 \dot{\omega}_0}$, hence

$$\left[\frac{\kappa^2 e^2}{hc}\right]\left[\frac{h}{\kappa^2 e^2 c}\right]c^2 = 1 \qquad \text{and} \qquad \left[\frac{\kappa^2 e^2}{2\pi h\dot{\omega}_0}\right]\left[\frac{2\pi h}{\kappa^2 e^2 \dot{\omega}_0}\right]\dot{\omega}_0^2 = 1 \tag{14}$$

which provides us with the permitivity and permeability

$$\epsilon_o = \frac{\kappa^2 e^2}{hc} \qquad \text{and} \qquad \mathring{\epsilon}_o = \frac{\kappa^2 e^2}{2\pi h\dot{\omega}_0}$$

$$\mu_o = \frac{h}{\kappa^2 e^2 c} \qquad \text{and} \qquad \mathring{\mu}_o = \frac{2\pi h}{\kappa^2 e^2 \dot{\omega}_0}$$

Assertions-5 and 6 are confirmed by: (recalling $\|\psi\| = c\|\phi\|$, $\|\mathring{\psi}\| = n\dot{\omega}_0\|\phi\|$, and $\|\phi\| = h/\kappa e$)

$$\epsilon_o\|\phi\|\|\psi\| = \epsilon_o c\|\phi\|^2 = h \qquad \text{and} \qquad \mathring{\epsilon}_o\|\phi\|\|\mathring{\psi}\| = \mathring{\epsilon}_o n\dot{\omega}_0\|\phi\|^2 = n\hbar \tag{15}$$

Now—with a bit of hindsight—all that remains, and depending on CODATA definitions, is to set

$$\kappa^2 = \frac{h}{|\mu_0| e^2 c} \qquad \text{or} \qquad \kappa^2 = \frac{1}{2\alpha}$$

where $|\mu_0| = 4\pi \times 10^{-7}$ without units and also using the numeric values of h, e and c. Alternatively κ is derived over the fine structure constant α. Equation (14) now gives the sought after result

$$\epsilon_0 = \frac{e^2}{2\alpha hc} \qquad \text{and} \qquad \mu_0 = \frac{2\alpha h}{e^2 c}$$

This concludes the proof that the field equation sets, repeated here:

$$\mathcal{M}(\nu,\phi,\psi) := \left\{ \nu = \frac{\phi\times\psi}{\phi\cdot\phi}, \quad \phi = \frac{\psi\times\nu}{\nu\cdot\nu}, \quad \psi = \nu\times\phi \right\}$$

and

$$\mathring{\mathcal{M}}(\omega,\phi,\mathring{\psi}) := \left\{ \omega = \frac{\phi\times\mathring{\psi}}{\phi\cdot\phi}, \quad \phi = \frac{\mathring{\psi}\times\omega}{\omega\cdot\omega}, \quad \mathring{\psi} = \omega\times\phi \right\}$$

are the quantised Maxwell field equations in vacuum, hierarchically superordinate to the non-quantised Maxwell field equations. This is because we can now replace $1/u^2$ in (7) with $\epsilon_0\mu_0$, having derived it independently rather than postulating it from experience.

4.1 Quantising Space

Importantly, the field equation sets $\mathcal{M}$ and $\mathring{\mathcal{M}}$, are defined here in terms of fluxes, is valid only for a quantised space element l_o^3. Beyond this, potentials and fields are derived see Section 5.2 (pp. 28).

The relations $\kappa^2 = 1/(2\alpha)$ and $h = \varrho\kappa e c l_o$, where $\varrho = 1\ \mathrm{kg\,C^{-1}}$, provide the key to determining the elementary length and time. Furthermore, the elementary angle $\vartheta_o = \dot\omega_0 t_o$ allows us to solve for $\mathring{\mathcal{I}}$ using equation (12).

$$l_o = \frac{h}{\varrho\kappa e c} \qquad \text{and} \qquad \mathring{\mathcal{I}} = \frac{c^2}{2\pi\dot\omega_0^2} = \frac{c^2}{4\pi^2}$$

Using the 2018 CODATA we get:

$$\kappa = 8.277\,559\,999\,29(62)$$

$$l_o = 1.666\,566\,299\,11(12) \times 10^{-24} \quad \text{metres}$$

$$t_o = 5.559\,066\,796\,49(42) \times 10^{-33} \quad \text{seconds} \qquad \text{using } l = ct.$$

From a symmetry point of view the above results beckons to hypothesise a gyration rate $\varpi = 2\pi/t_o$ that cannot be exceeded (one revolution per elementary distance travelled)

$$\varpi = 1.130\,259\,005\,19(08) \times 10^{33} \quad \text{radian per second}$$

4.2 The Electromagnetic- and Yang–Mills Mass Gap

The preceding theorem enables the determination of the electromagnetic mass gap, denoted Δ_0, to CODATA-level precision. This mass gap represents the minimum energy required for vacuum excitation.

Equations (15) demonstrate that a photon possesses an electromagnetic mass gap. Consider the interaction of a single elementary EM-soliton, $\mathcal{M}(\nu,\phi,\psi) \cup \mathring{\mathcal{M}}(\omega,\phi,\mathring{\psi})$, integrated over one second. Equation (12) defines elementary action as electromagnetic momentum times distance l_o. Consequently, we define the ratio $r_o = t_o/t_{\mathrm{H}}$, where $t_{\mathrm{H}} = 1$ second. The energy qE of an elementary EM-soliton is then expressed as:

$$E = \frac{h r_0}{t_{\mathrm{H}}} + hf$$

When $f = 0$ the lowest energy state corresponds to

$$\Delta_{\mathrm{EM}} = \frac{h r_0}{t_{\mathrm{H}}} = 3.683\,476\,656\,21(28) \times 10^{-66} \quad \text{joule.}$$

Given that Δ_{EM} is an energy quantum then we can calculate the radial velocity quantum

$$\omega_q = 3.492\,864\,681\,73(26) \times 10^{-32} \quad \text{radian per second}$$

Extending the electromagnetic mass gap concept to the full framework developed in this work—including its generalised field structures and rotational invariants—provides a natural explanation for the existence of a positive Yang–Mills mass gap. Within the $R(3)SO(3)$ formalism, this lowest-energy excitation does not arise from spontaneous symmetry breaking or confinement assumptions, but emerges necessarily from the intrinsic quantised geometry and solitonic topologies of the field system.

4.3 Magnetic flux quantum disparity

The astute reader would have noticed that the standard definition for the fundamental magnetic flux quantum is: $\Phi_0 = h/2e$, which differs from the elementary magnetic flux quantum defined in this work as $\|\phi\| = h/\kappa e$, which is demonstrably smaller than the established value. This is not a contradiction; it is a context-related difference:

$\Phi_0 = {}^{h}/_{2e}$—*Superconductor context*

- Originates from the Bardeen-Cooper-Schrieffer theory of superconductivity, where charge carriers are Cooper pairs with charge $2e$.
- The quantisation reflects a *collective quantum state* of many particles (Cooper pairs) within a macroscopic wavefunction constrained by boundary conditions (such as in a superconducting ring).
- The factor $2e$ emerges due to the pairing mechanism, making this quantisation inherently linked to the condensed matter environment and the properties of Cooper pairs.

$\phi = {}^{h}/_{e\kappa}$—*Vacuum context*

- The $R(3)SO(3)$ framework and the soliton equation system $\mathcal{M}$ enabled a vacuum-based quantisation rooted in fundamental constants: h, e, c, α.
- The dimensionless coupling factor $\kappa = {}^{1}/\sqrt{2\alpha}$ couples electric action to mechanical action over a unit conversion constant ($1\text{kg}\,\text{C}^{-1}$) and relates to the topology of the quantised vacuum element l_o^3.
- The relation of $\kappa = {}^{1}/\sqrt{2\alpha}$ intriguingly ties the vacuum quantisation to the fine structure constant, suggesting deeper connections to electromagnetic interactions in vacuum rather than a condensed matter environment.
- This quantisation implies a fundamental unit of magnetic flux in the vacuum, not dependent on particle pairing mechanisms but rather on the discrete structure of the vacuum and field interactions at a fundamental level.

4.4 Discussion

Elementary charge carried by EM*-soliton*

Theorem 4.1 (pp. 20) Assertion-4 asserts that an elementary soliton described by $\mathcal{M} \cup \mathring{\mathcal{M}}$ carries an elementary charge e. Let's consider an electron (e) positron (p) annihilation producing two gamma-rays. The convention has it that charge is annihilated, but Theorem 4.1 (pp. 20) Assertion-4 now preserves the charge in the gamma-rays, therefore in the framework that is developed here, we have:

$$\mathrm{e} + \mathrm{p} \longleftrightarrow \gamma_p + \gamma_e$$

preserving charge, momentum and energy.

What Defines the Speed of Light?

Traditionally, the speed of light in vacuum, denoted by c, is defined as

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}},$$

where ϵ_0 is the vacuum permittivity and μ_0 the vacuum permeability. However, in the derivation of Theorem 4.1 (pp. 20), these constants themselves are shown to emerge from the quantised structure of space, conditioned on the prior assertion that the speed of light is fixed by the relation

$$l_o = c\, t_0,$$

where l_o and t_0 are elementary quanta of length and time, respectively.

This leads to a circular reasoning: c is defined via ϵ_0 and μ_0, but these constants are themselves derived from the assumption of a fixed c. To resolve this foundational issue, we introduce the following proposition:

PROPOSITION 4.1: *Transportivity as a Fundamental Property of Space.* We postulate the existence of a primitive, irreducible property of space called *transportivity*, denoted $\mathcal{T}$, defined in vacuum by

$$\mathcal{T} := c^2.$$

Transportivity encodes the maximal causal velocity permitted by the structure of space, independent of any specific field equations. It is not derived from electromagnetic properties such as ϵ_0 or μ_0, but rather serves as a generative principle from which such constants may emerge.

This reformulation allows us to avoid circular definitions and treat c as an emergent quantity rooted in the more fundamental transportivity $\mathcal{T}$. The deeper physical origins of $\mathcal{T}$ are not yet fully understood; however, the role it plays in defining inertial response, field propagation, and causal structure will become evident in the derivation of gravitational and Coulomb fields in the concluding sections of this work.

PART II

Towards a Unified Field Theory

In Part I, we established the $R(3)SO(3)$ mathematical framework, from which the quantised Maxwell field equations in vacuum naturally emerge. This foundation revealed how the internal structure of space, encoded in cross-product cyclicity and rotational symmetry, gives rise to coherent field dynamics.

In Part II, we extend this framework to structured fields, aiming to describe quantised topological electromagnetic solitons embedded within a truly nilpotent Universe. This involves the aggregation of solitons, the emergence of inertial and energising fields, and the formulation of interaction principles governed by internal field geometry and curvature.

5 Aggregation, Fields and Interactions

Consider the solution of the field equation system

$$\mathring{\mathcal{M}}(\omega, \phi, \mathring{\psi}) := \left\{ \omega = \frac{\phi \times \mathring{\psi}}{\phi \cdot \phi}, \quad \phi = \frac{\mathring{\psi} \times \omega}{\omega \cdot \omega}, \quad \mathring{\psi} = \omega \times \phi \right\},$$

describing a quantised electromagnetic soliton:

$$\mathring{\Upsilon} \xrightarrow[\text{by}]{\text{dsc}} \begin{bmatrix} \omega \\ \phi \\ \mathring{\psi} \end{bmatrix} = \begin{bmatrix} c_\emptyset c_w & s_\emptyset c_w & -s_w \\ -c_s s_\emptyset + s_s c_\emptyset s_w & c_s c_\emptyset + s_s s_\emptyset s_w & s_s c_w \\ s_s s_\emptyset + c_s c_\emptyset s_w & -s_s c_\emptyset + c_s s_\emptyset s_w & c_s c_w \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix},$$

which represents a stationary three-dimensional gyration of a space quantum l_o^3. Its energy is given by

$$E_\Upsilon = \hbar\sqrt{\omega_s{}^2 + \omega_o{}^2 + \omega_w{}^2}.$$

In this section, we investigate the aggregation (fusion) of such solitons, define their field structure, and describe their interactions.

We define four quantum numbers:

- m $\mathsf{m} \in \mathbb{N}$ is a quantum length multiplier, scaling the space quantum to $(\mathsf{m}l_o)^3$.
- f $\mathsf{f} \le \mathsf{m}^2$ is a fill or occupancy number, it denotes the number of magnetic flux quanta in the area $(\mathsf{m}l_o)^2$.
- v $\mathsf{v} = \mathsf{m}^2/\mathsf{f} \ge 1$, the vacancy ratio of the scaled space quantum.
- r $\mathsf{r} \in \mathbb{Z}$, representing the radial distance from the soliton's centre, $r = \mathsf{r}\, l_o/2$, in integer steps of $l_o/2$.

Aggregating (fusing) two solitons, $\Upsilon' = 2\Upsilon$, requires an enlarged space quantum of volume $(2l_o)^3$, which could accommodate four Υ solitons, since the cross-sectional area $(2l_o)^2$ quadruples. However, populating this space with only $f = 2$ solitons yields an energy of

$$E'_\Upsilon = \hbar \mathsf{f}\sqrt{\omega_s{}^2 + \omega_o{}^2 + \omega_w{}^2},$$

and a vacancy ratio of $\mathsf{v} = 2$.

5.1 Understanding Gyrations and Aggregated Gyrations

The aggregated soliton

$$\mathring{\Upsilon}(\mathsf{m}) \xrightarrow[\text{by}]{\text{dsc}} \begin{bmatrix} \omega \\ \phi \\ \mathring{\psi} \end{bmatrix} = \begin{bmatrix} c_\emptyset c_w & s_\emptyset c_w & -s_w \\ -c_s s_\emptyset + s_s c_\emptyset s_w & c_s c_\emptyset + s_s s_\emptyset s_w & s_s c_w \\ s_s s_\emptyset + c_s c_\emptyset s_w & -s_s c_\emptyset + c_s s_\emptyset s_w & c_s c_w \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}.$$

- If $\mathsf{m} = 1$, $\mathring{\Upsilon}$ represents a stationary, three-dimensional gyration of a space quantum $(l_o)^3$. As defined in Section 1.2 (pp. 8), this soliton exhibits three distinct types of rotational motion: spin gyration (ω_s) about the x axis, orbital gyration (ω_o) around the initial y axis, and nutational gyration (ω_w) about the initial z axis. These combined gyrations define the internal topological structure of the soliton; there is no physical or translational motion.

  This motion may be visualised as a gyroscope (radius $l_o/2$) spinning rapidly around its own x axis (spin), while the spinning disc itself flips forward around the y axis (orbital), and simultaneously performs a sideways flipping or precessional motion around the z axis (nutation). Alternatively, one may think of it as a spherical structure exhibiting a structured, three-dimensional vortex-like resonance of the magnetic and electric components.
- If $\mathsf{m} > 1$, the above representation is not scaled like a lighthouse beam that reaches into the distance. Rather, the enlarged structure $(\mathsf{m}l_o)^3$ now consists of m^3 space

quanta, each in a three-dimensional gyratory mode but with different phasing. At a position $\vec{r} = r_\mathsf{x}\hat{\mathsf{x}} + r_\mathsf{y}\hat{\mathsf{y}} + r_\mathsf{z}\hat{\mathsf{z}}$, there exists a space quantum

$$\mathring{\Upsilon}(\vec{r}) \xrightarrow[\text{by}]{\text{dsc}} \begin{bmatrix} \omega \\ \phi \\ \mathring{\psi} \end{bmatrix} = \begin{bmatrix} c^\mathsf{x}_ø c^\mathsf{x}_w & s^\mathsf{y}_ø c^\mathsf{y}_w & -s^\mathsf{z}_w \\ -c^\mathsf{x}_s s^\mathsf{x}_ø + s^\mathsf{x}_s c^\mathsf{x}_ø s^\mathsf{x}_w & c^\mathsf{y}_s c^\mathsf{y}_ø + s^\mathsf{y}_s s^\mathsf{y}_ø s^\mathsf{y}_w & s^\mathsf{z}_s c^\mathsf{z}_w \\ s^\mathsf{x}_s s^\mathsf{x}_ø + c^\mathsf{x}_s c^\mathsf{x}_ø s^\mathsf{x}_w & -s^\mathsf{y}_s c^\mathsf{y}_ø + c^\mathsf{y}_s s^\mathsf{y}_ø s^\mathsf{y}_w & c^\mathsf{z}_s c^\mathsf{z}_w \end{bmatrix} \begin{bmatrix} \hat{\mathsf{x}} \\ \hat{\mathsf{y}} \\ \hat{\mathsf{z}} \end{bmatrix},$$

where, for example, $c^\mathsf{x}_w := \cos(\omega_w t + \omega_w r_\mathsf{x}/c)$, and $s^\mathsf{y}_s := \sin\left(\omega_s t + \omega_s r_\mathsf{y}/c\right)$, etc., and $r = \mathsf{r}\, l_o/2$.

Therefore, $\mathring{\Upsilon}(\mathsf{m})$ is a collection of m^3 quanta, symmetrically arranged around a central point (the origin of the internal reference frame), each gyrating synchronously with all the others but with specific phasing depending on its position.

Furthermore, the energy does not scale with the volume but with the cross-sectional area, that is,

$$\mathring{E}_\Upsilon(\mathsf{m}) = \hbar \mathsf{m}^2 \sqrt{\omega_s{}^2 + \omega_o{}^2 + \omega_w{}^2}.$$

Each magnetic flux quantum occupies a cross-section of area l_o^2 and volume l_o^3, meaning that only one flux quantum may exist per space quantum. If the structure is only partially populated with magnetic flux quanta—that is, fewer than m^2 flux quanta occupy the available cross-sectional area—then the energy becomes

$$\mathring{E}_\Upsilon(\mathsf{f}) = \hbar \mathsf{f} \sqrt{\omega_s{}^2 + \omega_o{}^2 + \omega_w{}^2}, \quad \text{with } \mathsf{f} < \mathsf{m}^2.$$

NOTATION (Hollowed Soliton). We introduce the term *hollowed soliton* to describe an aggregated soliton configuration in which the cross-sectional structure is only partially populated with magnetic flux quanta. Each such flux quantum requires a cross-sectional area l_o^2 and a volume l_o^3. The energy of this configuration is reduced to

$$\mathring{E}_\Upsilon(\mathsf{f}) = \hbar \mathsf{f} \sqrt{\omega_s{}^2 + \omega_o{}^2 + \omega_w{}^2}, \quad \text{with } \mathsf{f} < \mathsf{m}^2.$$

A complete description requires specifying the corresponding field structure, which will be developed in subsequent sections.

5.2 The ℵ Function Defines Fields and Potentials

We generalise the scalar Planck constant $\hbar$ to a spatially dependent, vector-valued action field $\hbar(\vec{r}) \in \mathbb{R}^3 \subset R(3)SO(3)$, which encodes directionally structured quantum inertia. For clarity, it is important to note that the magnitude of this vector-valued field is constant, such that $\|\hbar(\vec{r})\| = \hbar$. This allows us to explore the spatial derivatives of $\hbar(\vec{r})$, specifically $\mathrm{d}^2\hbar(\vec{r})/\mathrm{d}\vec{r}^2$, while maintaining the fundamental scale of quantum action.

Energy is then defined through frequency modulation of this field, for example:

$$\mathring{E}_{\Upsilon_\text{eff}} = \|\hbar(\vec{r})\| \cdot \sqrt{\omega_s^2 + \omega_o^2 + \omega_w^2},$$

or, in the case of structured solitons such as photons with orbital angular momentum,

$$\breve{E} = \hbar(\vec{r}) \cdot \omega_\mathsf{S}(1 + cø).$$

The $\aleph$ function characterises the radial and compositional dependence of a soliton's normalised quantum field, which is an inertial field, $\mathcal{F}^{\mathrm{I}}$, both within and beyond its boundary. It ensures continuity, balance, and conservation across the entire radial extent. The defining condition is

$$\aleph(r) = \aleph'(r) + \aleph''(r) = 0 \quad \text{that is,} \quad \int_{\mathrm{m}}^{0} F(\mathrm{r})\,\mathrm{dr} + \int_{\mathrm{m}}^{\infty} F(\mathrm{r})\,\mathrm{dr} = 0.$$

This ensures that when a soliton A traverses the radial path from the centre of soliton B to infinity, both solitons return to their original state. That is, the sum of all quantum forces is zero, and neither soliton gains nor loses energy along the path; the structural compositions at both ends remain unchanged. This reflects the principle of the nilpotent universe, whereby every physical interaction occurs within a globally balanced system, and each soliton's contribution is offset by its surrounding field such that the total configuration remains identically null.

This condition of nilpotency is not merely conceptual, but is dynamically realised through the curvature of the action field. It is not the action alone but its structured variation under effective frequency that governs soliton dynamics. The second spatial derivative $\mathrm{d}^2\hbar(\vec{r})/\mathrm{d}\vec{r}^2$ expresses the curvature of the action field within $R(3)SO(3)$, and is instrumental in ensuring that quantum forces cancel over radial paths. It governs how momentum gradients are spatially balanced and ensures that interactions—though locally non-trivial—globally sum to zero. In this way, nilpotency emerges from the spatial structure of the vector-valued action field and its modulation by effective frequency.

LEMMA 5.1: *Curvature Criterion for Topological Stability.*
The condition of topological stability for a soliton field configuration is dynamically realised through the curvature of the vector-valued action field $\hbar(\vec{r}) \in \mathbb{R}^3 \subset R(3)SO(3)$*. Although the magnitude* $\|\hbar(\vec{r})\| = \hbar$ *remains constant, its spatial structure encodes directional information relevant to the soliton's internal gyration modes.*

The second spatial derivative

$$\frac{\mathrm{d}^2\hbar(\vec{r})}{\mathrm{d}\vec{r}^2}$$

acts as a stability operator that quantifies the resistance of the soliton's field structure to smooth deformations. Under transformations in the $R(3)SO(3)$ *symmetry group, the preservation or covariant transformation of this curvature ensures that the soliton's internal structure remains invariant. Consequently, nilpotency and topological stability are enforced not abstractly, but via the curvature of the generalised action field.*

Thus, topological stability is realised when the curvature profile of $\hbar(\vec{r})$ *remains structurally consistent under all admissible transformations in the* $R(3)SO(3)$ *symmetry.*

Having established the role of curvature in maintaining topological stability, we now turn to the geometric consequences of this structure: how internal field vectors give rise to soliton trajectories, including curved and spherular paths.

LEMMA 5.2: *Kinematics and Resonances from Internal Field Geometry.*
Within the $R(3)SO(3)$ framework, the internal field vectors ϕ and ψ, defined by the triple product in the field equation system

$$\mathcal{M}(v,\phi,\psi) := \left\{ v = \frac{\phi\times\psi}{\phi\cdot\phi}, \quad \phi = \frac{\psi\times v}{v\cdot v}, \quad \psi = v\times\phi \right\},$$

generate a spatial velocity vector v that defines the soliton's trajectory. When this velocity is integrated over time,

$$\vec{r}(t) = \vec{r}_0 + \int v(t)\,\mathrm{d}t,$$

the resulting path is determined by the local orientation and interaction of the internal field vectors.

The waveform illustrated in Figure 1 (pp. 9) exemplifies this curvature mechanism in contrast to a conventional travelling plane wave. This structure allows for the emergence of curved propagation modes—including straight, circular, and spherular paths—without requiring external potentials or imposed geometric constraints. The spherular waveform illustrated juxtaposes a structured, soliton-based curvature against a conventional plane wave, demonstrating that coherent, curved wave propagation arises naturally from the internal field geometry.

Analogously, the internal field vectors ϕ and $\mathring{\psi}$, defined by the triple product in the field equation system

$$\mathring{\mathcal{M}}(\omega,\phi,\mathring{\psi}) := \left\{ \omega = \frac{\phi\times\mathring{\psi}}{\phi\cdot\phi}, \quad \phi = \frac{\mathring{\psi}\times\omega}{\omega\cdot\omega}, \quad \mathring{\psi} = \omega\times\phi \right\},$$

provide the corresponding kinematic structure for the three-dimensional gyration vector ω. These internal rotational cycles form coherent vortices that encode the soliton's spin, orbital, and nutational structure within its localised space. As with the translational case, these vortices do exhibit topological richness, including axial symmetries and directionally structured geometric resonances that may be one-, two-, or three-dimensional in character, fully determined by the self-consistent interaction of the internal field vectors.

The internal structure defined above not only determines the soliton's trajectory but also encodes its spatial energy distribution, as formalised in the following corollary.

COROLLARY 5.2.1: *Energy Gradient from Structured Gyration.*
Let $\Omega = \omega_s\hat{x} + \omega_\emptyset\hat{y} + \omega_w\hat{z}$ denote the internal gyration rate vector of a soliton in the $R(3)SO(3)$ framework, representing spin, orbital, and nutational components. Then the spatial energy distribution of the soliton is given by

$$E(\vec{r}) = \vec{\hbar}(\vec{r})\cdot\Omega,$$

where $\vec{\hbar}(\vec{r})$ is the vector-valued action field. This energy gradient governs the curvature of the soliton's path and determines whether its trajectory is open (e.g., photon-like), circular, or closed and spherular, depending on the resonance conditions among the components of Ω.

Closed trajectories arise when the internal gyration frequencies satisfy resonance conditions such that the phase structure completes in integral multiples of 2π. The *resulting motion is not imposed externally but emerges naturally from the local structure of the action field and the internal field geometry.*

NOTE. The periodicity of internal motion and the resonance conditions associated with closed trajectories are expressed through the sinusoidal modulation present in the solution matrix S. These encode the time-dependence of the soliton's internal rotational modes.

DEFINITION 5.1: *The* ℵ *Function.* The ℵ function defines a soliton's radial quantum field structure through a pair of integrals representing the inward and outward contributions:

- $\aleph'(\mathrm{v}, r)$ for $0 \leq r \leq \mathrm{m}\, l_o/2$, integrates the quantum field $F(\mathrm{r})$ from the soliton's boundary inward to the centre. This defines the efficacy of a soliton
- $\aleph''(\mathrm{v}, r)$ for $r \geq \mathrm{m}\, l_o/2$, continues the integration from the soliton's boundary outward to infinity. This defines the presence of a soliton.

Recalling that $\mathrm{v} = \mathrm{m}^2/\mathrm{f}$, the ℵ components take the general form:

$$\aleph'(\mathrm{v},\mathrm{r}) = \int_{\mathrm{m}}^{0} \frac{\mathrm{m}}{\mathrm{f}} \left(\frac{\mathrm{r}}{\mathrm{m}}\right)^{\frac{\mathrm{m}^2}{\mathrm{f}}-1} \mathrm{dr} = -1, \qquad \text{for } \mathrm{r} \leq \mathrm{m},\ r \in \mathbb{N},$$

$$\aleph''(\mathrm{v},\mathrm{r}) = \int_{\mathrm{m}}^{\infty} \frac{\mathrm{m}}{\mathrm{f}} \left(\frac{\mathrm{m}}{\mathrm{r}}\right)^{\frac{\mathrm{m}^2}{\mathrm{f}}+1} \mathrm{dr} = 1, \qquad \text{for } \mathrm{r} \geq \mathrm{m},$$

giving

$$\aleph(\mathrm{v},\mathrm{r}) = \aleph'(\mathrm{v},\mathrm{r}) + \aleph''(\mathrm{v},\mathrm{r}) = 0.$$

The cancellation condition imposed by the ℵ-function leads directly to a global conservation law for energy flow along soliton trajectories.

THEOREM 5.1: *Nilpotent Energy Conservation along Soliton Trajectories.*
Let a soliton be characterised by the vector-valued action field $\hbar(\vec{r}) \in \mathbb{R}^3 \subset R(3)SO(3)$ *and its internal gyration structure* $\Omega = \omega_s \hat{\mathrm{x}} + \omega_\emptyset \hat{\mathrm{y}} + \omega_w \hat{\mathrm{z}}$, *such that the spatial energy density is given by the projection:*

$$E(\vec{r}) = \hbar(\vec{r}) \cdot \Omega.$$

The field configuration satisfies the nilpotency condition defined by the ℵ*-function:*

$$\aleph(\vec{r}) = \aleph'(\vec{r}) + \aleph''(\vec{r}) = 0,$$

which guarantees that the inward and outward quantum field contributions cancel exactly:

$$\int_0^m F(r)\,\mathrm{d}r + \int_m^\infty F(r)\,\mathrm{d}r = 0.$$

Then, the total energy flux along any soliton trajectory—whether linear, circular, or spherular—is exactly zero when integrated across the complete radial domain. That is,

$$\int_{\mathbb{R}^3} \nabla \cdot (E(\vec{r})\, v(\vec{r}))\, \mathrm{d}^3\vec{r} = 0.$$

This implies that solitons propagate in curved or closed orbits without radiative energy loss. The internal energy exchange is locally dynamic but globally conservative, in full agreement with the nilpotent structure of the universe. In particular, this establishes the physical basis for radiation-free rotational motion, essential to modelling bound particle states such as electrons.

PROOF. The spatial energy density is defined as $E(\vec{r}) = \hbar(\vec{r})\cdot\Omega$, where Ω is constant for a given soliton type. Since $\hbar(\vec{r})$ is normalised by the $\aleph$-function to ensure global cancellation across the soliton's domain, the net flux of $E(\vec{r})\nu(\vec{r})$ over space vanishes under divergence integration. This follows directly from the integral expression for $\aleph(\vec{r})$ and its property $\aleph = 0$. Thus, curved or closed soliton paths do not result in radiative energy loss.

We now formalise how such energy densities arise from quantised fields and their associated potentials.

DEFINITION 5.2: *Fields and Potentials.* The relationship between the potential $\mathcal{V}(r)$ and its corresponding field $\mathcal{F}(r)$ is given by

$$\mathcal{F}(r) = -\,\mathrm{d}\mathcal{V}(r)/_{\mathrm{d}r}.$$

The $\aleph$ function imposes a quantised radial structure on the soliton's field:

$$\mathcal{F}(\mathsf{r}) := \begin{cases} \mathsf{m}\left(\dfrac{\mathsf{r}}{\mathsf{m}}\right)^{\frac{\mathsf{m}^2}{\mathsf{f}}-1}, & \text{if } \mathsf{r} \le \mathsf{m},\ r \in \mathbb{N} \\ \mathsf{m}\left(\dfrac{\mathsf{m}}{\mathsf{r}}\right)^{\frac{\mathsf{m}^2}{\mathsf{f}}+1}, & \text{if } \mathsf{r} \ge \mathsf{m}. \end{cases}$$

having multiplied by f, since the $\aleph$ function is normalised, the potential takes the form:

$$\mathcal{V}(\mathsf{r}) := \begin{cases} \mathsf{m}\left(\dfrac{\mathsf{r}}{\mathsf{m}}\right)^{\frac{\mathsf{m}^2}{\mathsf{f}}}, & \text{if } \mathsf{r} \le \mathsf{m},\ r \in \mathbb{N} \\ \mathsf{m}\left(\dfrac{\mathsf{m}}{\mathsf{r}}\right)^{\frac{\mathsf{m}^2}{\mathsf{f}}}, & \text{if } \mathsf{r} \ge \mathsf{m}. \end{cases}$$

For hollowed solitons, where the vacancy ratio satisfies $\mathsf{v} > 1$, the quantum field within the region $\mathsf{m}l_o$ forms a potential well. Beyond this region, the field decays rapidly as $1/_{r^n}$ with $n > 2$, leading to quantum forces of shorter range than the Coulomb interaction, which decays as $1/_{r^2}$. This structural feature provides a natural mechanism for short-range interactions and may offer insight into confinement phenomena in QED and QCD.

5.3 Soliton Interactions

Quantised electromagnetic solitons Υ exhibit topological stability through invariant field configurations arising from three-dimensional rotations in $R(3)SO(3)$, ensuring that their distinct interaction properties are preserved under continuous transformations.

Any solution to the field equation system $\mathcal{M}$ or $\mathring{\mathcal{M}}$ describes a quantised topological electromagnetic soliton Υ. These solutions define the spatial structure of Υ, while the $\aleph(r)$ function characterises the corresponding quantum field strength. The field configuration at a position $\vec{r}$ reflects the internal structure of the soliton.

For example, the potential $\mathcal{V}(\Upsilon)$ of a soliton Υ at a position $\vec{r} = r_\mathsf{x}\hat{\mathsf{x}} + r_\mathsf{y}\hat{\mathsf{y}} + r_\mathsf{z}\hat{\mathsf{z}}$ is given by:

$$\mathcal{V}(\Upsilon(\vec{r})) \xrightarrow[\text{by}]{\text{dsc}} \begin{bmatrix} \omega \\ \mathcal{V}(\phi) \\ \mathcal{V}(\mathring{\psi}) \end{bmatrix} = \mathcal{V}\left(\frac{2r}{l_o}\right) \begin{bmatrix} c_\varnothing^\mathsf{x} c_w^\mathsf{x} & s_\varnothing^\mathsf{y} c_w^\mathsf{y} & -s_w^\mathsf{z} \\ -c_s^\mathsf{x} s_\varnothing^\mathsf{x} + s_s^\mathsf{x} c_\varnothing^\mathsf{x} s_w^\mathsf{x} & c_s^\mathsf{y} c_\varnothing^\mathsf{y} + s_s^\mathsf{y} s_\varnothing^\mathsf{y} s_w^\mathsf{y} & s_s^\mathsf{z} c_w^\mathsf{z} \\ s_s^\mathsf{x} s_\varnothing^\mathsf{x} + c_s^\mathsf{x} c_\varnothing^\mathsf{x} s_w^\mathsf{x} & -s_s^\mathsf{y} c_\varnothing^\mathsf{y} + c_s^\mathsf{y} s_\varnothing^\mathsf{y} s_w^\mathsf{y} & c_s^\mathsf{z} c_w^\mathsf{z} \end{bmatrix} \begin{bmatrix} \hat{\mathsf{x}} \\ \hat{\mathsf{y}} \\ \hat{\mathsf{z}} \end{bmatrix},$$

where, for example, $c_w^\mathsf{x} := \cos(\omega_w t + \omega_w r_\mathsf{x}/c)$, and $s_s^\mathsf{y} := \sin\big(\omega_s t + \omega_s r_\mathsf{y}/c\big)$), etc., and $r = \mathrm{r}\, l_o/2$.

Importantly, the quantum fields do not carry energy; the energy is confined within the soliton itself. Thus, the associated quantum fields are instantaneously available throughout space, even as the soliton propagates.

The quantum interaction between two solitons Υ_1 and Υ_2 produces a quantum force, given by:

$$I_\mathrm{Q}^{1-2}(\vec{r}) = \frac{\mathring{\varepsilon}_0(\phi_1 \times \mathcal{V}(\mathring{\psi}_2))}{\phi_1 \cdot \phi_2}, \qquad I_\mathrm{Q}^{2-1}(\vec{r}) = \frac{\mathring{\varepsilon}_0(\phi_2 \times \mathcal{V}(\mathring{\psi}_1))}{\phi_2 \cdot \phi_1}.$$

These interactions arise from the cross-coupling of one soliton's magnetic flux with the other's electric potential . Since I_Q has units of action per radian (angular momentum), the associated interaction torque is given by

$$T_\mathrm{Q}^{1-2}(\vec{r}) = \frac{\mathrm{d}I_\mathrm{Q}^{1-2}(\vec{r})}{\mathrm{d}t}.$$

This torque is not aligned with the relative position vector $\vec{r}$; its direction is determined by the ternary cross products in $R(3)SO(3)$. This defines a quantum electrodynamic torque distinct from both Coulomb and gravitational forces. A detailed analysis is beyond the scope of this work. However, in the context of optics, this torque provides a possible explanation for phenomena such as the dispersion of incoherent (white) light beams, optical resonances (e.g., in free-electron lasers), and laser-based particle beam cooling, among others.

5.4 Translational Motion

The soliton

$$\mathring{\Upsilon} \xrightarrow[\text{by}]{\text{dsc}} \begin{bmatrix} \omega \\ \phi \\ \mathring{\psi} \end{bmatrix} = \begin{bmatrix} c_\varnothing c_w & s_\varnothing c_w & -s_w \\ -c_s s_\varnothing + s_s c_\varnothing s_w & c_s c_\varnothing + s_s s_\varnothing s_w & s_s c_w \\ s_s s_\varnothing + c_s c_\varnothing s_w & -s_s c_\varnothing + c_s s_\varnothing s_w & c_s c_w \end{bmatrix} \begin{bmatrix} \hat{\mathsf{x}} \\ \hat{\mathsf{y}} \\ \hat{\mathsf{z}} \end{bmatrix},$$

describes a stationary, three-dimensional gyration of a quantised space unit l_o^3. As defined in Section 1.2 (pp. 8), this soliton exhibits three distinct types of rotational motion: spin gyration (ω_s) about the x axis, orbital gyration (ω_o) around the initial y axis, and nutational gyration (ω_w) about the initial z axis. These combined gyrations define the internal topological structure of the soliton; there is no physical or translational motion.

The gyration vector is

$$\Omega = \omega_s \hat{\text{x}} + \omega_ø \hat{\text{y}} + \omega_s \hat{\text{z}},$$

and the energy of the soliton is given by

$$\mathring{E}_\Upsilon = \hbar\sqrt{\omega_s^2 + \omega_o^2 + \omega_w^2}.$$

Through rotational transformations, the topological structure of the soliton's magnetic and electric fields is altered, transforming the stationary soliton Υ into a propagating entity such as a gamma particle traversing space at the speed of light $|v|$:

$$\vec{\Upsilon} \xrightarrow[\text{by}]{\text{dsc}} \begin{bmatrix} v \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} & (1 & 0 & 0) \\ e^{c\omega_ø t} e^{m\omega_W t} & (0 & \cos\omega_S t & \sin\omega_S t) \\ e^{c\omega_ø t} e^{m\omega_W t} & (0 & -\sin\omega_S t & \cos\omega_S t) \end{bmatrix} \begin{bmatrix} \hat{\text{x}} \\ \hat{\text{y}} \\ \hat{\text{z}} \end{bmatrix},$$

which possesses spin angular momentum (SAM, ω_s), orbital angular momentum (OAM, ω_o), and nutational angular momentum (NAM, ω_w). The energy remains invariant:

$$E_\Upsilon = \hbar\sqrt{\omega_s^2 + \omega_o^2 + \omega_w^2}.$$

These rotational transformations arise from the quantum interactions discussed in the previous section. However, such transformations may also occur incrementally:

$$\mathring{\Upsilon}^{\langle\vec{\theta}\rangle} = \cos\vec{\theta}\,\mathring{\Upsilon} + \sin\vec{\theta}\,\vec{\Upsilon},$$

resulting in a soliton $\mathring{\Upsilon}^{\langle\vec{\theta}\rangle}$ that possesses both a gyrational component $\mathring{\Upsilon}$ and a translational component $\vec{\Upsilon}$, with velocity $\vec{v} = \sin\vec{\theta}\, v\hat{\text{x}}$.

AXIOM 5.1: *Orthogonality of Gyrating and Translational Energy.*
In the $R(3)SO(3)$ framework, energy contributions from gyrating and translational soliton states are orthogonal components of a single conserved structure. The structural energy of a soliton undergoing such a transition is expressed as a complex sum:

$$\mathring{E}_\Upsilon^{\langle\vec{\theta}\rangle} = \mathring{E}_\Upsilon + i\vec{E}_\Upsilon,$$

where $\mathring{E}_\Upsilon$ is the gyrating energy and $\vec{E}_\Upsilon$ is the translational energy arising from field-induced structural rotations resulting in motion. The imaginary unit i encodes the structural orthogonality of these energy modes within the complexified soliton formalism of $R(3)SO(3)$. The physical, or effective, energy is given by

$$E_{\Upsilon\text{eff}} = \left\| \mathring{E}_\Upsilon^{\langle\vec{\theta}\rangle} \right\| = \sqrt{\mathring{E}_\Upsilon^2 + \vec{E}_\Upsilon^2}.$$

With this framework, the complexified energy becomes:

$$E_\Upsilon^{\langle\vec{\theta}\rangle} = \hbar\cos\vec{\theta}\sqrt{\omega_s^2 + \omega_o^2 + \omega_w^2} + i\hbar\sin\vec{\theta}\sqrt{\omega_s^2 + \omega_o^2 + \omega_w^2},$$

yielding an effective total energy:

$$E_{\Upsilon\text{eff}} = \hbar\sqrt{\omega_s^2 + \omega_o^2 + \omega_w^2}.$$

Although the above derivation focuses on linear motion, it extends naturally to orbital and spherular motion (orbit plus nutation):

$$\underset{\text{photonic motion}}{\begin{bmatrix}1 & 0 & 0\\ 0 & c_s & s_s\\ 0 & -s_s & c_s\end{bmatrix}} \quad \underset{\text{orbital motion}}{\begin{bmatrix}c_\emptyset & 0 & -s_\emptyset\\ s_s s_\emptyset & c_s & s_s c_\emptyset\\ c_s s_\emptyset & -s_s & c_s c_\emptyset\end{bmatrix}} \quad \underset{\text{spherular motion}}{\begin{bmatrix}c_\emptyset c_w & s_\emptyset c_w & -s_w\\ -c_s s_\emptyset + s_s c_\emptyset s_w & c_s c_\emptyset + s_s s_\emptyset s_w & s_s c_w\\ s_s s_\emptyset + c_s c_\emptyset s_w & -s_s c_\emptyset + c_s s_\emptyset s_w & c_s c_w\end{bmatrix}}$$

5.5 Inertial and Energising Fields; Revisiting Charge

Returning to the compound soliton state

$$\mathring{\Upsilon}^{\langle\vec{\theta}\rangle} = \cos\vec{\theta}\,\mathring{\Upsilon} + \sin\vec{\theta}\,\vec{\Upsilon},$$

with its associated complexified energy,

$$E_{\Upsilon}^{\langle\vec{\theta}\rangle} = \hbar\cos\vec{\theta}\sqrt{\omega_s^2+\omega_o^2+\omega_w^2} + i\hbar\sin\vec{\theta}\sqrt{\omega_s^2+\omega_o^2+\omega_w^2},$$

we observe that this configuration results from a soliton–soliton interaction. No energy is exchanged; instead, the solitons respond to each other through their quantum field configurations. These are termed *inertial fields*: they influence structure and dynamics without transferring energy.

However, physical processes also require *energising fields*—fields capable of mediating the transfer and redistribution of energy. This leads us to a fundamental classification:

DEFINITION 5.3: *Inertial and Energising Fields.* We distinguish two fundamental classes of fields within the $R(3)SO(3)$ framework:

- *Inertial Fields*, generated by electromagnetic solitons (denoted $\mathcal{F}^{\mathrm{I}}$), associated with structural potentials $\mathcal{V}^{\mathrm{I}}$. These fields do not convey energy; instead, they mediate topological and configurational influences, as in the Aharonov–Bohm effect. Their effects are non-local, affecting soliton configuration and phase.
- *Energising Fields*, responsible for actual energy exchange. These are not produced by electromagnetic solitons directly, but by structural extensions whose field geometry enables energy propagation—interactions observed classically as work, force, or radiation.

This distinction prompts a deeper question: if solitonic fields are inherently inertial, what mediates energy transfer? The resolution lies in re-evaluating the notion of electric charge. Specifically, we must distinguish between:

- The scalar charge e, which governs atomic-scale field binding (e.g., Coulomb interactions);
- A directional, structured quantity ℓ that mediates energy transfer at a distance through structured fields.

This leads naturally to a new ansatz.

AXIOM 5.2: *Structured Charge Duality.*
The charge associated with atomic-scale interactions (e.g., bound electrons) is structurally distinct from the effective charge responsible, for example, for translational

energy transfer in electric currents. Though numerically equal in conventional units, these charges fulfil fundamentally different roles and must not be treated as interchangeable.

The elementary charge e, treated as a scalar constant, governs Coulombic interactions. However, energising interactions—those involving the transfer of energy through fields—require a separate structure: the load, *denoted $\ell \in \mathbb{R}^3 \subset R(3)SO(3)$, a directional object intrinsic to energy-carrying configurations.*

This load serves as the dynamical counterpart to e, coupling with the spatially structured action field $\hbar(\vec{r})$ to generate quantised magnetic flux and energising soliton interactions. Its vectorial character enables the emergence of flux pairs, structured resonance, and energy exchange mechanisms absent in scalar theories.

This axiom is supported by theoretical considerations and by the proposed experiment outlined in Appendix 4 (pp. 91), which is specifically designed to distinguish between the roles of scalar charge and directional load in electromagnetic processes.

NOTE. The distinction introduced here allows for a coherent treatment of both bound field configurations and dynamic energy transfer. While the charge e governs Coulomb-type field interactions, the directional charge responsible for energy propagation will be defined separately as a *load* in the next step.

NOTATION (Load as Operational Charge). To mediate dynamic interactions and directional energy transfer between solitons, we introduce the vector-valued *load*, denoted $\ell \in \mathbb{R}^3 \subset R(3)SO(3)$. This object plays the role of a coupling vector in field-based energy transport and current-like interactions, distinct from the scalar charge e associated with localised electromagnetic fields.

The load ℓ appears in all expressions involving soliton kinematics and flux transport, particularly those based on the field equation system $\mathcal{M}$. The quantum number ℓ, reserved for orbital quantisation, remains unaffected and is not to be confused with the load.

To describe magnetic flux consistently within the structured charge framework, we first express it in terms of the local action field:

DEFINITION 5.4: *Vector-Valued Magnetic Flux.* The local magnetic flux vector associated with a soliton is defined as

$$\phi(\vec{r}) := \frac{\hbar(\vec{r})}{\kappa\ell}, \qquad \text{with reference to (13) in Theorem 4.1 (pp. 20):} \quad \|\phi\| = \frac{h}{\kappa e},$$

where $\hbar(\vec{r}) \in \mathbb{R}^3 \subset R(3)SO(3)$ is the vector-valued action field, and $\ell \in \mathbb{R}^3 \subset R(3)SO(3)$ is the structured load. This formulation captures the orientation and magnitude of quantised magnetic flux and governs all cross-product and dot-product interactions in solitonic field dynamics.

Promoting the quantum load ℓ to a directional object necessitates a refined definition of flux:

DEFINITION 5.5: *Structured Quantum Charge.* The structured quantum load $\ell \in \mathbb{R}^3 \subset R(3)SO(3)$ is defined as a directional coupling vector, such that interaction-specific charges (e.g., electromagnetic) correspond to distinct components or projections. The magnetic flux associated with a soliton is then expressed as

$$\phi(\vec{r}) := \frac{\hbar(\vec{r})}{\ell}.$$

This orientation-dependent coupling between the local action field and the quantum load enables multiple interaction types to coexist within a unified geometric framework.

Having established the vectorial structure of the action field $\hbar(\vec{r})$ and the direction-dependent load ℓ, we now define load polarisation and the symmetry breaking it induces:

DEFINITION 5.6: *Load Polarity and Symmetry Breaking.* The quantum load $\ell \in \mathbb{R}^3 \subset R(3)SO(3)$ admits two distinct energetic polarisations:

- Positive polarity:

$$\ell^+ := \ell_0\, e^{c(\pi/2+\delta_c)},$$

- Negative polarity:

$$\ell^- := \ell_0\, e^{-c(\pi/2+\delta_c)},$$

where $\ell_0 \in \mathbb{R}$ is a scalar, and δ_c denotes small symmetry-breaking angles.

Crucially, the sum of these polarisations is non-zero:

$$\ell^+ + \ell^- \neq 0,$$

indicating a fundamental asymmetry in the energetic configuration of structured loads. This symmetry breaking underlies the emergence of long-range residual forces, including gravitation and the weak nuclear interaction.

We now define the fundamental flux pairing associated with structured load polarity:

LEMMA 5.3: *Quantised Flux Pairing and Structural Neutrality.*
Magnetic flux within the $R(3)SO(3)$ framework is represented not as continuous field lines but as quantised vector pairs $\{\phi, \bar{\phi}\}$, where each flux component is defined by:

$$\phi(\vec{r}) := \frac{\hbar(\vec{r})}{\kappa \ell^+}, \qquad \bar{\phi}(\vec{r}) := \frac{\hbar(\vec{r})}{\kappa \ell^-},$$

with $\hbar(\vec{r}) \in \mathbb{R}^3 \subset R(3)SO(3)$ the structured action field and ℓ^+, ℓ^- representing the polarised structured loads.

The pair forms a flux-conserving structure satisfying:

$$\phi + \bar{\phi} \approx 0, \qquad \|\phi\| = \|\bar{\phi}\|,$$

and represents the minimal unit of magnetic flux in solitonic interactions.

This quantised pairing provides the mechanism by which neutral structures, such as atoms, maintain topological and energetic stability. Analogous behaviour appears *in type-II superconductors, where Abrikosov flux vortices manifest as boundary-bound source–sink pairs, reflecting this same pairing mechanism at the macroscopic level.*

Quantised flux, therefore, is not an emergent or averaged quantity, but a foundational entity in the topological dynamics of soliton coherence, energy conservation, and field interaction.

REMARK. We are working with rotating field vectors. For example, ϕ denotes a rotating vector that acts as the source of a north-pointing elementary magnetic flux. Importantly, $-\phi$ still acts as a source of a north-pointing flux, but with reversed direction or 180 degree rotation.

To distinguish between flux emission and absorption, we introduce $\bar{\phi}$ as the magnetic field vector that absorbs a north-pointing flux. This leads to the superposition relations:

$$\phi + \bar{\phi} = 0, \qquad \phi - \bar{\phi} = 2\phi.$$

Below is a visual representation of this distinction, where the symbol Ⓢ denotes the flux source or sink—that is, the point of origin of the soliton that emits or absorbs the flux.

$$\phi \mapsto \texttt{S}Ⓢ\!\longrightarrow\!\texttt{N}, \qquad -\phi \mapsto \texttt{N}\!\longleftarrow\!Ⓢ\texttt{S}$$
$$\bar{\phi} \mapsto \texttt{N}Ⓢ\!\longleftarrow\!\texttt{S}, \qquad -\bar{\phi} \mapsto \texttt{S}\!\longrightarrow\!Ⓢ\texttt{N}$$

COROLLARY 5.3.1: *Electric Flux Lattices and Structured Field Coherence.*
The electric flux vector $\psi \in \mathbb{R}^3 \subset R(3)SO(3)$, defined by the field equation system,

$$\psi(\vec{r}) = \nu \times \frac{\hbar(\vec{r})}{\kappa \ell_e}, \qquad \mathring{\psi}(\vec{r}) = \omega \times \frac{\hbar(\vec{r})}{\kappa \ell_e}$$

exhibits the same quantised structure and orientation-dependent coherence as magnetic flux. Conjugate pairings of ψ and $\bar{\psi}$ form discrete electric flux units that aggregate into lattices.

Such flux lattices appear in a range of physical systems: in Type-II superconductors as magnetic induced vortex lattice, topological solitons in nematic liquid crystals electrically induced, or in Bose Einstein condensates mechanically induced. These phenomena, though dimensionally distinct, reflect the same geometric principles and field quantisation.

This corollary extends the unified soliton framework to structured electric interactions and identifies a common quantised flux architecture underlying both microscopic and mesoscopic domains.

REMARK 5.1: *Flux as Interaction Unit.* The pairs $\{\phi, \bar{\phi}\}$ and $\{\psi, \bar{\psi}\}$ define the minimal, discrete units of interaction in the soliton framework. These flux structures enable local field coherence, maintain quantised conservation laws, and underlie all inertial and energising interactions in the $R(3)SO(3)$ framework.

This reconceptualisation of flux as an orientable, quantised vector pair replaces the classical continuum of field lines with a discrete, geometric principle—revealing flux as a structural agent of coherence and not merely a field strength descriptor.

NOTE. The separation between atomic charge and structured load, as introduced in Axiom 5.2 (pp. 35), frames all subsequent flux expressions. Energy transfer no longer follows field carriers but is instead mediated through structural coupling to quantised flux defined by the local action field. This forms the foundation for emergent interactions in the nilpotent framework.

Furthermore, this separation of atomic charge and structured load not only resolves, for example, the physical origin of energy flow (electric current), but also provides the foundation for gravity as an emergent interaction arising from residual symmetry breaking.

DEMONSTRATION 5.1: *Gravity and Weak Forces from Symmetry-Breaking.* Consider two conjugate load elements:

$$\ell^+ = e^{i(\pi/2+\delta)}, \quad \ell^- = e^{-i(\pi/2+\delta)},$$

where $\delta \in \mathbb{R}$ quantifies a small symmetry-breaking deviation from perfect phase opposition.

Define interaction strengths via inner products:

$$\begin{aligned}
(\ell^+, \ell^+) &= -e^{2i\delta} & \lim_{\delta\to 0}(\ell^+, \ell^+) &= 1 & &\text{(electromagnetism)}\\
(\ell^-, \ell^-) &= -e^{-2i\delta} & \lim_{\delta\to 0}(\ell^-, \ell^-) &= 1 & &\text{(electromagnetism)}\\
(\ell^+, \ell^-) &= 1, & & & &\text{(electromagnetism)}\\
(\ell^+\ell^-, \ell^+\ell^-) &= 4\sin^2\delta. & & & &\text{(gravity emerges)}
\end{aligned}$$

That is, for small $|\delta| \ll 1$, a residual atomic interaction emerges, captured by the total:

$$(\ell_1^+, \ell_2^+) + (\ell_1^-, \ell_2^-) + (\ell_1^+, \ell_2^-) + (\ell_1^-, \ell_2^+) = (\ell^+\ell^-, \ell^+\ell^-) = 4\sin^2\delta.$$

This residual coupling—suppressed by many orders of magnitude compared to electromagnetic strength—corresponds to *gravitational attraction* between neutral bound systems (e.g., atoms).

Takeaway. This one-dimensional proof-of-concept illustrates the emergence of gravity as a residual effect of symmetry breaking within the $R(3)SO(3)$ framework. In subsequent sections, this idea is extended to a fully symmetrical structure comprising three component charges—load, fervour, and vigour—whose mutual couplings within compounded particles give rise to the phenomena conventionally associated with the *strong* and *weak interactions*. These forces are thus reinterpreted not as fundamental axioms, but as emergent features arising from broken symmetries within a unified, nilpotent field structure.

6 The Nilpotent Universe

By retaining a linear and fully differentiable algebra, the principle of superposition enables the nine-dimensional space $\mathbb{R}^9$ to be radially reduced into three subspaces:

Y, C, and M, a process denoted by $R(3)$. These subspaces are related by $M = Y \times C$, and are governed by the special orthogonal gauge group $SO(3\times 3)$. While $SO(3\times 3)$ retains the core properties of $SO(3)$, it operates independently of the fact that each of its defining axes is itself a radially reduced space—each equivalent to a three-dimensional spatial direction. This structure ensures the viability of $R(3)SO(3)$, maintaining rotational symmetries while embedding transformations across the coupled subspaces.

When working within the space Y using $R(3)SO(3)$, the spaces C and M contribute *complex* axes to the real space Y, thereby extending $R(3)SO(3)$ into a fully differentiable and cyclic algebraic framework.

Physically, the space Y represents the observable three-dimensional world in which interactions take place. Electric *charge* resides in C, while the *mass*-binding strong force arises in M. The full configuration space W binds these components together.

We are now in a position to formalise the governing field equation system for the entire Universe. This system arises as the aggregate of all quantised electromagnetic solitons defined in the radially reduced subspaces Y, C, M, each equipped with a $R(3)SO(3)$ structure and collectively embedded in the extended symmetry group $SO(3\times 3)$.

AXIOM 6.1: *Nilpotent Universe Field System in $SO(3\times 3)$.*
Let $\mathcal{U}$ denote the universal field equation system, defined as the aggregate of all quantised topological electromagnetic solitons within the radially reduced subspaces Y, C, M, each equipped with $R(3)SO(3)$ structure and collectively embedded in the extended symmetry group $SO(3\times 3)$. The system is given by the union of the following two coupled formulations:

$$\mathcal{U}(\nu_U, \phi_U, \psi_U) := \left\{ \nu_U = \frac{\phi_U \times \psi_U}{\phi_U \cdot \phi_U}, \quad \phi_U = \frac{\psi_U \times \nu_U}{\nu_U \cdot \nu_U}, \quad \psi_U = \nu_U \times \phi_U \right\},$$

$$\mathcal{U}(\omega_U, \phi_U, \mathring{\psi}_U) := \left\{ \omega_U = \frac{\phi_U \times \mathring{\psi}_U}{\phi_U \cdot \phi_U}, \quad \phi_U = \frac{\mathring{\psi}_U \times \omega_U}{\omega_U \cdot \omega_U}, \quad \mathring{\psi}_U = \omega_U \times \phi_U \right\}.$$

We now define $\overline{\mathcal{U}}$ as the contra-universe field equation system*, structurally mirroring $\mathcal{U}$ but composed of solitons with negative energy—termed* contra-matter*. These are not mapped one-to-one to solitons in $\mathcal{U}$; instead, the aggregate of all contra-solitons $\sum \overline{\mathcal{U}}_m$ counterbalances the aggregate of all solitons $\sum \mathcal{U}_n$, where the indices m and n need not match.*

The nilpotent condition *is expressed as:*

$$\mathcal{U} + \overline{\mathcal{U}} = 0,$$

meaning the total field content of the Universe and contra-Universe—across energy, topology, and structure—sums to zero. This reflects a fundamental balance embedded within the full field symmetry: the totality of all solitons and contra-solitons exactly cancels, even though their spatial distributions, modes, or indexing may differ.

While $\mathcal{U}$ describes the physical universe we observe, $\overline{\mathcal{U}}$ exists as a contra-physical domain that is not accessible to direct observation. The deeper cosmological implications of this duality—including the divergence of $\mathcal{U}$ and $\overline{\mathcal{U}}$ following their initial overlap—are addressed later in the context of inflationary separation *(see Section 17 (pp. 84))**.*

Since $\mathcal{U}$ and $\overline{\mathcal{U}}$ span the same $R(3)SO(3)$ symmetry across the subspaces Y, C, M, their combined structure preserves the identity:

$$SO(3\times 3) = R(3)SO(3),$$

and the condition $\det\left(\mathcal{U}+\overline{\mathcal{U}}\right) = 0$ encodes the nilpotency of the total Universe field system. This formulation extends conservation principles beyond classical limits and provides a structural basis for interpreting quantum non-locality, entanglement, and the ultimate energy balance between the observable and contra-physical domains.

6.1 Nilpotency and Contra-Matter as Structural Reflection

The nilpotent condition

$$\mathcal{U}+\overline{\mathcal{U}} = 0$$

together with

$$\det\left(\mathcal{U}+\overline{\mathcal{U}}\right) = 0$$

implies a deeper symmetry beyond simple cancellation: the contra-Universe field system $\overline{\mathcal{U}}$ is not merely an inverse of $\mathcal{U}$, but a structural reflection across the full $SO(3\times 3)$ configuration space. This reflection occurs not in coordinate space but within the field-theoretic symmetry group, $R(3)SO(3) \subset \mathbb{R}^9$, where each axis represents an embedded subspace direction with topological field content.

DEFINITION 6.1: *Contra-Matter as Structural Reflection.* Contra-matter refers to solitons of negative energy residing in the contra-Universe field system $\overline{\mathcal{U}}$, defined as a structural reflection of the Universe field system $\mathcal{U}$ within the symmetry algebra of $SO(3\times 3)$. The reflection is defined by:

$$\overline{\mathcal{U}} := -\mathcal{U},$$

with the sign reversal acting on all field vectors and phase components such that their topological, energetic, and algebraic contributions cancel exactly.

This formulation introduces a symmetric duality: for every energetic configuration in $\mathcal{U}$, there exists a reflected configuration in $\overline{\mathcal{U}}$, not necessarily pointwise, but such that the aggregated field system remains nilpotent. Unlike anti-matter, which differs only in charge, contra-matter is defined by opposite energy, phase, and propagation signature.

PROPOSITION 6.1: *Nilpotency as Generalised Conservation.* The nilpotent condition $\mathcal{U}+\overline{\mathcal{U}} = 0$ extends Noether's theorem to the full field system, embedding energy and structural conservation into the topological algebra. This means:

- The total energy of the Universe and contra-Universe is zero.

- The total angular momentum, field flux, and interaction action are null.
- Entanglement and non-locality are algebraic consequences of structural coupling across $R(3)SO(3)$, not violations of locality.

This interpretation allows a rethinking of cosmogenesis: the observable Universe $\mathcal{U}$ and the contra-Universe $\overline{\mathcal{U}}$ may have originated as a symmetric pair within a single initial domain. The inflation of $\mathcal{U}$ corresponds to the contraction (or collapse) of $\overline{\mathcal{U}}$, preserving nilpotency while establishing an arrow of time and an observable energy asymmetry.

This framework lays the groundwork for interpreting gravitational, weak, and dark phenomena as broken symmetries or residual effects arising from the structural divergence of $\mathcal{U}$ and $\overline{\mathcal{U}}$, to be developed in later sections.

REMARK 6.1: *On Maxwell, Contra-Matter, in the Field System $\mathcal{U}$.* The universal field equation system $\mathcal{U}$, together with its contra-symmetric counterpart $\overline{\mathcal{U}}$, encodes the full aggregate of quantised solitons across the radially reduced subspaces Y, C, M, encompassing both the structural and dynamical content of the Universe. It extends the classical Maxwell framework into a fully quantised, rotationally invariant field theory within $SO(3\times 3)$, embedding topological stability and conservation laws directly into its algebraic structure.

In this construction, Maxwell's equations are not approximations but rather the foundational structure from which all field interactions—electromagnetic and otherwise—emerge. For example, let the subspace C govern electric charge interactions. The subspaces M and Y introduce additional quantised field domains, governed by the same Maxwell equations, but distinct in topology and physical effect. Interaction with these domains gives rise to field-theoretic mechanisms responsible for binding phenomena analogous to the strong interaction.

The presence of $\overline{\mathcal{U}}$, as the contra-Universe field system, ensures that all physical quantities—energy, momentum, angular momentum, and field action—are conserved via the nilpotent condition $\mathcal{U} + \overline{\mathcal{U}} = 0$.

This supports Poincaré's insight that either everything is electromagnetic in origin, or our understanding is merely epistemological. Within this framework, electromagnetism—or rather a global *Maxwellism*—constitutes the complete structural basis of all physical fields. Contra-matter provides the necessary counter-structure to restore universal balance, affirming that topological field symmetry—not substance—is the true foundation of physical reality.

REMARK 6.2: *Chirality and Field Handedness.* Because $\overline{\mathcal{U}} = -\mathcal{U}$, the contra-Universe field system inherits the opposite handedness to that of $\mathcal{U}$. If the solitons in $\mathcal{U}$ are right-handed— following a standard orientation of cross products in three-space— then those in $\overline{\mathcal{U}}$ are inherently left-handed.

This intrinsic chirality implies that the nilpotent Universe is not only structurally and energetically balanced, but also chirally symmetric. The observable dominance of right-handed fields in our Universe may thus reflect a cosmological chirality-breaking event: the divergence of $\mathcal{U}$ and $\overline{\mathcal{U}}$ during early inflation. This offers a new topological perspective on parity violation and handedness in quantum field theory.

COROLLARY 6.1: *CPT Symmetry under Nilpotency.*
The nilpotent field structure $\mathcal{U}+\overline{\mathcal{U}}=0$*, together with the determinant condition* $\det\left(\mathcal{U}+\overline{\mathcal{U}}\right)=0$*, implies that the total Universe—comprising both* $\mathcal{U}$ *and* $\overline{\mathcal{U}}$*—is exactly CPT-symmetric. Charge (C), parity (P), and time-reversal (T) symmetries are preserved globally when considered over the full dual field system defined in* $SO(3\times 3)$.

However, within the observable domain $\mathcal{U}$*, the CPT-conjugate configurations exist exclusively in the contra-domain* $\overline{\mathcal{U}}$*, which is causally and energetically divergent from* $\mathcal{U}$*. Consequently, searches for CPT-conjugate particles—such as CPT-symmetric photons—within the* $\mathcal{U}$ *domain are necessarily null.*

This corollary affirms that the Universe is not CPT-violating, but rather CPT-partitioned. The full symmetry is preserved in the nilpotent structure, even though it remains observationally incomplete from within any single domain.

THEOREM 6.1: *Multiplicative Conjugacy of the Contra-Universe Field System.*
In addition to satisfying the nilpotent condition $\mathcal{U}+\overline{\mathcal{U}}=0$*, the contra-Universe field system* $\overline{\mathcal{U}}$ *also satisfies a multiplicative conjugacy:*

$$\mathcal{U}\cdot\overline{\mathcal{U}}=1.$$

This identity expresses that $\overline{\mathcal{U}}$ *is not merely the additive inverse of* $\mathcal{U}$*, but its full algebraic conjugate under the extended field symmetry. Importantly, this relationship does not imply a one-to-one correspondence between individual solitons in* $\mathcal{U}$ *and* $\overline{\mathcal{U}}$*. Instead, it holds at the level of the aggregate field structure, whereby the total configuration of* $\overline{\mathcal{U}}$ *conjugates that of* $\mathcal{U}$ *across the full algebra.*

This field symmetry is further characterised by the $\aleph$ *function (see Definition 5.1 (pp. 31) and Section 5.2 (pp. 28)), which governs the balance and continuity of soliton fields across radial trajectories. In particular, Theorem 5.1 (pp. 31) establishes that energy is conserved through field curvature and inertial action, ensuring that the conjugacy between* $\mathcal{U}$ *and* $\overline{\mathcal{U}}$ *applies not only algebraically, but also dynamically across space.*

This form of conjugate duality implies that the complete field content of $\mathcal{U}$ *is mirrored in* $\overline{\mathcal{U}}$*—not individually, but structurally—in such a way that both additive cancellation and multiplicative closure are preserved. It reinforces the CPT symmetry of the combined system and affirms the universal field framework as closed, self-dual, and algebraically consistent.*

7 Quantum Entanglement

7.1 Quantum Entanglement as Nilpotent Conservation

All mono- or multi-body quantum interactions, $\Upsilon_n \longrightarrow \Upsilon_m$, result in an entangled output state Υ_m that preserves the total quantum structure of the initial configuration Υ_n. Within the nilpotent universe framework, this is expressed as the invariance of the full field system under transformation:

$$\mathcal{U}_n+\overline{\mathcal{U}}_o=\mathcal{U}_m+\overline{\mathcal{U}}_p=0.$$

Here, $\overline{\mathcal{U}}_o$ and $\overline{\mathcal{U}}_p$ denote the contra-Universe field content corresponding to the initial and final interaction states, respectively. In cases of local and symmetric interactions (e.g., spontaneous parametric down-conversion), the changes may be entirely balanced within the observable sector $\mathcal{U}$, leaving $\overline{\mathcal{U}}$ unchanged. However, for interactions involving large-scale solitonic aggregates— those whose energy or structural complexity influences the global nilpotent balance—the contra-Universe sector reflects these changes to ensure the total field system remains invariant and nilpotent.

A representative case is the process $\Upsilon_0 \longrightarrow \Upsilon_1 + \Upsilon_2$, where a pump photon Υ_0 is converted into two daughter photons, Υ_1 and Υ_2. This conversion may occur via spontaneous parametric down-conversion (SPDC) or via a delayed emission pathway, in which the pump photon excites an electron that decays through an intermediate level, emitting two correlated photons.

The conservation laws implied by Noether's theorem extend to all such field interactions. In this formulation, quantum entanglement encodes conservation of the original soliton's full quantum state across all degrees of freedom. Specifically:

$\operatorname{cyc} S_0 = \big(\operatorname{cyc}(R_1 S_1) + \operatorname{cyc}(R_2 S_2)\big)/2$	cyclicity preservation (soliton symmetry)
$\breve{E}_0 = \breve{E}_1 + \breve{E}_2$	structured energy preservation
$\vec{p}_0 = \vec{p}_1 + \vec{p}_2$	momentum preservation
$\Omega_0 = \Omega_1 + \Omega_2$	angular momentum preservation
$R_0 = R_1 + R_2$	internal symmetry (phase) preservation

Here, R_1 and R_2 are zero-energy internal rotations, structurally induced by the interaction of the solitons with the potential fields of the birefringent medium.

Following this entangling event, any subsequent local interaction affecting either Υ_1 or Υ_2 must be reflected in the other in a manner that maintains the nilpotency condition:

$$(\mathcal{U}_1 + \mathcal{U}_2) + (\overline{\mathcal{U}}_1 + \overline{\mathcal{U}}_2) = 0.$$

This reciprocity ensures that entanglement remains a structural conservation constraint.

EXPERIMENT 1: *Bell Test Discriminator for Entanglement Origin.* The $R(3)SO(3)$ framework predicts that entanglement arises from structural field coherence rather than from probabilistic collapse. To test this prediction, we propose a decisive experiment using Type I spontaneous parametric down-conversion (SPDC), in which a vertically polarised pump photon decays into two horizontally polarised daughter photons. This constitutes a nilpotent process: the signal is rotated in a positive direction, and the idler in a negative direction, yielding two daughter photons that are both horizontally polarised.

A Bell test is then performed both before and after the insertion of quarter-wave plates (QWPs), allowing a distinction to be drawn between latent entanglement and dynamically emergent entanglement.

Specifically, the prediction is:

- *Before inserting the QWPs*, the signal and idler photons remain linearly polarised and phase-locked to the macroscopic pilot wave of the originating beam. The Bell test will detect *no violation*, not because the photons reconfigure internally, but because coherence is preserved externally—the condition $\mathcal{U}+\overline{\mathcal{U}}=0$ is satisfied by environmental adjustment rather than by solitonic freedom.
- *After inserting the QWPs*, the signal and idler photons enter circular polarisation states $|R\rangle$ and $|L\rangle$, respectively. This imparts internal phase flexibility, allowing dynamic solitonic shifts that satisfy the nilpotent field condition $\mathcal{U}+\overline{\mathcal{U}}=0$, thereby enabling observable entanglement as internal coherence is maintained.

This prediction offers a clean falsification of the Copenhagen interpretation. If correct, it demonstrates that Bell violations require not wavefunction collapse, but the ability of solitons to self-align phases, thereby preserving the nilpotent Universe of $R(3)SO(3)$. This self-aligning ability requires that the photon be guided in a circularly polarised field and governed by a non-local hidden variable that maintains nilpotency during SPDC pair production and throughout subsequent interactions.

See Appendix B (pp. 92) for a detailed diagram and theoretical explanation.

7.2 Quantum Coherence, Decoherence, and Entanglement Entropy

Within the $R(3)SO(3)$ framework, coherence and entanglement are not abstract quantum phenomena, but manifestations of the structural integrity of solitons and their internal symmetries. These symmetries, represented by zero-energy rotations R_i, preserve the soliton's cyclicity and phase—ensuring the soliton remains in a coherent state.

Coherence as Phase Preservation Quantum coherence is sustained when the internal phase rotations R_i of the soliton remain invariant under interaction. These phase symmetries are conserved in all entangling interactions as a direct consequence of Noether's theorem, as shown in Corollary 6.1 (pp. 43) and Proposition 6.1 (pp. 41). Thus, coherent propagation corresponds to structural invariance within the $R(3)SO(3)$ configuration space.

Decoherence as Structural Deformation Decoherence occurs when a soliton's internal phase structure is irreversibly modified by its environment. In this model, decoherence is interpreted not as probabilistic collapse but as a topological deformation within the soliton's internal symmetry, leading to loss of phase correlation. This does not destroy information, but transfers it into the broader field context.

Entanglement and Field Entropy The entanglement of solitons preserves total information even when subsystems appear incoherent. Since all interactions are embedded within the nilpotent field condition $\mathcal{U}+\overline{\mathcal{U}}=0$, any apparent increase in entanglement entropy reflects only the redistribution of phase information across subsystems. Globally, no entropy is lost.

This suggests a new interpretation of entanglement entropy—not as disorder, but as a measure of topological phase displacement within the soliton configuration space. A formal definition of soliton field entropy will be addressed in future work, extending from the internal symmetry degrees of freedom in $R(3)SO(3)$.

7.2.1 Recoherence and Contra-Entanglement

In the $R(3)SO(3)$ framework, coherence and entanglement are not probabilistic features but arise from structural configurations of solitonic fields that collectively satisfy the nilpotency condition:

$$\mathcal{U} + \overline{\mathcal{U}} = 0.$$

This implies that any local change in the field structure (such as a measurement, phase shift, or interaction) must be globally balanced across both $\mathcal{U}$ and $\overline{\mathcal{U}}$. The coherence of quantum systems is therefore not lost in measurement—it is redistributed across the field domains.

Recoherence. Conventional interpretations describe decoherence as an irreversible entanglement of a system with its environment. In contrast, the $R(3)SO(3)$ framework permits recoherence when the structural conditions are reversed or symmetrically restored. This accounts for observed effects in quantum erasers, delayed-choice experiments, and cavity-based quantum optics, where systems can regain coherence after apparent measurement.

Recoherence occurs not because the system returns to its prior quantum state, but because the structural alignment of the soliton field returns to a configuration compatible with its prior state. The field symmetry allows re-entry of coherence under cyclic or controlled symmetry restoration.

Contra-Entanglement. Entanglement, too, finds a new expression in this framework. While two solitons may appear entangled within $\mathcal{U}$, their conservation properties may in fact require corresponding field arrangements in $\overline{\mathcal{U}}$. These contra-solitons, residing in the contra-universe field system, mirror the structural adjustments of their $\mathcal{U}$ counterparts to maintain global nilpotency:

$$\mathcal{U}_{\text{local}} + \overline{\mathcal{U}}_{\text{contra}} = 0.$$

This mechanism provides a deterministic foundation for nonlocality and may serve as a conceptual basis for otherwise unaccounted conservation balances in apparent vacuum fluctuations, dark sector phenomena, or unmeasurable entanglement traces.

PROPOSITION 7.1: *Recoherence and Contra-Entanglement.* Within the nilpotent field framework, coherence and entanglement are preserved globally via redistribution across $\mathcal{U}$ and $\overline{\mathcal{U}}$. Measurement-induced decoherence corresponds to local reconfiguration of $\mathcal{U}$, with the coherence redistributed in $\overline{\mathcal{U}}$. Under symmetric field conditions, coherence may re-emerge, leading to recoherence. Additionally, entangled solitons may require corresponding contra-entangled solitons in $\overline{\mathcal{U}}$ to preserve the nilpotent field structure.

8 Solutions of $\mathcal{M}$ describe Quantised Topological Electromagnetic Solitons

DEFINITION 8.1: *Waves.* In order for a physical phenomenon to be considered a wave, its mathematical representation must lead to a specific second-order partial differential equation, specifically the d'Alembert wave equation.

DEFINITION 8.2: *Topological Stability.* Within the $R(3)SO(3)$ framework, *topological stability* refers to the invariance of field configurations under continuous 3D rotations acting on soliton fields defined in $\mathbb{R}^3$, whose internal structure is represented by rank-2 tensors embedded in a 9-dimensional real vector space. This space arises from the soliton's spatial embedding and its internal rotational degrees of freedom, encoded in the structured tensor basis associated with the $R(3)SO(3)$ symmetry.

This form of stability does not arise from conventional topological invariants (e.g., winding numbers or homotopy classes), but from the structural preservation of the soliton's electromagnetic field tensors under smooth transformations of the internal rotational degrees of freedom.

A soliton is said to exhibit topological stability if its field configuration is preserved (up to equivalence under smooth transformations within the $R(3)SO(3)$ symmetry group)—that is, if deformations that respect the group action leave its physical structure and interactions invariant.

The mathematical condition for such structural preservation will later be expressed in terms of the spatial curvature of a generalised action field, denoted $\hbar(\vec{r})$, whose second derivative plays a key role in ensuring the soliton's stability under smooth deformations (see Lemma 5.1 (pp. 29)).

DEFINITION 8.3: *Topological Soliton.* In order for a physical phenomenon to be considered a topological soliton, its mathematical representation must satisfy the following conditions:

1. It must lead to a specific second-order partial differential equation, namely the d'Alembert wave equation.

2. It must exhibit a topological structure that is stable under propagation.

3. The topological structure must be in resonance with the wave solution derived in *1.*, ensuring consistent and sustained propagation without dispersion or dissipation.

LEMMA 8.1: *$\mathcal{M}$ and the d'Alembert Wave Equation.*
Let $\{\mathcal{M},\ \Omega\}$ reside in the quantised orthogonal gauge group, $R(3)SO(3)$. Then solutions of

$$\mathcal{M}(\nu,\phi,\psi) := \left\{ \nu = \frac{\phi \times \psi}{\phi \cdot \phi}, \quad \phi = \frac{\psi \times \nu}{\nu \cdot \nu}, \quad \psi = \nu \times \phi \right\}$$

and

$$\mathring{\mathcal{M}}(\omega,\phi,\mathring{\psi}) := \left\{ \omega = \frac{\phi \times \mathring{\psi}}{\phi \cdot \phi}, \quad \phi = \frac{\mathring{\psi} \times \omega}{\omega \cdot \omega}, \quad \mathring{\psi} = \omega \times \phi \right\}$$

parameterised by

$$\Omega = \hat{x}\,\omega_s + \hat{y}\,\omega_\emptyset + \hat{z}\,\omega_n$$

are also solutions of the d'Alembert wave equation.

PROOF. We seek a solution for a gyration that propagates along the path $\vec{s} = \int \vec{u}\mathrm{d}t$ and $u^2 = \vec{u}\cdot\vec{u}$ The d'Alembert wave equation in this context is given by:

$$\frac{\partial^2\phi}{\partial\vec{s}^2} - \frac{1}{u^2}\frac{\partial^2\phi}{\partial t^2} = 0.$$

We describe ϕ simultaneously as a product of squared spatial and temporal components:

$$\phi = f(\vec{s})^2 = g(t)^2 \qquad \text{which gives:} \quad \phi = f(\vec{s})g(t),$$

where $f(\vec{s})$ depends on position and $g(t)$ depends on time. Substituting this form into the wave equation gives:

$$\frac{\partial^2 f(\vec{s})g(t)}{\partial\vec{s}^2} - \frac{1}{u^2}\frac{\partial^2 f(\vec{s})g(t)}{\partial t^2} = 0.$$

Dividing by $\phi = f(\vec{s})g(t)$ and multiplying by u^2, we obtain:

$$\frac{u^2}{f(\vec{s})}\frac{\partial^2 f(\vec{s})}{\partial\vec{s}^2} - \frac{1}{g(t)}\frac{\partial^2 g(t)}{\partial t^2} = 0.$$

The first and the second terms are now independent of one another. Since $f(\vec{s})$ and $g(t)$ are independent functions of position and time, respectively, the derivatives are total derivatives. For the above to hold, both terms must equal a constant. Anticipating the solution, we introduce the constant $-\omega^2/4$:

$$\frac{u^2}{f(\vec{s})}\frac{\mathrm{d}^2 f(\vec{s})}{\mathrm{d}\vec{s}^2} = \frac{1}{g(t)}\frac{\mathrm{d}^2 g(t)}{\mathrm{d}t^2} = -\frac{\omega^2}{4}.$$

The resulting ordinary differential equations resemble simple harmonic oscillators with known solutions:

$$f(\vec{s}) = e^{i\omega\vec{s}/2u}, \quad g(t) = e^{i\omega t/2}.$$

Thus, the solution for ϕ becomes:

$$\phi = e^{i\omega\vec{s}/2u}e^{i\omega t/2} = e^{i\omega\vec{s}/u} = e^{i\omega t} = e^{ik\vec{s}/2+\omega t/2},$$

where $k = \omega/u$, and $\omega = \|\Omega\|$.

Since this solution holds if $\vec{s} = \int \vec{u}\mathrm{d}t$, it follows that the parametrisation by Ω satisfies the d'Alembert wave equation.

Therefore, by mapping $\vec{u} \mapsto v$ or $\vec{u} \mapsto \omega$ ensures that the solutions of $\mathcal{M}$ and $\mathring{\mathcal{M}}$ are also solutions of the d'Alembert wave equation, which proves the lemma. □

DEFINITION 8.4: *Resonances and Eigenvalues in Quantised Space.* With Lemma 8.1 (pp. 47), the field equation system was reduced to the one-dimensional d'Alembert wave equation:

$$\frac{\partial^2\phi}{\partial\vec{s}^2} - \frac{1}{u^2}\frac{\partial^2\phi}{\partial t^2} = 0.$$

Here, u may denote either a translational velocity v or a gyration rate ω.

By mechanical analogy, an infinitely long guitar string supports vibrations of arbitrary frequency. To illustrate, consider an impulse disturbance composed of a superposition of all frequencies. The Fourier Transform of a continuous-time impulse function, denoted $\delta(t)$, is given by:

$$F(\xi) = \mathcal{F}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t) e^{-i\xi t} \mathrm{d}t,$$

where:

- $F(\xi)$ is the Fourier Transform in the frequency domain.
- $\delta(t)$ is the Dirac delta function (impulse).
- ξ is the angular frequency.

Utilising the sifting property of the Dirac delta function:

$$\int_{-\infty}^{\infty} \delta(t-a) f(t) \mathrm{d}t = f(a)$$

In our case, we have $\delta(t) = \delta(t-0)$ and $f(t) = e^{-i\xi t}$. Applying the sifting property:

$$F(\xi) = \int_{-\infty}^{\infty} \delta(t) e^{-i\xi t} \mathrm{d}t = e^{-i\xi(0)} = e^0 = 1$$

Therefore, the Fourier Transform of the impulse function $\delta(t)$ is:

$$\mathcal{F}\{\delta(t)\} = 1$$

for all frequencies ξ.

In the mechanical analogy, the impulse propagates along the string at velocity u, with the medium (guitar string) sustaining all frequencies ξ. In $R(3)SO(3)$, the spatial structure is treated as infinite, admitting all ξ for the impulse, but also provides further dimensions to spin the impulse. This provides a model for a photon as a spinning impulse:

$$\Upsilon_{\text{photon}} = \mathcal{F}\{\delta(t)\} e^{i\omega_s t},$$

with no restriction on the spin rate ω_s, as all frequencies are allowed in an infinite Universe.

This continuous spectral behaviour ω_s characterises photons. In contrast, particles exhibit discrete energy levels, indicating the presence of resonance conditions. Clamping a guitar string between two fixed points results in a discrete set of resonance frequencies—eigenvalues—inversely proportional to the distance between the clamps. Analogously, in the quantised spatial framework $R(3)SO(3)$, fixed spacial quanta act as boundary conditions for gyrations, producing discrete resonant modes.

Therefore, the quantisation of space yields a spectrum of eigenvalues that define the allowed frequencies—or radial rates—of solitonic excitation. These resonance conditions, or eigenvalues

$$\{\omega_s, \omega_\emptyset, \omega_w\},$$

uniquely characterise each particle by assigning a fixed triple of quantised frequency components, determining rest energy and internal structure.

THEOREM 8.1: *Quantised Topological Electromagnetic Solitons* Υ *and* $\mathring{\Upsilon}$.
Let S *denote the solution matrix satisfying the field equation systems* $\mathcal{M}$ *and* $\mathring{\mathcal{M}}$. *An example solution is given by:*

$$S = \begin{bmatrix} c_ø c_w & s_ø c_w & -s_w \\ -c_s s_ø + s_s c_ø s_w & c_s c_ø + s_s s_ø s_w & s_s c_w \\ s_s s_ø + c_s c_ø s_w & -s_s c_ø + c_s s_ø s_w & c_s c_w \end{bmatrix} \quad \text{where} \quad \begin{matrix} c_s := \cos(\omega_s t) \\ s_ø := \sin(\omega_ø t) \\ \text{etc.} \end{matrix} \tag{16}$$

A quantised topological electromagnetic soliton Υ *and its gyrated counterpart* $\mathring{\Upsilon}$ *are defined by:*

$$\Upsilon \xrightarrow[by]{dsc} \mathcal{Y} = S\mathcal{A} \qquad \text{and} \qquad \mathring{\Upsilon} \xrightarrow[by]{dsc} \mathring{\mathcal{Y}} = S\mathcal{A},$$

- *where:*

$$\mathcal{Y} := \begin{bmatrix} \nu \\ \phi \\ \psi \end{bmatrix} \qquad \text{and} \qquad \mathring{\mathcal{Y}} := \begin{bmatrix} \omega \\ \phi \\ \mathring{\psi} \end{bmatrix} \tag{17}$$

with the following components:

ν *translational velocity vector of the soliton,*
ϕ *magnetic field vector component,*
ψ *electric field vector component,*
ω *the gyration rate vector,*
$\mathring{\psi}$ *gyro-electric field vector component.*

- *and where*

$$\mathcal{A} := \begin{bmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \end{bmatrix}, \qquad \text{with } \hat{c} = \hat{a} \times \hat{b} \tag{18}$$

and the axes $\hat{a}, \hat{b}$ *are drawn from:*

$$\{\hat{a}, \hat{b}\} \in \{\hat{y}_x, \hat{y}_y, \hat{y}_z, \hat{c}_x, \hat{c}_y, \hat{c}_z, \hat{m}_x, \hat{m}_y, \hat{m}_z\}$$

The pair $\{\hat{a}, \hat{b}\}$ *must satisfy the structural condition* $\hat{c} = \hat{a} \times \hat{b}$, *with* $\hat{m} = \hat{y} \times \hat{c}$ *reflecting the relation* $M = Y \times C$.

PROOF. From Lemma 8.1 (pp. 47), we have shown that $\mathcal{M}$ and $\mathring{\mathcal{M}}$ satisfies the radially reduced d'Alembert wave equation. Additionally:

1. Theorems 2.1 (pp. 17) and 4.1 (pp. 20) have proven that $\mathcal{M}$ and $\mathring{\mathcal{M}}$ are solutions of the Maxwell field equations in vacuum.
2. Theorem 4.1 (pp. 20) demonstrated the quantisation of both the Maxwell equations and the space itself.

Combining these results, we conclude that the composite field system formed by $\mathcal{M}$ and $\mathring{\mathcal{M}}$ constitutes a quantised electromagnetic topological soliton. □

REMARK 8.1 The soliton structure defined by the composite field system $\mathcal{M} \cup \mathring{\mathcal{M}}$ is preserved under continuous, differentiable transformations within the $R(3)SO(3)$ symmetry group. These transformations act on the internal rotational configuration without altering the soliton's identity or interaction characteristics, thereby establishing its topological stability as defined in Definition 8.3 (pp. 47) .

9 Information and Information Conservation

Having established the general structure of quantised topological electromagnetic solitons in Theorem 8.1 (pp. 50), we now turn to the interpretive significance of these configurations: each soliton encodes not merely a field solution, but a fundamental unit of information structured by the $R(3)SO(3)$ algebra.

DEFINITION 9.1: *Information in the $R(3)SO(3)$ Framework.* Information within the $R(3)SO(3)$ framework is defined as the minimal, topologically stable configuration of field structure—encoded in quantised solitonic states—that remains invariant under the transformations of the extended gauge group SO(3×3) and is conserved under nilpotent field dynamics.

Formally, a unit of information is realised through the triadic soliton structure:

$$\Upsilon \xrightarrow[\text{by}]{\text{dsc}} \mathcal{Y} = S\mathcal{A},$$

where $\mathcal{Y}$, S and $\mathcal{A}$ are defined in equations (17), (16), and (18), respectively.

Information is therefore:

- *Quantised*: bounded to finite, stable field configurations;
- *Topological*: resistant to continuous deformation;
- *Conserved*: preserved under solitonic interaction (no loss or duplication);
- *Non-local*: defined by relational invariants across Y, C, M;
- *Directional*: mediated by the structured load ℓ and action field $\hbar(\vec{r})$.

Hence, information in $R(3)SO(3)$ is not abstract, but a concrete, structural invariant embedded within the physical field system—manifesting as distinct, recognisable solitonic configurations that encode state, interaction history, and future potential for work.

This definition clarifies how information emerges from and is maintained within the solitonic architecture. We are now in a position to formalise its conservation across both local interactions and the global nilpotent Universe–Contra-Universe framework.

THEOREM 9.1: *Information Conservation in the Nilpotent Field System.*
In the $R(3)SO(3)$ framework, information is structurally defined by the solitonic triad

$$\Upsilon \xrightarrow[by]{dsc} \mathcal{Y} = S\mathcal{A},$$

where the field vector $\mathcal{Y}$, the solution matrix S, and the axis configuration $\mathcal{A}$ are given in equations (17), (16), *and* (18), *respectively.*

This structural definition implies that:

- *Information is* quantised, topologically protected, *and* relational*;*
- *Information is* non-locally *encoded via the full triadic symmetry $Y, C, M \subset \mathbb{R}^9$;*

- *Information is* conserved *under all solitonic transformations permitted by the field systems* $\mathcal{M}$ *and* $\mathring{\mathcal{M}}$.

  Let Υ_1, Υ_2 *be two solitons evolving via*

  $$\mathcal{M}(\Upsilon_1, \Upsilon_2), \qquad \mathring{\mathcal{M}}(\mathring{\Upsilon}_1, \mathring{\Upsilon}_2),$$

then the total structural information is invariant:

$$\mathcal{I}[\Upsilon_1] + \mathcal{I}[\Upsilon_2] = \mathcal{I}[\Upsilon_1'] + \mathcal{I}[\Upsilon_2'],$$

where primed quantities denote post-interaction states. This principle generalises to all mono- or multi-body quantum interactions, $\Upsilon_n \longrightarrow \Upsilon_m$, *such that*

$$\sum_n \mathcal{I}[\Upsilon_n] = \sum_m \mathcal{I}[\Upsilon_m']$$

This principle extends to the entire Universe–Contra-Universe duality:

$$\mathcal{U} + \bar{\mathcal{U}} = 0 \qquad \Longrightarrow \qquad \mathcal{I}_{\text{Universe}} + \mathcal{I}_{\text{Contra-Universe}} = 0.$$

Thus, the total information content of the nilpotent Universe is a conserved, structurally balanced invariant embedded in the field algebra of $R(3)SO(3) \subset SO(3\times 3)$*. No information is created, destroyed, or duplicated—only transformed in accordance with solitonic interaction symmetries.*

PART III

Everything in the Universe is of Electromagnetic Origin

To summarise Part I of this paper:

- Poincaré once pondered: *"Either everything in the universe is of electromagnetic origin… or a mere epiphenomenon, something due to our methods of measurement."*
- We established a special orthogonal gauge group $R(3)SO(3) \subset \mathbb{R}^9$, which:
  - is an extension of SO(3), inherently preserving all SO(3)-invariant symmetries;
  - is inherently singularity-free, and therefore continuous, differentiable, and integrable;
  - provides a classical algebraic framework within which the Maxwell field equations naturally operate.
- The field equation systems

  $$\mathcal{M}(\nu, \phi, \psi) := \left\{ \nu = \frac{\phi \times \psi}{\phi \cdot \phi}, \quad \phi = \frac{\psi \times \nu}{\nu \cdot \nu}, \quad \psi = \nu \times \phi \right\},$$

  together with

  $$\mathring{\mathcal{M}}(\omega, \phi, \mathring{\psi}) := \left\{ \omega = \frac{\phi \times \mathring{\psi}}{\phi \cdot \phi}, \quad \phi = \frac{\mathring{\psi} \times \omega}{\omega \cdot \omega}, \quad \mathring{\psi} = \omega \times \phi \right\}$$

  reside within the $R(3)SO(3)$ framework.

- The classical Maxwell field equations in vacuum emerge from $\mathcal{M}$ (Theorem 2.1 (pp. 17)), and analogously, we identified a set of gyratory Maxwell field equations from $\mathring{\mathcal{M}}$, which describe field gyrations or vortices.
- Using the first expressions in $\mathcal{M}$ and $\mathring{\mathcal{M}}$, we obtained a quantised formulation of the Maxwell equations—Theorem 4.1 (pp. 20)—and demonstrated that space itself is quantised, thereby establishing a fundamental quantum length l_o.
- Section 8 (pp. 46) defines wave solutions, topological stability, and topological solitons in the $R(3)SO(3)$ framework. Because the d'Alembert wave equation emerges from the Maxwell equations, solutions of $\mathcal{M}$ and $\mathring{\mathcal{M}}$ satisfy wave dynamics and are thus classified as quantised topological electromagnetic solitons.
- Section 6 (pp. 39) establishes the conservation principles.
- Definition 8.3 (pp. 47), provides the formal mathematical definition for these solitons.

We now continue with the demonstration that the photon—along with all its quantum properties—is described naturally as a quantised topological soliton within the $R(3)SO(3)$ framework. This is followed by the formulation of fields beyond the soliton's domain and an analysis of soliton–soliton interactions.

To facilitate the efficient representation of particles and their transformations within $R(3)SO(3)$, we introduce the following notational tool:

DEFINITION 9.2: *Row-by-Row Scaling Operator.* Consider a solution matrix $\mathcal{S}$ whose third row is defined as the cross product of the first two rows, that is,

$$\mathcal{S}_3 = \mathcal{S}_1 \times \mathcal{S}_2.$$

We define a row-wise scaling operator triad $\mathcal{X} = \langle a, b, ab\rangle$, where:

- a is a scalar multiplier applied to the first row of $\mathcal{S}$,
- b is a scalar multiplier applied to the second row of $\mathcal{S}$,
- ab scales the third row, preserving the cross pruducts

$$\mathcal{S}1 = \frac{\mathcal{S}_2 \times \mathcal{S}_3}{\mathcal{S}_2 \cdot \mathcal{S}_2}, \qquad \mathcal{S}_2 = \frac{\mathcal{S}_3 \times \mathcal{S}_1}{\mathcal{S}_1 \cdot \mathcal{S}_1}, \qquad \mathcal{S}_3 = \mathcal{S}_1 \times \mathcal{S}_2$$

defined in the structure of the field equation system

$$\mathcal{M}(\nu, \phi, \psi) := \left\{ \nu = \frac{\phi \times \psi}{\phi \cdot \phi}, \quad \phi = \frac{\psi \times \nu}{\nu \cdot \nu}, \quad \psi = \nu \times \phi \right\},$$

The action of $\mathcal{X}$ on $\mathcal{S}$ is denoted as:

$$\mathcal{X} \diamond \mathcal{S} = \begin{bmatrix} a(\star & \star & \star) \\ b(\star & \star & \star) \\ ab(\star & \star & \star) \end{bmatrix}$$

where $\mathcal{S}$ is the original (unscaled) matrix indicated by starred entries.

To apply this operator meaningfully to solitonic solutions, we fix the following notational convention for the trigonometric components of the solution matrix:

$$\begin{bmatrix} c_ø c_w & s_ø c_w & -s_w \\ -c_s s_ø + s_s c_ø s_w & c_s c_ø + s_s s_ø s_w & s_s c_w \\ s_s s_ø + c_s c_ø s_w & -s_s c_ø + c_s s_ø s_w & c_s c_w \end{bmatrix} \text{how}$$

where

$$\begin{cases} c_s := \cos \mathsf{h}\omega_s t & s_s := \sin \mathsf{h}\omega_s t \\ c_ø := \cos \mathsf{o}\omega_ø t & s_ø := \sin \mathsf{o}\omega_ø t \\ c_w := \cos \mathsf{w}\omega_w t & s_w := \sin \mathsf{w}\omega_w t \end{cases} \quad \text{and where} \quad \begin{cases} \gcd(\omega_s, \omega_ø) = 1 \\ \gcd(\omega_s, \omega_w) = 1 \\ \gcd(\omega_ø, \omega_w) = 1 \end{cases}$$

REMARK. To preserve the cross-product structure of the solution matrix, we define the row-scaling operator as

$$\mathcal{X} = \langle a, b, ab \rangle := \langle a, b, * \rangle = \langle a, *, ab \rangle = \langle *, b, ab \rangle,$$

where the starred component is implicitly determined by the requirement that it equals the product of the other two entries. This construction reflects the cross-product relations in $\mathcal{M}$, and more generally, expresses the principle that the third component is always given by the product of the first two.

10 Photons as a Quantised Topological Soliton in $R(3)SO(3)$

The objective here is to present the solution S to the field equation system, modelling the photon as a quantised topological electromagnetic soliton.

DEFINITION 10.1: *Photonic Structure in $R(3)SO(3)$.* Within the $R(3)SO(3)$ framework, quantum mechanical properties emerge as rotational transformations. These are rendered in upright serif font to emphasise their quantum nature and structural origin.

d	Direction, where $d \in \{-1, 1\}$.
h	Helicity, where $h \in \{-1, 1\}$.
s	Spin, given by the product of helicity and direction: $\mathsf{s} = \mathsf{hd}$.
ω_q	Fundamental spin rate quantum, see Section 4.2 (pp. 24).
n	Spin rate aggregator, $\mathsf{n} \in \mathbb{N}$, defining the quantised Einstein–Planck relation $E = \hbar \mathsf{n} \omega_q$.
$\omega_\mathsf{s} = \mathsf{nh}\omega_q$	Spin rate, determining $E = \hbar\omega_\mathsf{s}$.
$\dot{\mathsf{o}}$	Orbital rate periodicity, $-1 < \dot{\mathsf{o}} < +1 \in \mathbb{Q}$, with ratio $(\omega_\mathsf{s} : \omega_ø) \in \mathbb{N}$.
$\omega_ø = \dot{\mathsf{o}}\omega_\mathsf{s}$	Orbital rate, providing orbital angular momentum. The subscripted crossed o distinguishes it from the spin quantum ω_q.
$\dot{\mathsf{w}}$	Notational rate periodicity, $-1 < \dot{\mathsf{w}} < +1 \in \mathbb{Q}$ (from German *wanken*).
$\omega_\mathsf{W} = \dot{\mathsf{w}}\omega_\mathsf{s}$	Notational rate, defining notational angular momentum, with $(\omega_\mathsf{s} : \omega_\mathsf{W}) \in \mathbb{N}$.
$\{\vartheta_\mathsf{y}, \vartheta_\mathsf{z}\}$	Degree of polarisation or ellipticity of the magnetic component, defined by $\varepsilon = \cos\vartheta$, along y and z, respectively.
$\{\omega_{B,y}, \omega_{B,z}\}$	Berry phase rates under cyclic evolution in polarisation along y and z, respectively.

$\{\vartheta_{c}, \vartheta_{m}\}$ Refractivity of the medium. The refractive index is defined by $n_r = c/\nu_r$, with $\nu_r = 1/\cos\vartheta_i$ for $i = c, m$.

REMARK. In the above definition $\omega_{\text{ø}}$ and ω_w are subharmonics of ω_s

Guided by Definition 1.7 the photonic ternary rotations allow us to model a photon as a particle, incorporating all of the above:

$$\gamma \xrightarrow[\text{by}]{\text{dsc}} \begin{bmatrix} \nu \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} \mathrm{d}e^{c\vartheta_c}\, e^{m\vartheta_m}\,(1 & 0 & 0) \\ e^{c\dot{\mathrm{o}}\omega_{\mathrm{S}}t}\, e^{m\dot{\mathrm{w}}\omega_{\mathrm{S}}t}\,(0 & e^{m\vartheta'_{\mathrm{y}}}\cos\mathrm{h}\omega_{\mathrm{S}}t & e^{m\vartheta_{\mathrm{z}}}\sin\mathrm{h}\omega_{\mathrm{S}}t) \\ \mathrm{d}e^{c(\dot{\mathrm{o}}\omega_{\mathrm{S}}t+\vartheta_c)}\, e^{m(\dot{\mathrm{w}}\omega_{\mathrm{S}}t+\vartheta_m)}\,(0 & -e^{m\vartheta_{\mathrm{z}}}\sin\mathrm{h}\omega_{\mathrm{S}}t & e^{m\vartheta'_{\mathrm{y}}}\cos\mathrm{h}\omega_{\mathrm{S}}t) \end{bmatrix} \begin{bmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{bmatrix}$$

where $\vartheta'_{\mathrm{y}} = \vartheta_{\mathrm{y}} + t\omega_{B,y}$ includes Berry phase evolution.

All known quantum states of the photon, to the author's knowledge, are classically captured within this structural representation.

The Einstein–Planck relation is extended to incorporate photons possessing both orbital angular momentum (OAM) and notational angular momentum (NAM) through the introduction of a complex energy representation, denoted as $\breve{E}$

$$\breve{E} = \hbar\omega_{\mathrm{S}}(1 + c\dot{\mathrm{o}} + w\mathrm{w}),$$

- where $\hbar\omega_{\mathrm{S}}$ represents the real energy contribution associated with the intrinsic spin angular momentum (SAM), and where
- the reactive component $\hbar\omega_{\mathrm{S}}(\dot{\mathrm{o}} + \dot{\mathrm{w}})$ accounts for the energy contributions arising from the OAM and NAM, respectively, and where
- $\{y, c, w\} \in R(3)SO(3)$ are the ternary number operators, see Definition 1.7 (pp. 12).

The effective energy becomes

$$E_{\text{eff}} = \left|\breve{E}\right| = \hbar\omega_{\mathrm{S}}\sqrt{1 + \dot{\mathrm{o}}^2 + \mathrm{w}^2},$$

resulting in a *momentum dilation*, where the photon's momentum is given by

$$\vec{p} = \hat{v}\frac{E_{\text{eff}}}{c} = \hat{v}\frac{\hbar\omega_{\mathrm{S}}\sqrt{1 + \dot{\mathrm{o}}^2 + \dot{\mathrm{w}}^2}}{c},$$

with $\hat{v}$ denoting the unit velocity vector.

This momentum dilation—attributable to the inclusion of OAM and NAM as reactive energy— is experimentally validated in spontaneous parametric down-conversion (SPDC).

OBSERVATION 10.1: *Effective Energy Increase in SPDC.* In spontaneous parametric down-conversion (SPDC), a pump photon with frequency ω_{pump} is converted into two daughter photons with lower frequencies ω_s and ω_i, such that:

$$\omega_{\text{pump}} = \omega_s + \omega_i.$$

However, when the daughter photons acquire opposite orbital angular momentum (OAM), their individual effective energies increase:

$$\left\|\breve{E}_s\right\| = \hbar\omega_s\sqrt{1 + \dot{\mathrm{o}}^2}, \qquad \left\|\breve{E}_i\right\| = \hbar\omega_i\sqrt{1 + \dot{\mathrm{o}}^2},$$

leading to a total effective energy

$$\|\breve{E}_s\| + \|\breve{E}_i\| > \hbar(\omega_s + \omega_i) = E_{\text{pump}}.$$

The momenta of the signal and idler photons are:

$$\|p_s\| + \|p_i\| = \frac{\|\breve{E}_s\|}{c} + \frac{\|\breve{E}_i\|}{c} > \frac{\hbar(\omega_s + \omega_i)}{c}.$$

This implies that the downconverted photons carry more effective energy—manifested as increased momentum magnitude, evident in the expanded light cone emitted during the SPDC process—than the pump photon alone. The origin of this additional effective energy is not yet fully understood. It may be attributed to field-structural borrowing or redistribution from the local environment, both of which point toward deeper mechanisms in electrodynamics made possible by a globally elevated potential—*i. e.*, vacuum energy.

10.1 Entanglement

Section 7 (pp. 43) addresses the foundational aspects of quantum entanglement. In addition to that discussion, and having analysed the properties of photons produced via spontaneous parametric down-conversion (SPDC), it is noted that the signal and idler photon pair are entangled not only in their polarisation states, but also through their orbital angular momentum phasing. The conservation of total orbital angular momentum ensures that their combined OAM remains zero.

The birefringent medium acts effectively as a recoil-free system. Consequently, SPDC interactions proceed without environmental back-action, analogous to the Mössbauer effect in solid-state physics. This requires symmetry between the signal and idler photons in order to preserve nilpotency.

11 Particles in $R(3)SO(3)$

Let's summarise: With Theorem 8.1 (pp. 50) we developed the notation that a soliton is described $\Upsilon \xrightarrow[\text{by}]{\text{dsc}} \mathcal{Y} = \mathcal{S}\mathcal{A}$ The components of the expression $\mathcal{Y} = \mathcal{S}\mathcal{A}$ are: the EM-*Action Field Triad* $\mathcal{Y}$ that defines dynamic relation between the magnetic and electric components over velocity or gyration; a solution matrix $\mathcal{S}$ satisfying the field equation system $\mathcal{M}$ and $\mathring{\mathcal{M}}$; and the axes $\mathcal{A}$; all residing in $R(3)SO(3)$.

EM-action field triads We now define the EM-action field triads. There are four fundamental variants:

$$\vec{\mathcal{Y}} := \begin{bmatrix} \nu \\ \phi \\ \psi \end{bmatrix}, \qquad \mathring{\mathcal{Y}} := \begin{bmatrix} \omega \\ \phi \\ \mathring{\psi} \end{bmatrix}, \qquad \bar{\vec{\mathcal{Y}}} := \begin{bmatrix} \nu \\ \bar{\phi} \\ \bar{\psi} \end{bmatrix}, \qquad \bar{\mathring{\mathcal{Y}}} := \begin{bmatrix} \omega \\ \bar{\phi} \\ \bar{\mathring{\psi}} \end{bmatrix}$$

The pairs $\vec{\mathcal{Y}}$ and $\mathring{\mathcal{Y}}$ represent particles satisfying Maxwell's equations for linear motion (over set arrow) and gyrational (vortical) dynamics (overset circle), respectively. The barred versions represent their conjugate pairs, corresponding to opposite polarity due to the reversal of the magnetic component from source to sink.

The Axes Next we define the reference axes satisfying Theorem 8.1 (pp. 50):

$$\mathcal{A} := \begin{bmatrix} \hat{\mathsf{x}} \\ \hat{\mathsf{y}} \\ \hat{\mathsf{z}} \end{bmatrix} := \begin{bmatrix} \hat{\mathscr{y}}_{\mathsf{x}} \\ \hat{\mathscr{y}}_{\mathsf{y}} \\ \hat{\mathscr{y}}_{\mathsf{z}} \end{bmatrix}, \qquad \mathcal{A}^{\ddagger} := \begin{bmatrix} \hat{\mathscr{y}}_{\mathsf{x}} \\ \hat{\mathscr{c}}_{\mathsf{y}} \\ \hat{\mathscr{m}}_{\mathsf{z}} \end{bmatrix}.$$

It is noted that the gyration and velocity vectors ω and v in both cases are defined within the real space Y. The spaces C and M serve as complexification spaces for v and ω, allowing their coefficients to be complex. For example, the observed real velocity of a photon appears slowed in a medium: $v_{\mathsf{x}} = e^{\mathscr{c}\vartheta_{\mathscr{c}}}$, although its effective velocity remains constant. The magnetic component is primarily defined in Y and C; it may be doubly complexified by M, and vice versa, as in:

$$\phi_{\mathsf{x},\mathscr{c}} = e^{\mathscr{c}\vartheta_{\mathscr{c}}}\, e^{\mathscr{m}\vartheta_{\mathscr{m}}}\, \hat{\mathsf{x}} + e^{\mathscr{m}\vartheta_{\mathscr{m}}}\, e^{\mathscr{y}\vartheta_{\mathscr{y}}}\, \hat{\mathscr{c}}_{\mathsf{x}}.$$

The cross product $v \times \phi$ places the electric component primarily into the spaces Y and M.

The Solution Matrix We define the following quantum numbers, which have their parallels in the description of photon quantum properties.

- d Direction, $\mathsf{d} \in \{-1, 1\}$.
- h Helicity, (spin rate direction) $\mathsf{h} \in \{-1, 1\}$.
- o Orbital rate direction, $\mathsf{o} \in \{-1, 1\}$.
- p Polarity of magnetic component, $\mathsf{p} \in \{-1, 1\}$.
- w Notational rate direction, $\mathsf{w} \in \{-1, 1\}$.
- s Spin, the product of direction and helicity, $\mathsf{s} = \mathsf{dh}$.

See Remark 1 (pp. 38) for further clarification on the quantum number p, which defines the polarity of the magnetic component in terms of its role as a source or sink of quantised flux.

We now consider the solution matrix S satisfying $\mathcal{M}$ and $\mathring{\mathcal{M}}$:

$$\mathring{\mathcal{S}}^{\S} := \begin{bmatrix} (& c_{\emptyset} c_w & s_{\emptyset} c_w & -s_w) \\ \mathsf{p}(& -c_s s_{\emptyset} + s_s c_{\emptyset} s_w & c_s c_{\emptyset} + s_s s_{\emptyset} s_w & s_s c_w) \\ \mathsf{p}(& s_s s_{\emptyset} + c_s c_{\emptyset} s_w & -s_s c_{\emptyset} + c_s s_{\emptyset} s_w & c_s c_w) \end{bmatrix} \ \mathsf{how} \tag{19}$$

$$\text{where} \begin{cases} c_s := \cos \mathsf{h}\omega_s t & s_s := \sin \mathsf{h}\omega_s t \\ c_{\emptyset} := \cos \mathsf{o}\omega_{\emptyset} t & s_{\emptyset} := \sin \mathsf{o}\omega_{\emptyset} t \\ c_w := \cos \mathsf{w}\omega_w t & s_w := \sin \mathsf{w}\omega_w t \end{cases} \text{and where} \begin{cases} \gcd(\omega_s, \omega_{\emptyset}) = 1 \\ \gcd(\omega_s, \omega_w) = 1 \\ \gcd(\omega_{\emptyset}, \omega_w) = 1 \end{cases}$$

$$\vec{\mathcal{S}}^{\S} := \begin{bmatrix} & \mathsf{d}(1 & 0 & 0) \\ \mathsf{p} e^{\hat{\mathscr{c}}_{\mathsf{y}} \mathsf{w}\omega_w} e^{\hat{\mathscr{m}}_{\mathsf{z}} \mathsf{o}\omega_{\emptyset}} & (0 & \cos \mathsf{h}\omega_{\mathsf{s}} t & \sin \mathsf{h}\omega_{\mathsf{s}} t) \\ \mathsf{pd} e^{\hat{\mathscr{c}}_{\mathsf{y}} \mathsf{w}\omega_w} e^{\hat{\mathscr{m}}_{\mathsf{z}} \mathsf{o}\omega_{\emptyset}} & (0 & -\sin \mathsf{h}\omega_{\mathsf{s}} t & \cos \mathsf{h}\omega_{\mathsf{s}} t) \end{bmatrix} \tag{20}$$

The matrix $\vec{\mathcal{S}}^{\S}$ is a rotational transformation of $\mathring{\mathcal{S}}^{\S}$ allowed within $R(3)SO(3)$. The three gyration rates $\omega_s, \omega_{\emptyset}, \omega_w$, along with their angular directions $\mathsf{h}, \mathsf{o}, \mathsf{w}$, respectively, are preserved—indicating conservation of information.

While $\mathring{\mathcal{S}}^{\S}$ describes a stationary gyration, $\vec{\mathcal{S}}^{\S}$ describes a propagating gyration at the speed of light, i.e., a gamma ray. This rotational transformation is notationally defined as:

$$\mathcal{S}^{(\vec{\theta})} = \cos\vec{\theta}\,\mathring{\mathcal{S}}^{\S} + i\sin\vec{\theta}\,\vec{\mathcal{S}}^{\S}, \qquad \delta \le \vec{\theta} \le \pi/2 - \delta$$

For $\delta = 0$, information is lost, reinforcing the notion discussed in the context of the neutrino rest mass.

The solution matrices $\mathring{\mathcal{S}}^{\S}$ and $\vec{\mathcal{S}}^{\S}$ reflect the gyration rate vectors

$$\mathring{\Omega} = \mathsf{h}\omega_s\hat{\mathsf{x}} + \mathsf{w}\omega_w\hat{\mathsf{y}} + \mathsf{o}\omega_ø\hat{\mathsf{z}}, \qquad \vec{\Omega} = e^{\hat{c}_{\mathsf{y}}\mathsf{w}\omega_w}\, e^{\hat{m}_{\mathsf{z}}\mathsf{o}\omega_ø}\,\mathsf{h}\omega_s\hat{\mathsf{x}}$$

11.1 Structural and Energetic Transformations of a Fundamental Soliton

We are now in a position to define a generic fundamental soliton $\mathfrak{F}$, whose energetic state is determined by four transformational rotation angles, grouped into two pairs. We designate the most elementary solitonic excitation in $R(3)SO(3)$ as a $\mathfrak{F}$.

The first pair, denoted $\mathring{\theta}$ and $\vec{\theta}$, corresponds to structural rotations that do not alter the energy content of the soliton. The second pair, $\mathring{\beta}$ and $\vec{\beta}$, describes transformations involving energetic interactions—such as those occurring in particle accelerators—which change the soliton's internal energy.

An illustrative example of an energy-neutral structural transformation is electron–positron annihilation. Two solitons—initially described by $\mathring{\mathcal{Y}} = \mathring{\mathcal{S}}^{\S}\mathcal{A}$—are transformed into gamma rays described by $\vec{\mathcal{Y}} = \vec{\mathcal{S}}^{\S}\mathcal{A}$. This interaction involves two key effects: (i) the conversion of gyration dynamics, denoted ω in $\mathring{\mathcal{Y}}$, into displacement dynamics ν in $\vec{\mathcal{Y}}$; and (ii) a transformation of the solution matrix $\mathring{\mathcal{S}}^{\S}$ into its complexified counterpart $\vec{\mathcal{S}}^{\S}$, representing a rotational restructuring consistent with the field equations.

The two structural rotation angles associated with energy-neutral transitions are defined as follows:

$\mathring{\theta}$ A structural rotation complexifying the gyration rate vector:

$$\breve{\omega} := e^{c\mathring{\theta}_c}\, e^{m\mathring{\theta}_m}\,\omega, \qquad \mathring{\theta} = \sqrt{\mathring{\theta}_c^2 + \mathring{\theta}_m^2}.$$

Notationally,

$$\mathfrak{F}(\mathring{\theta}) = \mathring{\Upsilon}^{\langle\mathring{\theta}\rangle}$$

$$\xrightarrow[\text{by}]{\text{dsc}} \mathring{\mathcal{Y}}^{\langle\mathring{\theta}\rangle} = \mathring{\mathcal{S}}^{\S}\mathcal{A}$$

$\vec{\theta}$ A structural rotation converting stationary gyration into translational gyration:

$$\omega \longrightarrow \cos\vec{\theta}\,\omega + i\sin\vec{\theta}\,\nu$$

Notationally,

$$\mathfrak{F}(\vec{\theta}) = \cos\vec{\theta}\,\mathring{\Upsilon} + i\sin\vec{\theta}\,\vec{\Upsilon}$$

$$\xrightarrow[\text{by}]{\text{dsc}} \cos\vec{\theta}\left(\mathring{\mathcal{Y}} = \mathring{\mathcal{S}}^{\S}\mathcal{A}\right) + i\sin\vec{\theta}\left(\vec{\mathcal{Y}} = \vec{\mathcal{S}}^{\S}\mathcal{A}\right)$$

Here, $\mathring{\Upsilon}$ denotes a gyration structure, and $\vec{\Upsilon}$ a photonic translation structure. These differ by their solution matrices $\mathring{\mathcal{S}}^{\S}$ and $\vec{\mathcal{S}}^{\S}$, as defined in equations (19) and (20). The combination $\mathfrak{F}(\mathring{\theta},\vec{\theta})$ is possible but beyond the scope of this introductory article.

Energetic transformations—such as those occurring in soliton accelerators—are described by the two rotation angles $\mathring{\beta}$ and $\vec{\beta}$:

$\mathring{\beta}$ A structural rotation coupled with an increase in energy. This is represented by a rotation of the axis:

$$\mathcal{A}^{\langle\mathring{\beta}\rangle} = \cos\mathring{\beta}\,\mathcal{A} + i\sin\mathring{\beta}\,\mathcal{A}^{\ddagger}$$

Hence,

$$\begin{aligned}\mathfrak{F}(\mathring{\beta}) &= \cos\mathring{\beta}\,\mathring{\Upsilon} + i\sin\mathring{\beta}\,\mathring{\Upsilon}^{\ddagger}\\ &\xrightarrow[\text{by}]{\text{dsc}} \cos\mathring{\beta}\left(\mathring{\mathcal{Y}} = \mathring{\mathcal{S}}^{\S}\mathcal{A}\right) + i\sin\mathring{\beta}\left(\mathring{\mathcal{Y}} = \mathring{\mathcal{S}}^{\S}\mathcal{A}^{\ddagger}\right)\end{aligned}$$

with the energy relation:

$$\breve{E} = \mathring{E} + i\frac{\sin\mathring{\beta}}{\cos\mathring{\beta}}\mathring{E}^{\ddagger}$$

where $\mathring{E}$ is the rest energy of the unmodified $\mathfrak{F}$. The soliton remains stationary; only the internal gyration rate increases:

$$E_{\text{eff}} = \left(1 + \frac{\sin^2\mathring{\beta}}{\cos^2\mathring{\beta}}\right)\mathring{E},$$

requiring scaling of all gyration rates:

$$\mathring{\Omega}^{\langle\mathring{\beta}\rangle} = \left(1 + \frac{\sin^2\mathring{\beta}}{\cos^2\mathring{\beta}}\right)\left(\mathsf{h}\omega_s\hat{\mathsf{x}} + \mathsf{w}\omega_w\hat{\mathsf{y}} + \mathsf{o}\omega_{\varnothing}\hat{\mathsf{z}}\right).$$

$\vec{\beta}$ A structural rotation coupled with both an energy increase and directional motion. The axis transformation is identical:

$$\mathcal{A}^{\langle\vec{\beta}\rangle} = \cos\vec{\beta}\,\mathcal{A} + i\sin\vec{\beta}\,\mathcal{A}^{\ddagger}$$

with additional translational transformation:

$$\omega \longrightarrow \cos\vec{\beta}\,\omega + i\sin\vec{\beta}\,v.$$

Notationally,

$$\begin{aligned}\mathfrak{F}(\vec{\beta}) &= \cos\vec{\beta}\,\mathring{\Upsilon} + i\sin\vec{\beta}\,\vec{\Upsilon}^{\ddagger}\\ &\xrightarrow[\text{by}]{\text{dsc}} \cos\vec{\beta}\left(\mathring{\mathcal{Y}} = \mathring{\mathcal{S}}^{\S}\mathcal{A}\right) + i\sin\vec{\beta}\left(\vec{\mathcal{Y}} = \vec{\mathcal{S}}^{\S}\mathcal{A}^{\ddagger}\right)\end{aligned}$$

with energy given by:

$$\breve{E} = \mathring{E} + i\frac{\sin\vec{\beta}}{\cos\vec{\beta}}\mathring{E}^{\ddagger}$$

and corresponding effective energy:

$$E_{\text{eff}} = \left(1 + \frac{\sin^2 \vec{\beta}}{\cos^2 \vec{\beta}}\right) \mathring{E}.$$

The gyration vector becomes:

$$\vec{\Omega}^{\langle \vec{\beta} \rangle} = \left(1 + \frac{\sin^2 \vec{\beta}}{\cos^2 \vec{\beta}}\right) \left(\cos \vec{\beta} \left(\mathsf{h}\omega_s \hat{\mathsf{x}} + \mathsf{w}\omega_w \hat{\mathsf{y}} + \mathsf{o}\omega_{\emptyset} \hat{\mathsf{z}} \right) + i \sin \vec{\beta} \left(e^{\hat{c}_{\mathsf{y}} \mathsf{w}\omega_w} e^{\hat{m}_{\mathsf{z}} \mathsf{o}\omega_{\emptyset}} \mathsf{h}\omega_s \hat{\mathsf{x}} \right) \right).$$

REMARK (On the Use of the Imaginary Operator). The introduction of the complex operator i within the $R(3)SO(3)$ framework does not imply an extension of the space from $\mathbb{R}^9$ to $\mathbb{C}^n$. Rather, i functions as a symbolic representation of structural orthogonality between solitonic transformation modes—such as gyration versus translation, or inertial versus energised configurations. The use of i symbolically represents a collection of internal orthogonal rotations that preserve nilpotency, maintain topological stability, and distinguish dynamically separable internal states without departing from the real-valued foundation of the $R(3)SO(3)$ framework.

11.2 The Load, Fervour, and Vigour

With Axiom 5.2 (pp. 35), Coulomb charge was separated from the quantum load, which is defined as a structured quantity. Definition 5.6 (pp. 37) introduces the load as a vectorial element $\ell \in \mathbb{R}^3 \subset R(3)SO(3)$, capable of generating both positive and negative charges. It is defined by

$$\ell := \hat{\mathsf{e}}_{y}^{\mathsf{o}c(\pi/2+\delta)+\mathsf{w}m\delta},$$

where o and w are the orbital and notational rate direction quantum numbers, respectively, previously defined. Combined with the spin $\mathsf{s} = \mathsf{dh}$, defined as the product of direction and helicity, this yields eight distinct charge flavours, consistent with the gyrational dynamics of solitonic particles:

$$\text{positive} \begin{cases} & & \mathsf{s}=1 & \mathsf{s}=-1 \\ \mathsf{o}=1 & \mathsf{w}=1 & \ell^{\widehat{+}} & \ell^{\underset{\smile}{\pm}} \\ \mathsf{o}=1 & \mathsf{w}=-1 & \bar{\ell}^{\widehat{+}} & \bar{\ell}^{\underset{\smile}{\pm}} \end{cases}$$

$$\text{negative} \begin{cases} & & \mathsf{s}=1 & \mathsf{s}=-1 \\ \mathsf{o}=-1 & \mathsf{w}=1 & \ell^{\widehat{-}} & \ell^{\underset{\smile}{=}} \\ \mathsf{o}=-1 & \mathsf{w}=-1 & \bar{\ell}^{\widehat{-}} & \bar{\ell}^{\underset{\smile}{=}} \end{cases}$$

The physical charge is generated through a structured load–magnetic field relation:

$$\phi_{\ell}(\vec{r}) := \frac{\hbar(\vec{r})}{\ell}.$$

To preserve the full internal symmetry of $R(3)SO(3)$, two further vectorial charges are defined:

1. The *fervour*, $\mathscr{f} \in \mathbb{R}^3 \subset R(3)SO(3)$, defined by

$$\mathscr{f} := \hat{\mathsf{e}}_{c}^{\mathsf{o}m(\pi/2+\delta)+\mathsf{w}y\delta},$$

with its associated structured fervour–magnetic field relation:

$$\phi_{f}(\vec{r}) := \frac{\hbar(\vec{r})}{f}.$$

2. The *vigour*, $v \in \mathbb{R}^3 \subset R(3)SO(3)$, defined by

$$v := \hat{e}_m^{\text{o}y(\pi/2+\delta)+\text{w}c\delta},$$

with an analogous structured vigour–magnetic field relation:

$$\phi_v(\vec{r}) := \frac{\hbar(\vec{r})}{v}.$$

This balanced triad of charges expresses a deeper structural symmetry intrinsic to the nilpotent framework. Specifically, the sum over all eight subcomponents of each charge satisfies the nilpotent condition:

$$\sum \ell + \sum f + \sum v = 0.$$

This internal symmetry of three charge types gives rise to the binding mechanism, which distinguishes solitary excitations ($\mathcal{F}$) from composite configurations.

Let the load be identified with electric charge and the corresponding electromagnetic force, which is well understood in contemporary physics. However, the roles of the fervour and vigour charges remain unrecognised in current models. Within the present framework, these two additional structured charges are not merely theoretical extensions but necessary components of a deeper nilpotent symmetry. Their physical manifestations are yet to be determined, but their symmetry with electric charge suggests that they govern fundamental interactions of a qualitatively different kind—potentially underpinning phenomena not presently accounted for in established theory.

11.3 Structural Conjugates

A fundamental soliton $\mathcal{F}$ is defined over two distinct sets of axes:

$$\mathcal{A} := \begin{bmatrix} \hat{\text{x}} \\ \hat{\text{y}} \\ \hat{\text{z}} \end{bmatrix} := \begin{bmatrix} \hat{y}_\text{x} \\ \hat{y}_\text{y} \\ \hat{y}_\text{z} \end{bmatrix}, \qquad \mathcal{A}^{\ddagger} := \begin{bmatrix} \hat{y}_\text{x} \\ \hat{c}_\text{y} \\ \hat{m}_\text{z} \end{bmatrix}.$$

The soliton is generally expressed as:

$$\mathcal{F}(\alpha) = \cos\alpha\,\mathring{\Upsilon} + i\sin\alpha\,\vec{\Upsilon}^{\ddagger}$$
$$\xrightarrow[\text{by}]{\text{dsc}} \cos\alpha\left(\mathring{\mathcal{Y}} = \mathring{\mathcal{S}}\mathcal{A}\right) + i\sin\alpha\left(\vec{\mathcal{Y}} = \vec{\mathcal{S}}\mathcal{A}^{\ddagger}\right),$$

where $\mathring{\mathcal{S}}$ and $\vec{\mathcal{S}}$ are the structural solution matrices defined over their respective axis sets. We now define its structural counterpart under axis exchange:

DEFINITION 11.1: *Structural Conjugates.* Given the nilpotent symmetry of the $R(3)SO(3)$ framework, the *structural conjugate* of a solitonic excitation $\mathcal{F}$ is defined as a soliton $\mathcal{P}$ with the same internal structure matrix $\mathcal{S}$, but evaluated over the conjugate axis set $\mathcal{A} \leftrightarrow \mathcal{A}^{\ddagger}$, yielding

$$\mathcal{P}(\alpha) = \cos\alpha\,\mathring{\Upsilon}^{\ddagger} + i\sin\alpha\,\vec{\Upsilon}$$

$$\xrightarrow[\text{by}]{\text{dsc}} \cos\alpha\big(\mathring{\mathcal{Y}} = \mathring{\mathcal{S}}\mathcal{A}^{\ddagger}\big) + i\sin\alpha\big(\vec{\mathcal{Y}} = \vec{\mathcal{S}}\mathcal{A}\big),$$

which reflects an internal axial transformation, preserving nilpotency under structural duality.

The fundamental structural conjugates $\mathscr{P}$ are referred to as *potentiators*. Each acts as a nonlocal structural partner to the fundamental soliton $\mathscr{F}$, transferring energy via internal field transformation rather than direct spatial interaction. Within this framework, the excitation $\mathscr{F}$ gains or loses energy, while its structural conjugate $\mathscr{P}$ undergoes a corresponding loss or gain, in full consistency with the nilpotent conservation principle:

$$\mathscr{F} \Longleftrightarrow \mathscr{P} = 0.$$

11.4 The Electric Phenomenon

With Axiom 5.2 (pp. 35), the Coulomb charge was separated from the quantum load, and the proposed experiment outlined in Appendix 4 (pp. 91) is expected to confirm this. We can now identify the charge carrier in electric conducting circuits as the *Voltron*—a photon-like particle, representing the structural conjugate of the photon. While photons exist and propagate in vacuum, the voltron requires atomic structures to manifest; conductive media inherently host latent voltron populations. Thus, being confined to matter, the voltron serves as the medium counterpart of the photon in vacuum.

The *Voltron*, denoted by Λ, is itself a topological soliton and functions as the mediator of power transport in conductive circuits, akin to the photon in vacuum. Whereas a photon is described by $\gamma \xrightarrow[\text{by}]{\text{dsc}} \mathcal{Y} = \mathcal{S}\,\mathcal{A}$, a voltron is represented as $\Lambda \xrightarrow[\text{by}]{\text{dsc}} \mathcal{Y}_g\mathcal{S}\,\mathcal{A}^{\ddagger}$. Specifically,

$$\Lambda^+ = \begin{bmatrix} \nu \\ \phi_g \\ \psi_g \end{bmatrix} \begin{bmatrix} \mathsf{d}(1 & 0 & 0) \\ (0 & c_s & s_s) \\ \mathsf{d}\,(0 & -s_s & c_s) \end{bmatrix} \mathsf{h} \begin{bmatrix} \hat{\mathcal{Y}}_{\mathsf{x}} \\ \hat{c}_{\mathsf{y}} \\ \hat{m}_{\mathsf{z}} \end{bmatrix} \qquad \Lambda^- = \begin{bmatrix} \nu \\ \bar{\phi}_g \\ \bar{\psi}_g \end{bmatrix} \begin{bmatrix} \bar{\mathsf{d}}(1 & 0 & 0) \\ (0 & c_s & s_s) \\ \bar{\mathsf{d}}(0 & -s_s & c_s) \end{bmatrix} \bar{\mathsf{h}} \begin{bmatrix} \hat{\mathcal{Y}}_{\mathsf{x}} \\ \hat{c}_{\mathsf{y}} \\ \hat{m}_{\mathsf{z}} \end{bmatrix}$$

The energy per voltron transferred is governed by the Planck–Einstein relation $E = \hbar\omega$. Accordingly, the electric potential is directly proportional to the radial frequency. In a conducting wire, electric current flows toward the load in both conductors; that is, Λ^+ and Λ^- share the same velocity vector direction along parallel wires. The resulting magnetic field directions, however, differ due to the conjugate pairing of ϕ_g and $\bar{\phi}_g$, which accounts for the opposing polarities of the voltrons.

When charging a capacitor, voltrons undergo a transition into *Fluxons*, denoted Θ and described as $\Theta \xrightarrow[\text{by}]{\text{dsc}} \mathring{\mathcal{Y}}_g\mathcal{S}\,\mathcal{A}^{\ddagger}$. These fluxon solitons project and absorb structured electric flux, defining the field between capacitor plates:

$$\Theta^+ = \begin{bmatrix} \omega \\ \phi_g \\ \psi_g \end{bmatrix} \begin{bmatrix} (0 & c_s & s_s) \\ \mathsf{d}\,(0 & -s_s & c_s) \\ \mathsf{d}(1 & 0 & 0) \end{bmatrix} \mathsf{h} \begin{bmatrix} \hat{\mathcal{Y}}_{\mathsf{x}} \\ \hat{c}_{\mathsf{y}} \\ \hat{m}_{\mathsf{z}} \end{bmatrix} \qquad \Theta^- = \begin{bmatrix} \omega \\ \bar{\phi}_g \\ \bar{\psi}_g \end{bmatrix} \begin{bmatrix} (0 & c_s & s_s) \\ \bar{\mathsf{d}}\,(0 & -s_s & c_s) \\ \bar{\mathsf{d}}(1 & 0 & 0) \end{bmatrix} \bar{\mathsf{h}} \begin{bmatrix} \hat{\mathcal{Y}}_{\mathsf{x}} \\ \hat{c}_{\mathsf{y}} \\ \hat{m}_{\mathsf{z}} \end{bmatrix}$$

For magnetic configurations:

$$\Theta^{\mathrm{N}} = \begin{bmatrix} \omega \\ \phi_g \\ \psi_g \end{bmatrix} \begin{bmatrix} \mathrm{d}(0 & -s_s & c_s) \\ \mathrm{d}(1 & 0 & 0) \\ (0 & c_s & s_s) \end{bmatrix} \mathrm{h} \begin{bmatrix} \hat{y}_{\mathrm{x}} \\ \hat{c}_{\mathrm{y}} \\ \hat{m}_{\mathrm{z}} \end{bmatrix} \qquad \Theta^{\mathrm{S}} = \begin{bmatrix} \omega \\ \bar{\phi}_g \\ \bar{\psi}_g \end{bmatrix} \begin{bmatrix} \bar{\mathrm{d}}(0 & -s_s & c_s) \\ \bar{\mathrm{d}}(1 & 0 & 0) \\ (0 & c_s & s_s) \end{bmatrix} \bar{\mathrm{h}} \begin{bmatrix} \hat{y}_{\mathrm{x}} \\ \hat{c}_{\mathrm{y}} \\ \hat{m}_{\mathrm{z}} \end{bmatrix}$$

While only representative configurations are shown, these definitions must be extended to incorporate the full symmetry group of $R(3)SO(3)$, as was established in the characterisation of the photon γ, and the fundamental particles $\mathcal{F}$ and $\mathcal{P}$.

At this point, all required elements are in place to describe the electromagnetic universe as a self-consistent, symmetrical, and nilpotent structure embedded in the extraordinary orthogonal gauge group $R(3)SO(3)$.

11.5 The $\mathcal{F}$ Interaction with the Field $\Theta^+ – \Theta^-$

Consider a pair of fluxons immobilised on opposing capacitor plates, thereby establishing an electric potential field $\mathcal{V}\left(\Theta^+, \Theta^-\right)$. By definition, the potential difference between two positions s_1 and s_2 is obtained by integrating the field gradient:

$$\mathcal{V}\left(\{\Theta^+, \Theta^-\}\right) = \int_{s_1}^{s_2} \nabla\{\Theta^+, \Theta^-\} \mathrm{d}s = sk\hat{\mathrm{x}},$$

where $\nabla\{\Theta^+, \Theta^-\}$ denotes the potential field, and $k \propto \hbar\omega$ is proportional to the energy carried by the fluxon dual $\{\Theta^+, \Theta^-\}$. A fundamental charged soliton, represented by a $\mathcal{F}$, responds dynamically to this potential configuration.

From Theorem 4.1 (pp. 20), Equation 15 (pp. 23) gives:

$$n\hbar = \epsilon_0 \frac{\phi \times \psi}{\phi \cdot \phi},$$

and the interaction of soliton one with the potential field of soliton two takes the form:

$$p = \epsilon_0 \frac{\phi_1 \times \nabla\psi_2}{\phi_1 \cdot \phi_2},$$

where $\nabla\psi_2$ denotes an electric potential field (flux divided by length), and the identity $(n\hbar)/r = p$ is recognised as the solitonic momentum.

Therefore, the field interaction $\mathcal{F} \overset{\text{field}}{\Longleftrightarrow} \nabla\{\Theta^+, \Theta^-\}$—where $\overset{\text{field}}{\Longleftrightarrow}$ denotes a nilpotent soliton–soliton interaction mediated by the soliton structure of one soliton and the field structure of the other within the $R(3)SO(3)$ framework—yields the momentum exchange relation:

$$\vec{p} = \epsilon_0 \frac{\phi \times \nabla\psi_g}{\phi \cdot \phi_g},$$

where ϕ, defined in $\{\hat{\mathrm{x}}, \hat{\mathrm{y}}, \hat{\mathrm{z}}\}$, is the magnetic component of the $\mathcal{F}$, and $\nabla\psi_g$, defined in $\{\hat{\mathrm{x}}, \hat{c}_{\mathrm{y}}, \hat{m}_{\mathrm{z}}\}$, is the electric potential associated with the fluxon dual $\{\Theta^+, \Theta^-\}$.

Since $\left\|\nabla\psi_g\right\|_{\mathrm{x}} \approx 0$, and the y and z components of ϕ and $\nabla\psi_g$ are always orthogonal, their cross product produces a time-independent vector along the $\hat{\mathrm{x}}$ axis. Consequently, the soliton $\mathcal{F}$ is accelerated along the $\hat{\mathrm{x}}$ direction. The direction of motion depends on the polarities of the interacting solitons. In this interaction, the $\mathcal{F}$ gains kinetic energy, while the fluxon dual loses potential energy. Crucially, this energy exchange is mediated at a distance because the fields themselves remain non-energetic within the $R(3)SO(3)$ formalism.

The force acting on the soliton is given by the time derivative of momentum:

$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}.$$

To evaluate the complete effect on the soliton $\mathfrak{F}$ and the fluxon dual $\{\Theta^+, \Theta^-\}$, one must examine the temporal evolution of this soliton–field interaction within the structured field framework.

THEOREM 11.1: *Relativistic Energy and Inertial Mass from Solitonic Structure. Let a soliton $\mathfrak{F}$ $(\vec{\beta})$ be a soliton rest state composed*

$$\mathfrak{F}(\vec{\beta}) = \cos\vec{\beta}\,\mathring{\Upsilon} + i\sin\vec{\beta}\,\vec{\Upsilon}^{\ddagger}$$
$$\xrightarrow[by]{dsc} \cos\vec{\beta}\left(\mathring{\mathcal{Y}} = \mathring{\mathcal{S}}^{\S}\mathcal{A}\right) + i\sin\vec{\beta}\left(\vec{\mathcal{Y}} = \vec{\mathcal{S}}^{\S}\mathcal{A}^{\ddagger}\right) \qquad \vec{\beta} \ll 1.$$

Then, under interaction with an fluxon dual field $\{\Theta^+, \Theta^-\}\}$ the soliton $\mathfrak{F}$ acquires momentum in the electric potential field spanning between Θ^+ and Θ^-, given by

$$\vec{p} = \epsilon\frac{\phi \times \nabla\psi_g}{\phi \cdot \phi_g}$$

and its energised structural energy is expressed as the complexified quantity (see Axiom 5.1 (pp. 34))

$$\breve{E} = \mathring{E} + i\frac{\sin\vec{\beta}}{\cos\vec{\beta}}\mathring{E}^{\ddagger}$$

where $\mathring{E}$ would be the resting state energy and $\mathring{E}^{\ddagger}$ the energy to maintain the momentum p, and where $v = c\sin\vec{\beta}$ is the effective translational velocity attained under the interaction.

The measurable energy, or effective energy, of $\mathfrak{F}$ becomes

$$E_{\mathfrak{F},\mathrm{eff}} = \mathring{E}_{\mathfrak{F}}\left(1 + \frac{\sin^2\vec{\beta}}{\cos^2\vec{\beta}}\right) = \mathring{E}_{\mathfrak{F}}\sqrt{1 + \frac{v^2}{c^2 - v^2}}.$$

By differentiating with respect to velocity we obtain the dynamic change in energy in the $R(3)SO(3)$ framework, which we compare to the temporal change in energy using classical Newtonian methods, providing us the two equations

$$\frac{\mathrm{d}E_{\mathrm{eff},\mathfrak{F}}}{\mathrm{d}v} = \mathring{E}_{\mathfrak{F}}\frac{cv}{(c^2 - v^2)^{3/2}} \qquad \textit{and} \qquad \frac{\mathrm{d}E_{\mathrm{Newton}}}{\mathrm{d}t} = m_i\frac{\mathrm{d}v}{\mathrm{d}t}v,$$

as $\mathrm{d}E_{\mathrm{eff},\mathfrak{F}} = \mathrm{d}E_{\mathrm{Newton}}$ above give the expression for inertial mass

$$m_i = \mathring{E}_{\mathfrak{F}}\frac{c}{(c^2 - v^2)^{3/2}}.$$

In the limit $v = 0$ the structural energy yields the inertial mass equivalence:

$$\mathring{E}_{\mathfrak{F}} = mc^2.$$

Thus, the energy of the boosted soliton $\mathcal{F}(\vec{\beta})$ is given by

$$E_{\mathcal{F}(\vec{\beta})} = \frac{mc^2}{\sqrt{1 - v^2/c^2}}.$$

This derivation emerges naturally from the nilpotent soliton structure defined by $\mathcal{M}$ and $\mathring{\mathcal{M}}$ in the $R(3)SO(3)$ framework, requiring no external postulates. The derivation also elucidates the origin of momentum and grounds Newton's laws within the algebraic and energetic coherence of structured soliton interactions.

Importantly, the inequality

$$\mathring{E} + \mathring{E}^{\ddagger} > \sqrt{\mathring{E}^2 + \mathring{E}^{\ddagger 2}}$$

indicates that the energy increase associated with solitonic acceleration necessarily results in a residual output—typically in the form of emitted radiation or other secondary excitations.

REMARK. Any increase in the energy of $E_{\mathcal{F}}$, including radiative losses, is instantaneously balanced by an equal energy decrease in the fluxon dual $\{\Theta^+, \Theta^-\}$. This energy transfer is mediated non-locally through the nilpotent structure of the Universe, as encoded in the $R(3)SO(3)$ framework, which ensures conservation without recourse to field energy or propagation delay.

As $\mathcal{F}$ is a quantised soliton, this energy increase must arise via enhancement of its internal field dynamics—specifically, by scaling the gyration rates. These rates transform as:

$$\mathring{\Omega}_{\mathcal{F}} = \omega'_s \hat{y}_{\mathsf{x}} + \omega'_{\emptyset} \hat{y}_{\mathsf{y}} + \omega'_w \hat{y}_{\mathsf{z}}, \qquad \vec{\Omega}_{\mathcal{F}} = \omega'_s e^{c\omega'_{\emptyset} t} e^{m\omega'_w t} \hat{y}_{\mathsf{x}},$$

with the boosted frequencies:

$$\omega'_s = \gamma \omega_s, \quad \omega'_{\emptyset} = \gamma \omega_{\emptyset}, \quad \omega'_w = \gamma \omega_w, \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

NOTE. This derivation reproduces the results of Einstein's special relativity— specifically, Lorentzian time dilation and relativistic energy scaling—but within the topologically consistent, quantised field framework of the nilpotent universe $R(3)SO(3)$. Unlike the axiomatic derivation from plane-wave solutions to the d'Alembert equation, the present approach grounds relativistic effects in the soliton–field dynamics of quantised, rotationally invariant field configurations. It provides not only a mathematical derivation of $E = mc^2$, but also a first-principles identification of momentum and a soliton-based interpretation of Newton's first law.

COROLLARY 11.1.1: *Mass as an Emergent Property of $R(3)SO(3)$.*
Within the nilpotent field structure of $R(3)SO(3)$, inertial mass is not a fundamental input but an emergent quantity derived from the soliton's internal field configuration and its structural energy expression.

From the structural energy formulation in Axiom 5.1 (pp. 34)and Theorem 11.1 (pp. 64), we find that

$$\mathring{E}_{\mathcal{F}} = mc^2,$$

where $\mathring{E}_{\mathscr{F}}$ *is the soliton's intrinsic (gyratory) energy. Thus, the rest mass* m *is identified directly with the energy stored in the soliton's topological configuration.*

Consequently, mass in $R(3)SO(3)$ *arises from the quantised, nilpotent field structure of solitons themselves, without reference to external fields such as the Higgs mechanism. It is a property of stability, rotation, and field configuration, governed entirely by the structural field equations* $\mathcal{M}$ *and* $\mathring{\mathcal{M}}$ *in the reduced configuration space* $\mathbb{R}^9$.

This corollary recasts mass not as an independent entity but as the energetic expression of a soliton's topological identity, making it inherently compatible with conservation laws, quantisation, and nilpotency.

REMARK 11.1: *Electric Charge as an Emergent Quantity.* Axiom 4.1 (pp. 19) equates electromagnetic action with mechanical action, thereby aligning the momentum-transfer framework of electrodynamics with the inertial structure of matter. However, since mechanical action itself emerges from the solitonic dynamics of the $R(3)SO(3)$ framework, it follows that the elementary electric charge e must also be understood as an emergent—not fundamental—quantity.

This interpretation is fully consistent with Axiom 5.2 (pp. 35), which distinguishes between the scalar charge e and the structured quantities ℓ, f, and v, each mediating distinct interactions. Within this duality, e represents a measurable scalar effect arising from a deeper geometric origin: the triad of symmetrical charge structures.

Consequently, both gravitational (Newtonian) and electric (Coulombic) forces are not independent primitives, but rather manifestations of a unified, emergent phenomenon governed by the solitonic field structure and conserved topological interactions within $R(3)SO(3)$.

REMARK 11.2: *Potential Wells.* In Definition 5.2 (pp. 32), *hollowed solitons* were introduced; structures that generate a confined potential region within their interior. Let A denote such a hollowed soliton, and let B be a second soliton captured within the potential well defined by A.

If B possesses kinetic energy, it will undergo oscillatory motion within the potential well of A, alternating between phases of acceleration and deceleration. During acceleration phases, radiation is emitted; however, this radiative energy loss is not recuperated during deceleration. As a consequence, energy is irreversibly transferred to the surrounding field environment.

Over successive cycles, this loss reduces the kinetic energy of B, ultimately bringing it to rest at the minimum of the potential well. The system thus evolves toward a lower-energy configuration, consistent with entropic decay and external energy dissipation.

11.6 The Electron and Positron

We adopt the notation for a fundamental soliton $\mathscr{F}$ described solely by its charge—for example, $\mathscr{F}^+_\ell := \ell^+$—consistent with the definitions of load, fervour, and vigour in Section 11.2 (pp. 60). A compounded particle consisting of four fundamental solitons is written as

$$\text{particle} = \left(\!\left(\ell^{\widehat{-}}, f^{\widehat{-}}, \ell^{\widehat{-}}, f^{\widehat{+}}\right)\!\right),$$

resulting in a net negative charge from the pair $(\ell^{\widehat{-}}, \bar{\ell}^{\widehat{-}})$, while the fervour charges cancel. The particle possesses net up-spin momentum, as well as net orbital and notational angular momenta. All components of the particle share the same solution matrix $\mathcal{S}$.

The resultant solitonic structure—comprising n^2, here 2^2, fundamental solitons—is whole and not hollow; see Section 5 (pp. 26).

The positive and negative charged fervours differ by conjugate magnetic vectors defined by the quantum number p; see the item "Solution Matrix" in Section 11 (pp. 56).

The forces acting on the individual solitons arise from both Coulomb interactions and quantum forces generated by the four-body coupling. Part IV shows that both the gravitational and Coulomb forces emerge as residual effects in the $R(3)SO(3)$ framework. For fundamental solitons assembling into compound structures, quantum forces dominate and overcome the Coulomb repulsion. Section 11.5 (pp. 63) derives the force between two solitons induced by quantum field interaction as

$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t},$$

where

$$\vec{p} = \epsilon_0 \frac{\phi_1 \times \nabla\psi_2}{\phi_1 \cdot \phi_2}.$$

By Definition 1.9 (pp. 13), cross products in the ternary system $R(3)SO(3)$ preserve length. Hence, the four-body interactions within the compound particle $\left(\!\left(\ell^{\widehat{-}}, f^{\widehat{-}}, \ell^{\widehat{-}}, f^{\widehat{+}}\right)\!\right)$ produce forces that overwhelm the Coulomb repulsion between the pair $(\ell^{\widehat{-}}, \bar{\ell}^{\widehat{-}})$.

The particle described above is identified as the electron in the $R(3)SO(3)$ framework:

$$\mathsf{e}^{\widehat{-}} = \left(\!\left(\ell^{\widehat{-}}, f^{\widehat{-}}, \ell^{\widehat{-}}, f^{\widehat{+}}\right)\!\right).$$

The internal structure of the electron, defined in this manner, is well-posed: it is a resonance structure characterised by the eigenvalues

$$\left\{\omega_s^{(e)}, \omega_\varnothing^{(e)}, \omega_w^{(e)}\right\},$$

corresponding respectively to intrinsic spin rate, orbital angular momentum, and notational (nutational) frequency.

These eigenvalues define the topologically quantised internal configuration of the electron and determine its rest energy via

$$\mathring{E}_{\mathsf{e}} = \hbar\sqrt{\left(\omega_s^{(e)}\right)^2 + \left(\omega_\varnothing^{(e)}\right)^2 + \left(\omega_w^{(e)}\right)^2}.$$

This eigenstructure encodes all observable quantum properties of the electron—including spin, magnetic moment, and charge polarity—and provides a geometric origin for rest mass through solitonic resonance. The electron thus emerges as the lowest-energy soliton compound, whose excitation modes satisfy the quantised field equations $\mathcal{M}$ and $\mathring{\mathcal{M}}$ within the $R(3)SO(3)$ framework.

Analogously, the positron is defined as the compound

$$\mathsf{e}^{\pm} = (\!(\ell^{\pm}, f^{\pm}, \ell^{\pm}, f^{\mp})\!)\,,$$

featuring down-spin. A neutral compound configuration is also definable:

$$\mathsf{e}^{\circ} = (\!(\ell^{\pm}, f^{\pm}, \ell^{\mp}, f^{\mp})\!)\,.$$

In this construction, each component $\ell^{\mp}$ represents a structured solitonic state bearing half the total Coulomb charge, such that their combination yields the observed elementary charge of the electron or positron.

11.7 The Proton and Neutron

Within the $R(3)SO(3)$ framework, the proton—like the electron—is realised as a structured compound. However, unlike the electron's quadruple composition, the proton is defined as a *hexadecuple* (sixteenfold) solitonic structure. This makes it a "whole" configuration—topologically closed and non-hollow—whose eigenstructure determines its rest mass:

$$\mathsf{p}^{\pm} = \left[\!\!\left[\begin{matrix} (\!(\ell^{\pm}, \upsilon^{\pm}, \ell^{\pm}, \upsilon^{\mp})\!) \\ (\!(\ell^{\pm}, \upsilon^{\pm}, \ell^{\pm}, \upsilon^{\mp})\!) \\ (\!(\ell^{\mp}, f^{\mp}, \ell^{\mp}, f^{\pm})\!) \\ (\!(\ell^{\pm}, \upsilon^{\pm}, \ell^{\mp}, f^{\mp})\!) \end{matrix}\right]\!\!\right]$$

Here, the first row contributes the net positive charge, while the remaining three rows are collectively neutral. The particle is stabilised by the predominance of *vigour* υ, which acts as a catalyst enabling larger-scale assembly. Notably, vigour components outnumber fervour components by five to three. In a hydrogenic universe, this structure ensures that the combined charge types satisfy the nilpotent condition:

$$\sum \ell + \sum f + \sum \upsilon = 0,$$

since the corresponding electrons are bound using fervour.

The neutron appears as a neutral variant of this configuration:

$$\mathsf{p}^{\circ} = \left[\!\!\left[\begin{matrix} (\!(\ell^{\pm}, \upsilon^{\pm}, \ell^{\mp}, \upsilon^{\mp})\!) \\ (\!(\ell^{\pm}, \upsilon^{\pm}, \ell^{\pm}, \upsilon^{\mp})\!) \\ (\!(\ell^{\mp}, f^{\mp}, \ell^{\mp}, f^{\pm})\!) \\ (\!(\ell^{\pm}, \upsilon^{\pm}, \ell^{\mp}, f^{\mp})\!) \end{matrix}\right]\!\!\right]$$

in which the first row's charge is neutralised.

The interaction of a neutron p° with a neutral electron e° gives rise to a nilpotent, energetically balanced transformation:

$$\left\{\mathsf{p}^{\circ} = \left[\begin{matrix} (\!(\ell^{\pm}, \upsilon^{\pm}, \ell^{\mp}, \upsilon^{\mp})\!) \\ \cdots \end{matrix}\right]\right\} \oplus \left\{\mathsf{e}^{\circ} = (\!(\ell^{\pm}, f^{\pm}, \ell^{\mp}, f^{\mp})\!)\right\}$$

$$\longleftrightarrow \left\{\mathsf{p}^{+} = \left[\begin{matrix} (\!(\ell^{\pm}, \upsilon^{\pm}, \ell^{\pm}, \upsilon^{\mp})\!) \\ \cdots \end{matrix}\right]\right\} + \left\{\mathsf{e}^{\mp} = (\!(\ell^{\mp}, f^{\mp}, \ell^{\mp}, f^{\pm})\!)\right\} + 2\nu$$

This yields a proton p^{+}, an electron e^{-}, and two neutrinos 2ν, thereby ensuring conservation of energy and nilpotent field symmetry.

Why two neutrinos? The first neutrino carries away excess binding energy released during the neutron-to-proton transition; this energy is conveyed via vigour and is structurally analogous to a photon, which transports energy via load. The second neutrino carries the binding energy difference between the neutral electron e° and its charged counterpart e^{-}; this energy is mediated via fervour.

Consequently, the neutron's half-life is not solely determined by intrinsic instability but is modulated by the availability—i.e., the local density—of neutral electrons e°, which act as mediators of the decay process.

REMARK 11.3: *Photonic Equivalence and Neutrino Interactions.* Within the triadic charge structure of the $R(3)SO(3)$ framework, a photon is understood as a *load-mediated* excitation that interacts via electromagnetic channels. Analogously, neutrinos emerge as *fervour*- or *vigour*-mediated excitations, depending on the structural origin of the energy transfer. Each of the three charge types thus possesses its own vector-borne energy carrier: photons (load), fervour-neutrinos, and vigour-neutrinos.

These entities differ fundamentally in their interaction cross sections. Photons exhibit relatively large cross sections in matter, energising electrons within atomic shells and thereby facilitating transitions between discrete energy levels. This interaction is broad and probabilistically robust—comparable to a dart reliably striking anywhere on a dartboard.

Neutrinos, by contrast, interact only through rare, highly constrained events—akin to a dart striking the bull's eye. These interactions occur only when the solitonic structure of a neutrino precisely overlaps with that of a nuclear target. Consequently, neutrinos are not directly observable in the same manner as photons; their presence is inferred through secondary signatures in highly controlled detection environments.

As such, fervour and vigour interactions lie beyond the reach of conventional electromagnetic instrumentation and require careful interpretation of anomalous nuclear events to be inferred.

This framework maintains a complete charge and structural balance throughout the interaction, with no need for fractional charge postulates or internal recombination mechanisms. The resulting composite particles emerge naturally from the soliton–glueonic architecture, setting the stage for the nuclear-scale packing principles explored in the following section.

12 Atomic Nuclear Packing

We introduce the notation for describing compound solitonic structures involving four components:

$$\langle 3,1\rangle := \left(\!\left(\mathscr{F}^{+},\mathscr{F}^{+},\mathscr{F}^{-},\mathscr{F}^{+}\right)\!\right),$$

where the ordered pair $\langle p,n\rangle$ counts the number of positive and negative Coulomb half-charges, respectively. In this representation, proton–neutron pairs emerge as balanced configurations of structured charges and their corresponding glueonic counterparts.

While the packing of electrons into atomic orbitals—classified by the familiar subshells $\{s, p, d, f\}$—is well established, we extend this concept to a nuclear analogue. We define *nuclear shells*, distinct from electronic ones, but retaining analogous terminology. These nuclear shells are indexed $n = 1,\ldots,8$, with each shell containing the subshells $\{s, p, d, f, g\}$, whose respective capacities (in proton–neutron pairs) are:

$$\{s, p, d, f, g\} = \{1, 3, 5, 7, 11\},$$

corresponding to a total of $4 \times \{1, 3, 5, 7, 11\}$ Coulomb half-charges per subshell, due to each proton–neutron pair contributing four such charges.

For example, the s-subshell supports four half-charges, accommodating either two electrons or one proton. The p-subshell holds twelve half-charges, sufficient for three protons or six electrons. Accordingly, nuclear shell packing follows a generalised pattern:

$$(s_1),\ (s_2, p_2),\ (s_3, p_3, d_3),\ (s_4, p_4, d_4, f_4),\ (s_5, p_5, d_5, f_5, g_5),$$

culminating in a total of 57 protons and neutrons. This configuration corresponds to either the iron isotope $^{57}\mathrm{Fe}^{26}$ or the cobalt isotope $^{57}\mathrm{Co}^{27}$, at which point the fifth shell is complete.

The half-charge configuration within each nuclear sub-shell follows from the solitonic pairing structure defined earlier. Each sub-shell accommodates Coulomb half-charges arranged in quantised pairings, represented using the notation $\langle p, n\rangle$,, where the ordered pair $\langle p, n\rangle$ denotes the positive and negative half-charge contributions, respectively. The charge configurations for each sub-shell are summarised as:

$$\begin{aligned}
s &\mapsto \langle 1,3\rangle,\ \langle 2,2\rangle,\ \langle 3,1\rangle \quad \text{yields net charges} \quad -1,\ 0,\ +1,\\
p &\mapsto \langle 3,1\rangle,\ \langle 5,3\rangle,\ \big\{\langle 7,5\rangle, \langle 9,3\rangle\big\} \quad \text{yields net charges} \quad 1,\ 1,\ \{1,\ 3\},\\
d &\mapsto p,\ \big\{\langle 9,7\rangle, \langle 11,5\rangle\big\},\ \big\{\langle 11,9\rangle, \langle 13,7\rangle\big\},\\
f &\mapsto d,\ \big\{\langle 13,11\rangle, \langle 15,9\rangle\big\},\ \big\{\langle 15,13\rangle, \langle 17,11\rangle\big\},\\
g &\mapsto f,\ \big\{\langle 17,15\rangle, \langle 19,13\rangle\big\},\ \big\{\langle 19,17\rangle, \langle 21,15\rangle\big\},\ \big\{\langle 21,19\rangle, \langle 23,17\rangle\big\},\ \big\{\langle 23,21\rangle, \langle 25,19\rangle\big\}.
\end{aligned}$$

Each sub-shell thus defines a closed combinatorial set of half-charge couplings that contribute to the emergent structure of nuclear matter. This packing preserves nilpotency and charge balance within the nuclear compound, and it echoes the duality observed in electron shells—now reinterpreted through the glueonic soliton formalism introduced in the $R(3)SO(3)$ framework.

The sixth nuclear shell marks a transition to higher-energy packing, characterised by a doubling of the subshell capacities:

$$\{s_6, p_6, d_6, f_6, g_6\} = 8 \times \{1, 3, 5, 7, 11\}.$$

This shell concludes with the cadmium isotope $^{111}\mathrm{Cd}^{48}$. The seventh shell undergoes a further doubling of capacity:

$$\{s_7, p_7, d_7, f_7, g_7\} = 16 \times \{1, 3, 5, 7, 11\},$$

terminating with the radon isotope $^{219}\mathrm{Rn}^{86}$, after which the eighth shell begins.

The author has observed that various shell closures and subshell symmetries coincide with local minima in nuclear binding energy curves. While the analysis of these energetic correlations is beyond the scope of this article, they are understood to arise from the nilpotent and symmetrical structure of nuclear matter in the $R(3)SO(3)$ framework.

REMARK. The symmetry and duality observed in nuclear shell packing—mirroring that of electronic shells—emerges naturally from the algebraic structure of the $R(3)SO(3)$ framework and the solitonic eigenvalues governing compounded matter.

13 Cosmic Background Potential

Potential differences are ubiquitous across various states of atomic and molecular matter, including crystalline solids and gaseous environments. For instance, within atomic structures, potential differences emerge between electron shells and subshells; in crystalline materials, the Seebeck effect exemplifies a potential difference generated between two dissimilar structures; and in gaseous systems, such differences underpin the build-up and discharge mechanisms leading to phenomena like lightning, whether in thunderclouds or volcanic-induced updrafts. The progressive variation in electric potential from the Earth's surface into space is also a well-documented phenomenon.

In Observation 10.1 (pp. 55) (*Effective Energy Increase in SPDC*), an apparent increase in the effective energy of down-converted photons was described—an anomaly not accounted for within the conventional quantum field framework. This observation implicitly suggests the influence of a globally elevated, ambient potential.

The existence of a *cosmic background potential* is here proposed as a universal energy reservoir—a medium into which excess energy may be deposited or from which a deficit may be compensated during localised interactions. An analogue of this concept is found in the Seebeck effect, wherein each crystalline structure possesses a well-defined heat capacity and emits a characteristic black-body spectrum, interpreted in this context not as continuous but as composed of discrete spectral components. These components correspond to modified electron transitions within the material's Fermi layer, generating quantised phonons that are absorbed and re-emitted internally.

At the interface between dissimilar crystalline materials, these phonons—analogous in behaviour to photons—require matching energy levels to be absorbed across the boundary. When such a match is energetically non-viable, the interaction necessitates the borrowing or depositing of energy from a background potential. In conductors, this mechanism manifests macroscopically as the Seebeck effect.

PROPOSITION 13.1: *Existence of a Cosmic Background Potential.* Within the $R(3)SO(3)$ framework, and consistent with the nilpotent structure of the universe described in Section 6 (pp. 39), there exists a spatially extended background potential field $\mathcal{V}_{\mathrm{CB}}$, termed the *cosmic background potential*. This potential acts as a universal energy reservoir, enabling localised quantum and thermodynamic processes—such as phonon-photon conversion or frequency shifts during parametric interactions—to proceed through transient energy exchange with the ambient field, without requiring local conservation closure.

In accordance with the nilpotency of the universe, this background potential is necessarily balanced by a corresponding contra-potential $^{-}\mathcal{V}_{\text{CB}}$ in the contra-universe, such that the total potential vanishes when considered across the full system. The observable asymmetry in energy exchange is thus a local phenomenon, arising from the projection of this nilpotent structure into the observable universe.

The absence of this background potential from standard quantum mechanical treatments leaves many observed interaction phenomena unexplained or artificially constrained. Its explicit inclusion in the theoretical framework allows a coherent account of energy flow in systems undergoing transition across structural or environmental boundaries.

PART IV

Principle of Relativity, Gravity and Coulomb's Law

In Part I, we established the $R(3)SO(3)$ mathematical framework, from which the quantised Maxwell field equations in vacuum naturally emerge. This foundation demonstrated how the internal structure of space—encoded in cross-product cyclicity and rotational symmetry—gives rise to coherent field dynamics.

In Part II, we extended this framework to include structured fields, enabling the description of quantised topological electromagnetic solitons embedded within a genuinely nilpotent Universe. This involved the aggregation of solitons, the emergence of inertial and energising fields, and the formulation of interaction principles governed by internal field geometry and curvature.

In Part III, we described particles as quantised topological electromagnetic solitons within the $R(3)SO(3)$ framework. We showed that mass arises as an emergent property and recovered the Einsteinian expressions for relativistic mass. Consequently, $R(3)SO(3)$ cannot be interpreted within the four-dimensional spacetime of special relativity, as such a reduction would lead to contradictions with physical experience.

In Part IV, we develop a new principle of relativity consistent with the nilpotent and quantised structure of the $R(3)SO(3)$ framework. From this foundation, both gravitational and Coulomb fields are derived as emergent field phenomena. These derivations yield testable predictions, including the precession of planetary orbits and time dilation effects. The latter is demonstrated using the Bohr model of the hydrogen atom, where shifting atomic orbital frequencies under field influence accounts for both relativistic time dilation and spectral blue shift. In this formulation, classical field theories are unified and reinterpreted through the internal geometry of quantised space.

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14 Principle of Relativity $R(3)SO(3)$

The $R(3)SO(3)$ framework is formulated within an absolute reference structure. However, in our empirical experience, we do not observe or measure phenomena with respect to such an absolute frame. Instead, our measurements are conducted relative to locally co-moving frames—typically defined by terrestrial or solar system scales. An important empirical exception arises from the observation of the cosmic microwave background (CMB) radiation. The dipole anisotropy of the CMB provides a means to determine the velocity of the Sun, and by extension the Earth, with respect to this radiation field.

Given that the CMB is isotropic and uniform across the celestial sphere, it serves as a natural candidate for a universal rest frame. The radiation's origin at the so-called big bang and its present uniformity suggest it is stationary with respect to the large-scale structure of the Universe. If this radiation were to possess a motion relative to some background, it would imply the existence of a higher-order embedding space within which our Universe itself is in motion. Such a hypothesis would necessitate a framework extending beyond $R(3)SO(3)$ to a larger gauge space, for instance $R(3)R(3)SO(3)$, with $R(3)SO(3)$ as a proper subspace.

However, such an ontological extension leads to an infinite regress of spaces and symmetries. In the author's view, these constructions—while mathematically tractable—are philosophically unproductive and do not enhance our understanding of physical reality. The $R(3)SO(3)$ framework, as constructed, offers a self-contained, nilpotent description of field and particle interactions without requiring recourse to an external embedding space.

The question of how motion is perceived relative to various frames has historically invoked concepts such as the ether drag hypothesis. This notion postulated that a stationary luminiferous ether was partially dragged by ponderable bodies, such as planets, thereby creating a locally co-moving reference frame for light propagation. However, the ether drag theory was ultimately rejected, in part due to its inability to account for the observed phenomenon of stellar aberration.

The classical criticism of ether drag was rooted in analogies: light propagating through the ether was compared to a swimmer being carried by a river's current. In this analogy, an observer in a boat moving with the river would perceive no aberration in the swimmer's trajectory. Similarly, a photon in a dragged ether would not show aberration relative to an Earth-based observer. Since stellar aberration is observed, the ether drag model was deemed insufficient.

However, these classical analogies predate the quantum understanding of light's wave–particle duality. Within the $R(3)SO(3)$ framework, photons are described as structured solitonic excitations. When a photon approaches a moving body, it begins interacting with that body's field structure. As this interaction develops, the photon acquires a velocity component in the direction of the body's motion. To conserve momentum within the composite field structure, the photon's propagation vector is altered, effectively compensating for the acquired velocity component.

This dynamic adjustment leads to an angular displacement in the photon's trajectory—precisely the mechanism responsible for the observed aberration of starlight. The aberration is thus interpreted as a field-interaction effect rooted in conservation principles and solitonic structure, rather than requiring a mechanical ether or invoking special relativity postulates.

The Earth–Moon system offers further support for this field-dragging perspective. The Moon is tidally locked and effectively entrained by Earth's field. Similarly, Earth is dynamically influenced by the Sun's field, which in turn is structured within the galactic field associated with the Milky Way. Each body is thus nested within the field structure of larger systems, and motion is expressed through these interlinked field relationships. The principle of relativity, in this context, is recast not as a symmetry of inertial frames, but as a statement of field-constrained dynamics within a nilpotent, structured universe.

14.1 Transportivity

Part I concluded with Proposition 4.1 (pp. 26), which introduced a fundamental property of space termed *transportivity*, denoted $\mathcal{T}$. In vacuum, this is defined by the square of the speed of light:

$$\mathcal{T} := c^2.$$

Transportivity expresses the maximal causal propagation speed permitted by the structure of space, independent of any particular field equations. Unlike the traditional formulation $c = 1/\sqrt{\epsilon_0 \mu_0}$, which links the speed of light to the permittivity and permeability of free space, the $R(3)SO(3)$ framework recognises c as a primitive constant, not derived from other field properties.

In Section 5.1 (pp. 31), the *aleph function* was introduced as a means of encoding the localised but energy-free field structure of solitons. Although these fields do not carry energy in the classical sense, they occupy space and define the vibrational eigenmodes of solitons. This occupancy imposes a constraint on space's intrinsic transport capacity. Specifically, the solitonic fields reduce the available transportivity through their structured magnetic configurations, such that:

$$\mathcal{T}_l = c^2 - \sum_n \mathcal{V}(\vec{r}_n),$$

where $\mathcal{V}(\vec{r}_n)$ denotes the effective scalar potential induced at the local reference point by solitons located at positions $\vec{r}_n$ elsewhere in space. These potentials arise from the squared magnitudes of the fields, since energy is proportional to ϕ^2 and its glueonic counterpart ϕ_g^2, and potential is their spatial derivative. Thus,

$$\mathcal{V}(\vec{r}_n) \propto \nabla \left(\phi^2 + \phi_g^2\right).$$

The accumulated effect across all solitonic excitations results in an effective reduction in local transportivity:

$$\mathcal{T}_l = c^2 - c_a^2,$$

where $c_a^2 := \sum_n \mathcal{V}(\vec{r}_n)$.

DEFINITION 14.1: *Ambient Transportivity Deficit.* The quantity c_a^2 is defined as the *ambient transportivity deficit* induced by the presence of structured solitonic fields throughout space. It quantifies the cumulative reduction of transportivity due to field-induced scalar potentials and is given by:

$$c_a^2 := \sum_n \mathcal{V}(\vec{r}_n),$$

where $\mathcal{V}(\vec{r}_n)$ denotes the effective scalar potential at the local point induced by the soliton located at position $\vec{r}_n$. These potentials arise from the magnetic and glueonic field intensities as:

$$\mathcal{V}(\vec{r}_n) \propto \nabla\left(\phi^2 + \phi_g^2\right).$$

The local speed of light, as experienced within such a structured field environment, is therefore:

$$c_l^2 = \mathcal{T}_l = c^2 - c_a^2.$$

This decomposition of transportivity has direct implications for the d'Alembert wave equation. Recall its canonical form:

$$c^2 \frac{\partial^2 \varphi}{\partial x^2} - \frac{\partial^2 \varphi}{\partial t^2} = 0.$$

Substituting the decomposition $c^2 = c_l^2 + c_a^2$, we obtain:

$$c_l^2 \frac{\partial^2 \varphi_l}{\partial x^2} + c_a^2 \frac{\partial^2 \varphi_a}{\partial x^2} - \frac{\partial^2 \varphi_l}{\partial t^2} - \frac{\partial^2 \varphi_a}{\partial t^2} = 0,$$

which separates into two independent wave equations—each governing a component of the field evolution under distinct contributions from local structure (c_l) and ambient modulation (c_a).

REMARK (On the Michelson–Morley Experiment). This decomposition offers a natural interpretation of the Michelson–Morley experiment. The interferometric apparatus is sensitive only to c_l, the local causal transportivity of light within the experimental frame. Since c_a is embedded in the field structure and manifests through space-filling solitonic background, it remains undetectable to local interferometric methods. The observed invariance of the speed of light is therefore a manifestation of the locality of c_l, not of a universal constancy of c across inertial frames.

PROPOSITION 14.1: *Relativity in the $R(3)SO(3)$ Framework.* In the $R(3)SO(3)$ framework, the principle of relativity is not characterised by the invariance of physical laws across inertial frames, but by the locality of transportivity governed by the solitonic field structure of space. The apparent constancy of the speed of light, as observed in experiments such as Michelson–Morley, reflects the locality of c_l, the transportivity available after reduction by the ambient transportivity deficit c_a^2. This reinterpretation of relativistic invariance replaces global symmetry with field-constrained causality in a nilpotent universe.

15 Gravity and Electrostatic Forces

The definition of gravity and electrostatic forces within the $R(3)SO(3)$ framework requires a heuristic-based development. This development must respect the principles of $R(3)SO(3)$ and bridge the microscopic domain of quantised topological solitons with the macroscopic regime, where force interactions are described in terms of the emergent property of mass.

In a perfectly symmetrical nilpotent universe, atomic structure would assemble in such a way that all forces cancel, resulting in an inert body devoid of fields and interaction mechanisms with any other body. Section 5.5 (pp. 35) established that the elementary charge e is a phenomenon distinct from the load ℓ. Subsequently, in Section 11 (pp. 56), its glueonic analogue—the glue $\not\ell$—was introduced. Demonstration 5.1 (pp. 39) showed that when there is an asymmetry between a positive and a negative load, both modelled as imaginary quantities in $R(3)SO(3)$, a small real residual force remains, proportional in magnitude to the asymmetry.

The aleph function is introduced in Definition 5.1 (pp. 31), where the solitonic field structure in $R(3)SO(3)$ is defined. When atomic matter assembles into a ponderable body M, the solitonic fields (comprising both load and glue components) largely cancel, reinstating a nilpotent state. However, residual asymmetries in these fields manifest as nonzero field structures, partially filling the surrounding space and signalling the presence of the body.

The mass of M is understood as an emergent property of solitons in $R(3)SO(3)$. Theorem 11.1 (pp. 64) derives relativistic energy and inertial mass from solitonic structure. Describing the interaction between massive bodies thus requires a transition to a macroscopic viewpoint.

THEOREM 15.1: *Transportivity and the Gravitational Phenomenon.*
In Section 4.4 (pp. 25), we introduced transportivity*, denoted by $\mathcal{T}$, as a fundamental property of space, defined as*

$$\mathcal{T} := c^2.$$

A neutral massive body M generates, at a distance r, a scalar potential $\mathcal{V}(r)$ consistent with $R(3)SO(3)$ potentials.

This potential is interpreted as a gyratory electromagnetic wave permeating the local space and manifesting as a reduction in transportivity:

$$\mathcal{T}(r) := c^2 - \mathcal{V}(r).$$

This local modulation of transportivity gives rise to the classical gravitational force:

$$F_G = \frac{GMm}{r^2}.$$

PROOF. For a composite soliton representing neutral atomic matter, we define an effective radius

$$\mathfrak{r} = \frac{GM}{c^2},$$

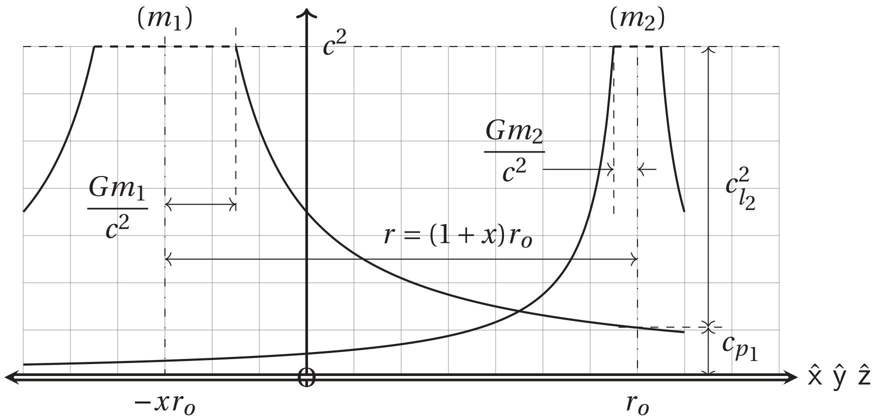

Figure 2: *Transportivity, Gravitational Interaction, and Orbits.* The horizontal axis represents the radially reduced space; the vertical axis represents transportivity, ranging from zero to c^2. The figure illustrates the variation in transportivity resulting from the presence of two massive bodies, m_1 and m_2, where $m_2 = xm_1$, positioned symmetrically about their barycentre. The body m_2 perceives a local speed of light $c_{l_2} = c^2 - c_{p_1}$, where c_{p_1} is the transportivity deficit induced by the gravitational potential of m_1. Although the diagram emphasises the effect of m_1 on m_2, the interaction is fully reciprocal: each body induces a transportivity deficit influencing the other, and the forces satisfy $F_{1_2} = F_{2_1}$ by construction.

which corresponds to half the Schwarzschild radius in general relativity, and is consistent with the nilpotent structure of the framework.

To model macroscopic fields, we introduce the macro-aleph function $\aleph_{\text{macro}}$, analogous to the quantum-scale definition in Section 5.1 (pp. 31). Normalised to $\mathcal{T}$, it is defined piecewise as

$$\aleph_{\text{macro}}(r) = \begin{cases} \aleph'_{\text{macro}}(r) = \dfrac{\mathcal{T}u}{\mathfrak{r}} \displaystyle\int_0^{\mathfrak{r}} \left(\frac{r}{\mathfrak{r}}\right)^{u-1} \mathrm{d}r = -\mathcal{T}, & \text{if } r \le \mathfrak{r} \\ \aleph''_{\text{macro}}(r) = \dfrac{\mathcal{T}u}{\mathfrak{r}} \displaystyle\int_{\mathfrak{r}}^{\infty} \left(\frac{\mathfrak{r}}{r}\right)^{u+1} \mathrm{d}r = \mathcal{T}, & \text{if } r \ge \mathfrak{r}. \end{cases}$$

This yields the nilpotent result $\aleph'_{\text{macro}}(r) + \aleph''_{\text{macro}}(r) = 0$. Setting $u = 1$ corresponds to a non-hollow soliton. The external component $\aleph''_{\text{macro}}(r)$ defines a scalar field

$$\mathcal{G}(r) = \frac{GM}{r^2}.$$

The relationship between the potential $\mathcal{V}(r)$ and its field $\mathcal{G}(r)$ is given by

$$\mathcal{G}(r) = -\frac{\mathrm{d}\mathcal{V}(r)}{\mathrm{d}r},$$

which integrates to yield

$$\mathcal{V}(r) = -\int \mathcal{G}(r)\mathrm{d}r = \frac{GM}{r}.$$

As with quantum fields, these macroscopic fields do not carry energy; rather, they instantaneously inform the remainder of the universe of the body's presence by modulating transportivity:

$$\mathcal{T}(r) = c^2 - \mathcal{V}(r) = c^2 - \frac{GM}{r}.$$

We define the *potential-induced transportivity deficit* c_p^2 as the reduction in transportivity experienced by a second body due to the scalar potential generated by a massive body M. That is,

$$c_p^2 := \mathcal{V}(r) = \frac{GM}{r}.$$

This allows the transportivity to be written compactly as

$$\mathcal{T}(r) = c^2 - c_p^2,$$

which in turn defines the local speed of light perceived by the body m at position r:

$$c_l^2 = \mathcal{T}(r) = c^2 - c_p^2 = c^2 - \frac{GM}{r}.$$

Figure 2 (pp. 77) illustrates this relationship by showing the variation in transportivity resulting from two massive bodies.

The energy of m is then given by

$$E_m = mc^2 = m\left(c_l^2 + \frac{GM}{r}\right),$$

which rearranges to

$$mc_l^2 = mc^2 - \frac{GMm}{r},$$

partitioning the energy into three components: inertial energy equals vacuous energy minus potential energy. The vacuous energy refers to the energy of m in an otherwise empty universe, where no other bodies are present to influence its transportivity. The vacuous energy also defines the body's gravitational mass, as well as its rest mass m.

Differentiating the inertial energy mc_l^2 with respect to r yields the force acting on m:

$$F_\text{G} = \frac{\mathrm{d}(mc_l^2)}{\mathrm{d}r} = \frac{GMm}{r^2},$$

which causes m to accelerate toward M, and vice versa. □

Unlike particles in accelerators, the gravitational force F_G does not contribute additional energy to either m or M; rather, their total energies remain constant.

REMARK. This framework may offer new insights into the dynamics of gas clouds. If such clouds behave as hollowed solitons with a potential well (i.e., $u > 1$), the tendency of gas clouds to contract around a central locus over time may be naturally explained.

COROLLARY 15.1.1: *Inertial and Gravitational Mass.*
Transportivity modulates inertial mass. If $c_l < c$ is the local speed of light, then a body with gravitational mass m possesses an inertial mass

$$m^{(i)} = m\frac{c_l^2}{c^2}.$$

DEFINITION 15.1: *Mass Types and Associated Energies.* Within the $R(3)SO(3)$ framework, the following mass and energy notions are distinguished:

- m denotes the *gravitational mass*, which also serves as the *rest mass*, defined by the body's vacuous energy.
- $m^{(i)}$ denotes the *inertial mass*, which varies with position through its dependence on the local transportivity $\mathcal{T}(r)$.
- mc^2 is the body's *vacuous energy*, representing the energy of the body in the absence of any other mass influencing transportivity.
- $m^{(i)}c^2 = mc_l^2$ is the *inertial energy*, where $c_l^2 = \mathcal{T}(r)$ denotes the local speed of light squared.
- $m^{(p)}c^2 = GMm/r$ is the *potential energy* associated with the gravitational interaction between the body and an external mass M.

While gravitational and inertial masses are experimentally indistinguishable in classical physics, the distinction becomes necessary in this framework due to the role of spatial transportivity in determining inertial behaviour.

COROLLARY 15.1.2: *Interaction of Charged Bodies.*
If the atomic matter of the body M is not neutral—that is, if it is ionised—it carries a net charge Q, meaning that the number of positive loads ℓ^+ are not balanced by the number of negative loads ℓ^-. As established in Section 5.5 (pp. 35), the load is treated in the $R(3)SO(3)$ framework as an imaginary quantity. Therefore, the method applied in the preceding gravitational derivation can be extended to account for charged bodies.

Let iQ represent the electric charge, modelled analogously to the mass M, by replacing GM with $k_e(iQ)$, where $k_e = 1/(4\pi\epsilon_0)$ is Coulomb's constant and i denotes the imaginary unit. This substitution leads to an effective radius

$$\mathfrak{r}'_q = \frac{k_e(iQ)}{c^2},$$

which has units of $\mathrm{m\,kg\,C^{-1}}$*. This quantity is used to define the electric potential $\mathcal{V}_e(r')$ analogously via the aleph function.*

As a final step, we normalise the radial variable by dividing r' by the charge-to-mass ratio constant ϱ, defined in Axiom 4.1 (pp. 19) (Electromagnetic Action and Coupling Constants),

$$r = \frac{r'}{\varrho}, \quad \text{where} \quad \varrho := 1\mathrm{kg\,C^{-1}}.$$

This yields the electric potential

$$\mathcal{V}_e(r) = \frac{k_e(iQ)}{r},$$

and the corresponding force becomes

$$F_E = \frac{k_e(iQ)(iq)}{r^2} = -\frac{k_e Qq}{r^2},$$

indicating that like charges exert repulsive forces, in contrast to the attractive nature of gravitational interaction.

The Moon's orbital dynamics are governed primarily by the Earth's gravitational field, which defines its immediate sphere of influence. However, both the Earth and Moon exist within a broader ambient domain governed by the Sun, which in turn resides within the gravitational and field structure of the Milky Way. Each of these nested systems contributes cumulatively to the ambient transportivity deficit experienced locally.

The preceding theorem and corollaries address local field interactions and gravitational effects under the assumption of an idealised vacuum background, thereby excluding the ambient deficit c_a^2 arising from large-scale field contributions. A comprehensive treatment of planetary motion within the $R(3)SO(3)$ framework requires that this ambient deficit be incorporated.

Accordingly, the total transportivity should be expressed as:

$$\mathcal{T}(r) := c^2 - c_a^2 - \mathcal{V}(r),$$

where c^2 is the universal vacuum transportivity, c_a^2 is the ambient transportivity deficit as defined in Definition 14.1 (pp. 74), and $\mathcal{V}(r)$ denotes the local potential generated by the body of interest.

In this expanded formulation, the aleph function must be normalised not to c^2, but to the reduced background value $c^2 - c_a^2$, reflecting the diminished causal propagation capacity available in the local environment.

For the purposes of the present work, we continue under the simplifying assumption that c_a^2 is excluded, and defer its full incorporation to an expanded treatment of the theory.

15.1 Precession of Planetary Orbits

This section applies the concept of transportivity to classical mechanics, specifically analysing a two-body system in circular orbit about their common barycentre, which serves as the origin for this analysis.

Let m_1 and m_2 be the masses of the two bodies, with $m_2 < m_1$. Define the mass ratio $x = m_2/m_1$, and let r_o be the distance from m_2 to the barycentre. (See Figure 2 (pp. 77).) Then, the distance from m_1 to the barycentre is $r_1 = xr_o$, and the total separation between the bodies is $r = (1+x)r_o$.

The transportivity at the positions of m_1 and m_2 is given by

$$\mathcal{T}_1 = c_{l_1}^2 = c^2 - \frac{Gm_2}{r}, \quad \mathcal{T}_2 = c_{l_2}^2 = c^2 - \frac{Gm_1}{r}, \tag{21}$$

where G is the gravitational constant.

Assuming $Gm_1/r \ll c^2$, the kinetic energies of the orbiting bodies can be related to the available potential energy Gm_1m_2/r as follows:

$$\frac{1}{2}m_2v_2^2 = \frac{1}{2}m_2(c^2 - c_{l_2}^2) = \frac{Gm_1m_2}{2(1+x)^2r_o},$$

$$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1(c^2 - c_{l_1}^2) = \frac{Gm_1m_2}{2\frac{(1+x)^2}{x}r_o}.$$

where $\{v_1, v_2\} << c$. Solving for the orbital velocities yields:

$$v_2 = \sqrt{\frac{Gm_1}{r_o(1+x)^2}}, \quad v_1 = \sqrt{\frac{Gm_2 x}{r_o(1+x)^2}} = x v_b.$$

The centrifugal forces acting on m_1 and m_2 are modified by the local transportivity (see Corollary 15.1.1 (pp. 78) *Inertial and Gravitational Mass*):

$$F_2^{\text{centri}} = \frac{c_{l_2}^2}{c^2} \frac{m_2 v_2^2}{r_o}, \quad F_1^{\text{centri}} = \frac{c_{l_1}^2}{c^2} \frac{m_1 v_1^2}{x r_o}.$$

The gravitational forces are given by:

$$F_2^{\text{grav}} = \frac{Gm_1 m_2}{r^2}, \quad F_1^{\text{grav}} = \frac{Gm_1 m_2}{r^2}.$$

The effective potential for the orbit of m_2 is:

$$V_2(r_o) = \frac{c_{l_2}^2}{c^2} \frac{L_2^2}{2m_2 r_o^2} - \frac{Gm_1 m_2}{r_o(1+x)}, \tag{22}$$

where $L_2 = m_2 v_2 r_o$ is the angular momentum of m_2.

Expanding $m_2 c^2$ using Equation (21) and substituting into Equation (22), we obtain:

$$V_b(r_o) = \frac{c_{l_2}^2}{c^2} \left(\frac{L_2^2}{2m_2 r_o^2} - \frac{Gm_1 m_2}{(1+x) r_o} \right) - \frac{Gm_1 L_b^2}{m_2 c^2 r_o^3}.$$

In the limit $x \to 0$, $m_1 \to M$, $m_2 \to \mu = \frac{mM}{m+M}$, and $r_o \to r$, the effective potential simplifies to:

$$V_{\text{eff}}(r) = \frac{L^2}{2\mu r^2} - \frac{GM\mu}{r} - \frac{GML^2}{\mu c^2 r^3}, \tag{23}$$

where μ is the reduced mass and $L = \mu v r$ is the angular momentum.

The additional third term $-GML^2/\mu c^2 r^3$ leads to a precession of elliptical orbits, an effect not predicted by Newtonian mechanics.

REMARK. The effective potential $V_{\text{eff}}(r)$ given in (23) also emerges from the post-Newtonian limit of the Schwarzschild metric, the first exact solution to Einstein's field equations in general relativity. This correction to the Newtonian potential leads directly to the celebrated result for the relativistic precession of planetary orbits. The corresponding precession angle per revolution, originally derived by Einstein in 1916 and famously applied to Mercury, is approximately:

$$\delta\varphi \approx \frac{6\pi G(M+m)}{c^2 A(1-e^2)},$$

where A is the semi-major axis and e is the orbital eccentricity.

16 Clocks

Modern atomic clocks achieve their exceptional precision by exploiting the highly stable and well-defined transition frequencies between quantised electron orbital

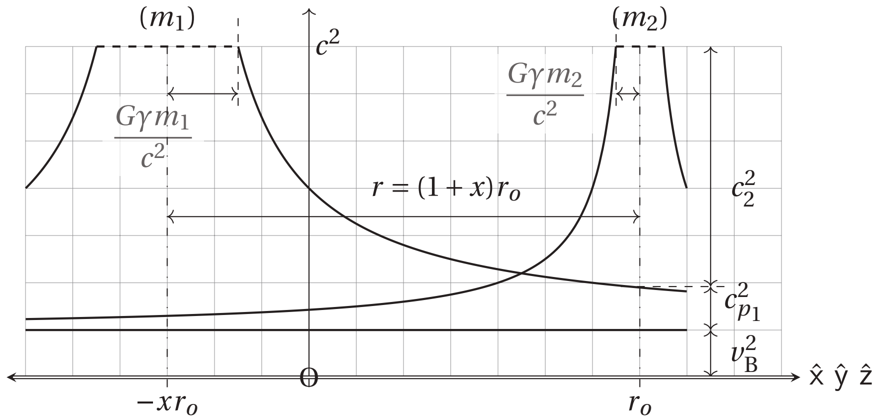

Figure 3: *Boosted Orbits.* This sketch builds on Figure 2 (pp. 77) to illustrate the effect of Lorentz boosting on a two-body system. Three key features are shown: (i) the boost-induced transportivity deficit v_{B}^2 appears as a constant offset; (ii) the energy levels of m_1 and m_2 undergo dilation due to boosting; and (iii) the transportivity curves flatten, becoming asymptotic to v_{B}^2 rather than to zero. The approach developed here for boosted orbits is structurally congruent with the treatment of ambient transportivity deficits. In environments with both ambient and boost-induced effects, the total deficit becomes $v_{\mathrm{B}}^2 \mapsto v_{\mathrm{B}}^2 + v_a^2$.

energy levels in atoms. In this section, the Bohr model of hydrogen is employed to simulate atomic clocks within the $R(3)SO(3)$ framework. However, it must first be acknowledged that, within atomic matter, classical orbits—as found in planetary systems—do not exist. Instead, such orbits serve as a useful duality, offering a conceptual analogue for analysing electron shells. In the $R(3)SO(3)$ framework, these shells are understood as discrete eigenstates of gyrations within solitonic field structures, rather than as trajectories of point particles.

Consider a microcosm consisting of hydrogen atoms that is Lorentz-boosted to a velocity v_{B} by an accelerating and energising force. The energy of the microcosm increases by the Lorentz factor

$$\gamma_{\mathrm{B}} = \frac{1}{\sqrt{1 - v_{\mathrm{B}}^2/c^2}},$$

and the transportivity relevant for the interaction between the proton and electron within a hydrogen atom in this boosted microcosm is given by

$$\mathcal{T}(r) := c^2 - v_{\mathrm{B}}^2 - \mathcal{V}(r),$$

where v_{B}^2 represents the ambient transportivity deficit internal to the boosted frame. This is illustrated in Figure 3 (pp. 82).

The electrostatic potentials generated by the proton and electron, respectively, are

$$\mathcal{V}^+(r) = \frac{k_e(i\gamma_{\mathrm{B}} q)}{r} \cdot \frac{c^2 - v_{\mathrm{B}}^2}{c^2}, \qquad \mathcal{V}^-(r) = \frac{k_e(-i\gamma_{\mathrm{B}} q)}{r} \cdot \frac{c^2 - v_{\mathrm{B}}^2}{c^2}.$$

The boosting of the electrostatic charge is a direct consequence of the $R(3)SO(3)$ framework, in which both the ℓ and $\not{\ell}$ contributions are enhanced under Lorentz transformation. The factor $\frac{c^2-v_{\mathrm{B}}^2}{c^2}$ arises due to the normalisation of the aleph function to the available transportivity $\mathcal{T} = c^2 - v_{\mathrm{B}}^2$.

The electron's potential energy in the hydrogen atom is determined by the product of the electrostatic charge of the electron and the potential due to the proton:

$$V_e = \left(-i\gamma_{\mathrm{B}} q\right)\left(\frac{k_e(i\gamma_{\mathrm{B}} q)}{r} \cdot \frac{c^2 - v_{\mathrm{B}}^2}{c^2}\right) = \frac{k_e q^2}{r}.$$

As in gravitational systems, the potential energy of the electron is partitioned into kinetic and electrostatic contributions. For a bound electron in the boosted microcosm, the kinetic energy involves the *boosted mass*, yielding

$$\frac{1}{2}\gamma_{\mathrm{B}} m_e v^2 = \frac{1}{2} \cdot \frac{k_e q^2}{(1+x)^2 r},$$

where x is the inverse of the proton-to-electron mass ratio.

Bohr's original postulate quantised the orbital angular momentum of the electron as an integer multiple of the reduced Planck constant $\hbar$, classically written as

$$m_e v r = n\hbar.$$

In the $R(3)SO(3)$ framework, this quantisation condition is reinterpreted: it defines the eigenvalues associated with electron shells as solitonic gyration states, and these eigenvalues remain invariant under Lorentz boosts. The dynamical solitonic state, however, evolves with the relativistically boosted mass $\gamma_{\mathrm{B}} m_e$, ensuring that energy relations and field interactions remain consistent with the relativistic structure of the theory.

Solving for the radius r, we obtain

$$r = \frac{\gamma_{\mathrm{B}} n^2 \hbar^2 (1+x)^2}{k_e m_e q^2},$$

Substituting this result into the expression for the potential energy yields

$$V_e = \frac{(k_e q^2)^2 m_e}{\gamma_{\mathrm{B}} n^2 \hbar^2 (1+x)^2}.$$

This expression shows that the potential energy associated with electron transitions in atomic matter *decreases* under Lorentz boosts. As a consequence, the energy difference between quantised states becomes smaller, leading to a *longer period* for oscillations—i.e., a dilation of the clock's ticking rate. This is in agreement with experimental observations of time dilation in moving atomic clocks, and is naturally reproduced within the $R(3)SO(3)$ framework.

No discussion of relativistic clocks would be complete without consideration of gravitational effects. In a gravitational field, the inertial mass of the electron decreases as a function of gravitational potential:

$$m_e^{(i)} = m_e \cdot \frac{c^2 - \frac{GM}{r}}{c^2}.$$

Therefore, for two identical stationary clocks positioned at different heights in the gravitational field of a body M, the ratio of their periods (time between ticks) is given by

$$\frac{\tau_1}{\tau_2} = \frac{c^2 - \frac{GM}{r}}{c^2 - \frac{GM}{r+h}},$$

where h is the height difference. The clock located closer to the centre of mass M ticks more slowly due to its reduced local transportivity.

REMARK (On Gravitational Blue Shift and Mass Duality in $R(3)SO(3)$). The gravitational shift of clocks described above is derived here from variations in inertial mass, as predicted within the $R(3)SO(3)$ framework. In this interpretation, the ticking rate of a clock is determined by the local transportivity, which in turn modulates the inertial mass of its constituent particles. This contrasts with conventional formulations, which treat gravitational redshift primarily as a spacetime curvature effect.

Consequently, physical clocks—through their measurable response to gravitational potential—serve as experimental evidence for a distinction between gravitational mass and inertial mass. Within the $R(3)SO(3)$ framework, these are not identical quantities but emerge from distinct aspects of solitonic field structure: gravitational mass is associated with the rest-state configuration, while inertial mass is modulated by the local field-induced transportivity.

17 Cosmogenesis

The $R(3)SO(3)$ framework permits a novel perspective on the origin and cyclic evolution of the universe. Under the assumption that the universe and contra-universe emerged together in the same space as a mathematically nilpotent system, the initial condition may be expressed as

$$\mathcal{U} + \overline{\mathcal{U}} = 0 \quad \text{and} \quad \mathcal{U} \cdot \overline{\mathcal{U}} = 1.$$

These dual systems, while jointly nilpotent, exhibit repulsive interactions. One expands while the other contracts—an inflationary phase driven by mutual opposition. If it is assumed that the contra-universe underwent contraction, it may now occupy a highly compressed region at the centre of the $R(3)SO(3)$ manifold, while our universe expanded outward, like the surface of a balloon.

In this view, the inflation of our universe is driven by the contracting contra-universe, balanced against a form of *surface tension* provided by gravitational forces. As stars and matter burn and decay, the effective gravitational tension diminishes, leading to an acceleration of expansion. In parallel, the contra-universe continues to shrink.

Eventually, a reversal occurs. The contra-universe undergoes a kind of supernova collapse and rebounds into a new inflationary phase, while our universe deflates and contracts. This suggests a cosmological cycle of expansion and contraction—an eternal dynamical equilibrium governed by the interplay between dual universes. In this framework, information is not preserved in the detailed configurations of matter, but instead encoded across cycles into larger-scale structures. These may persist as imprints visible in the cosmic microwave background.

The $R(3)SO(3)$ manifold itself is nine-dimensional, whereas the physical universe we inhabit is three-dimensional. This disparity allows for the possibility that our three-dimensional universe is a surface embedded within a higher-dimensional space—perhaps the boundary or hypersurface of a nine-dimensional sphere. In such a topology, the universe may not be spatially infinite, but instead relatively compact, with light paths tracing curved or closed geodesics. Under this condition, it becomes conceivable that distant, highly redshifted objects we observe—interpreted as primordial galaxies—could in fact be earlier images of our own Milky Way or local structures, observed along extended causal loops through the curved geometry of space.

18 Beyond the Standard Forces

> "In biology we are faced with an entirely different situation. … We are here obviously faced with events whose regular and lawful unfolding is guided by a 'mechanism' entirely different from the 'probability mechanism' of physics."
>
> — Erwin Schrödinger, *What is Life?*

The standard model of physics recognises four fundamental forces: electromagnetic, weak, strong, and gravitational. Within the $R(3)SO(3)$ framework, the electromagnetic force arises naturally from the structured charge ℓ, whose role as the generator of electric fields is retained and extended. However, the introduction of two additional structured charges—*fervour* f and *vigour* v—necessitates a reevaluation of force classification. These charges are not abstract artefacts but emerge from the same algebraic and solitonic principles that give rise to electromagnetic and gravitational phenomena.

It is here proposed that the missing manifestations of these forces are not to be found in conventional particle interactions but in the origin and organisation of life itself. The force arising from f may underlie the processes of the plant kingdom: growth, structural patterning, and energy assimilation. In parallel, v may drive the internal and external dynamism of the animal kingdom: movement, cognition, and the directional use of energy. Or, both may contribute to both the plant and animal kingdom.

In the $R(3)SO(3)$ formalism, photons are understood as load-mediated solitons, transferring energy via interactions with electron shells. However, as elaborated in Remark 11.3 (pp. 69), neutrinos are the structural equivalents of photons for the f and v charges—each corresponding to quantised solitonic excitations that transfer energy through field coupling rather than direct electromagnetic means.

Photons provide heat and energise electrons, catalysing photosynthesis and supporting a wide range of biochemical processes, among others. But if life arises from a deeper solitonic coherence, it should also draw not only on electric (load) dynamics but also on the persistent, low-interaction fields associated with f and v. In this expanded view, neutrinos—long considered ghostlike due to their small cross sections in standard electroweak theory—are reinterpreted: those that carry fervour and vigour interact through entirely different field channels and could exhibit a *large* effective cross section with living systems. Whereas—in conventional thinking—neutrinos are like darts striking a bull's eye, in life-coupled interactions they could behave more like darts hitting a wide-open dartboard: absorbed, transferred, and exchanged as integral components of biological structure and function.

If correct, this interpretation suggests that life is not an emergent anomaly, but a structurally compelled phenomenon—a product of the full symmetry encoded in the $R(3)SO(3)$ manifold. The structured interplay of load, fervour, and vigour provides the field environment in which life could emerge as a coherent solitonic organisation. Life may not a rare exception, but the natural expression of field-theoretic completeness. Its presence on Earth is not merely compatible with physics—it is required by it.

Conclusion

This work has introduced the $R(3)SO(3)$ framework as a new ontological and mathematical foundation for physical theory, grounded in the postulate of a nilpotent universe. At its core, $R(3)SO(3)$ replaces conventional operator-based formulations of quantum mechanics with a structurally constructive and geometrically constrained system based on topologically quantised solitons. Within this framework, all physical phenomena—fields, particles, interactions, and motion—emerge as manifestations of local field structures embedded in a causally consistent, non-energetic background.

A central innovation in this approach is the development of a *ternary number system* within a framework that reinstates classical algebra in the formulation of the special orthogonal gauge group $R(3)SO(3)$, deliberately avoiding the use of Lie algebras. This decision is not an aesthetic divergence but a foundational shift: it enables the modelling of continuous field deformations and gauge-consistent dynamics without recourse to operator-based symmetry algebras. The algebraic structures introduced here are intrinsically compatible with the discrete and quantised character of physical reality as described by solitonic field entities. Within this setting, fields do not carry energy in the classical sense; rather, they define the geometric and topological conditions under which solitonic structures exist, propagate, and interact.

The recovery of quantum mechanics arises naturally from the intrinsic structure of soliton eigenstates. Rather than treating quantum properties as probabilistic or axiomatic, the $R(3)SO(3)$ framework derives them from structural resonances and constraints within a quantised, causal space. This eliminates the need for operator formalism, Hilbert spaces, or wavefunction collapse, and replaces them with mathematically grounded field dynamics. Entanglement, quantised angular momentum, and energy levels are shown to emerge from geometric properties and conservation laws in the solitonic field lattice.

Moreover, key features of special and general relativity are recovered not through geometric curvature or Lorentz symmetry alone, but as emergent effects of transportivity—an intrinsic property of space governing causal propagation. Time dilation, gravitational redshift, and the invariance of the local speed of light arise as natural consequences of modulated inertial mass and field-structured transport deficits. The gravitational interaction, in particular, is recast as a manifestation of field-structured potential variation, not spacetime curvature, and permits direct reinterpretation of the Einstein precession and gravitational clock effects.

Together, these elements define a unified mathematical and physical architecture capable of subsuming quantum, relativistic, and classical results without the need for quantisation rules, symmetry postulates, or geometric reification. The $R(3)SO(3)$ framework presents not merely a reinterpretation of known physics, but a proposal for a coherent, causal, and quantised field theory rooted in topological structure and algebraic consistency.

Importantly, this framework also elevates the role of structured charges beyond electromagnetism. The introduction of *fervour* and *vigour*—two additional structured charge types within $R(3)SO(3)$ —points to the existence of forces not recognised within the Standard Model. If these charges are indeed fundamental, their corresponding forces may not manifest in traditional particle interactions, but in the processes of life itself. Neutrinos, long considered elusive and marginal, may in fact carry these life-related charges. Unlike photons, which energise matter via well-characterised electromagnetic interactions, neutrinos may act as agents of biological coherence, absorbed and exchanged through structures tuned to the fields of vigour and fervour. The suggestion that life depends on such field-based interactions would reframe biology not as an emergent exception, but as an intrinsic feature of field structure—a natural outgrowth of the Universe's deeper symmetry.

Future work will extend this foundation to multi-body dynamics, spin–statistics relationships, and cosmological-scale structure, as well as further develop the implications of the nilpotent constraint on conservation laws, gauge freedom, and the emergence of physical constants.

In closing, it is worth recalling the prescient words of Henri Poincaré[1], whose philosophical insight anticipated the dilemma at the heart of modern physics:

> "If we were to admit the postulate of relativity, we would find the same number in the law of gravitation and the laws of electromagnetism—the speed of light—and we would find it again in all other forces of any origin whatsoever. This state of affairs may be explained in one of two ways: either everything in the universe would be of electromagnetic origin, or this aspect—shared, as it were, by all physical phenomena—would be a mere epiphenomenon, something due to our methods of measurement. How do we go about measuring? The first response will be: we transport solid objects considered to be rigid, one on top of the other. But that is no longer true in the current theory if we admit the Lorentzian contraction. In this theory, two lengths are equal, by definition, if they are traversed by light in equal times.
> Perhaps if we were to abandon this definition Lorentz's theory would be as fully overthrown as was Ptolemy's system by Copernicus's intervention. Should that happen someday, it would not prove that Lorentz's efforts

1 M. H. Poincaré (1906). Sur la dynamique de l'électron. *Rendiconti del Circolo matematico di Palermo* 21.1, pp. 129–175. Translated by Scott Walter In J. Renn (ed.), The Genesis of General Relativity *Vol. 3: Theories of Gravitation in the Twilight of Classical Physics; Part I*

were in vain, because regardless of what one may think, Ptolemy was useful to Copernicus."

— Henri Poincaré, *Sur la dynamique de l'électron*

It is in this spirit that the $R(3)SO(3)$ framework has been developed—not to discard the insights of relativity and quantum theory, but to reinterpret them within a deeper constructive ontology. By replacing measurement-centric formulations with field-constrained causality and solitonic structure, $R(3)SO(3)$ aims to restore a physical basis to the shared features of all interactions, and to offer a coherent foundation from which both electromagnetic and gravitational phenomena may be understood not as epiphenomena, but as emergent expressions of a unified field framework. In doing so, this work aspires to continue, rather than overturn, the legacy of those who sought not just predictive power, but conceptual clarity.

By admitting our shortcomings, we give our children hope that they too can contribute to the advancement of scientific thought and discovery.

Outlook: Foundations for the Future

The $R(3)SO(3)$ framework not only recasts fundamental physics within a structurally quantised and causally coherent paradigm—it opens new avenues of inquiry in both natural sciences and technological innovation. Historically, revolutions in physical understanding have preceded profound shifts in technical capability: from Newton's mechanics enabling celestial navigation, to Maxwell's unification fostering the electrical age, to quantum mechanics catalysing semiconductors and lasers. If $R(3)SO(3)$ succeeds in reshaping our understanding of mass, charge, and interaction, then a similar wave of innovation may follow.

In particular, the framework may serve as a new lens through which to interpret complex systems—such as magnetic confinement plasmas, biological signalling, or gravitational anomalies—not merely through statistical or macroscopic models, but as emergent phenomena grounded in coherent solitonic structure. For example, the ITER fusion project relies heavily on simulations rooted in Maxwellian and magnetohydrodynamic approximations. Yet, the Sun—our natural prototype—exhibits mass ejections, field instabilities, and flux rope formations that suggest a richer topological and field-based dynamics. If such behaviours arise from underlying quantised field structures, then present fusion models may be omitting critical solitonic effects.

Moreover, the introduction of structured field charges—ℓ, $\not{f}$, and υ —implies that there exist field modes complementing the electromagnetic spectrum. These modes may become technologically exploitable if methods of resonance coupling or synthetic generation can be discovered. In this way, biological and quantum systems alike could become not only better understood but also more precisely engineered. Technologies that today seem out of reach—field-mediated communication, inertia modulation, or ultra-high-efficiency energy transfer—may prove accessible when designed with the topological structure of solitons in mind.

On the biological side, if the conjecture that neutrinos serve as agents of life's coherence is substantiated, it would reframe biology as a field-synchronous process, not simply a molecular one. Understanding life as a resonance in $R(3)SO(3)$ field structure suggests new methods for investigating developmental pathways, consciousness, and even regenerative phenomena—fields often regarded as empirical rather than foundational.

Ultimately, the promise of $R(3)SO(3)$ is not merely explanatory—it is generative. It provides a canvas for further mathematical construction, a toolkit for physical exploration, and a basis upon which new science and new technologies may be built. Its predictions, once tested, could inform not just the questions we ask but the instruments with which we ask them.

The constructive ontology presented here is incomplete—yet deliberately so. It invites collaboration, challenges consensus, and recognises that any enduring scientific revolution must also be an invitation: to question, to reimagine, and to build anew.

19 Epilogue

It is customary to conclude academic works with acknowledgments of funding and support. Regrettably, this project was undertaken without external financial assistance or institutional backing. My initial, perhaps unconventional, ideas—ideas that ultimately formed the foundation of the mathematical structures presented herein—were often met with scepticism. I faced criticism for challenging established paradigms, which unfortunately limited opportunities to present preliminary versions or to discuss ansätze related to this work within academic forums. This endeavour was a solitary and, at times, arduous and socially painful journey spanning over twenty-five years.

Furthermore, I wish to address the notion of mathematical "discovery." It is my firm belief that mathematics, like painting, music composition, or literary creation, is an art form. It is the product of deep, logical thought and structured composition. The assertion that mathematical formulae are "discovered," as if they exist independently awaiting unearthing, seems to me a misapprehension. Mathematical abstractions—especially novel ones—do not pre-exist in a Platonic realm; they are created through intellect and insight.

As an electrical engineer, I approached this work with a sceptical perspective on many existing physical explanations. Drawing on the analytical tools developed over my engineering career, I sought recurring patterns that span the microscopic and macroscopic realms. This investigation led to the invention of extensions to classical mathematical methods and the formulation of new frameworks—most notably, the ternary number system—whose internal consistency and expressive capacity allowed for visual and analytic representations aligned with physical observations.

A particular challenge moving forward is the establishment of a consistent and accessible vocabulary. The development of new mathematical and physical structures often demands a corresponding linguistic shift. However, I do not presume to be the final arbiter of terminology. Rather, I hope this work serves as an initial proposal, and I appeal to the physics and mathematics communities for collaborative refinement. Consensus on vocabulary is essential if the $R(3)SO(3)$ framework is to evolve into a shared language capable of supporting further theoretical and experimental development.

My next step is to expand this introductory work into a comprehensive book. Yet I continue to face the practical obstacle of institutional independence: I am not affiliated with any university or research body, a prerequisite for many public funding mechanisms. As such, I have no alternative but to ask—if you are interested in supporting the completion and expansion of this work in any way, information on how to contribute can be found at my website: `https://r3so3.com/`. Any assistance, intellectual or material, would be warmly welcomed.

Despite the solitary path this project has taken, the horizon it opens is necessarily collective. The $R(3)SO(3)$ framework is not offered as a final doctrine, but as a conceptual beginning—an invitation to rethink foundations, to unify what has long been divided, and to rediscover the physical world through structures that are both simpler and deeper. In an age where physics often leans on abstraction for its own sake, this work aspires to restore meaning, coherence, and causality to the heart of theory. If these ideas find resonance in others, then the effort has already begun to succeed—not in closure, but in the possibility of dialogue, expansion, and shared understanding.

APPENDIX

A Experimental Proposal: Testing Drude's Hypothesis

In 1897, Thomson discovered the electron. Three years later, Drude proposed that electric current arises from a fluid of free electrons drifting through an atomic lattice. In 1927, Sommerfeld extended Drude's model by incorporating Fermi–Dirac statistics, establishing a quantum mechanical explanation for electrical conduction in metals—a view still widely accepted.

However, observations from particle accelerators, particularly energy recovery LINACs, raise questions about the validity of this model.

The proposed experiment is a simplified analogue of energy recovery LINACs, consisting of a back-to-back accelerator and decelerator powered by isolated DC voltage sources, as illustrated in Figure 4 (pp. 91). The isolation of the voltage sources, along with the neutral path B–C, is central to the experiment, as it cleanly partitions the accelerating and decelerating stages, eliminating any cross-coupling. The entire setup is enclosed within magnetic and electric shielding to minimise external interference. Two identical, isolated voltage sources (batteries B_1 and B_2) create opposing electric

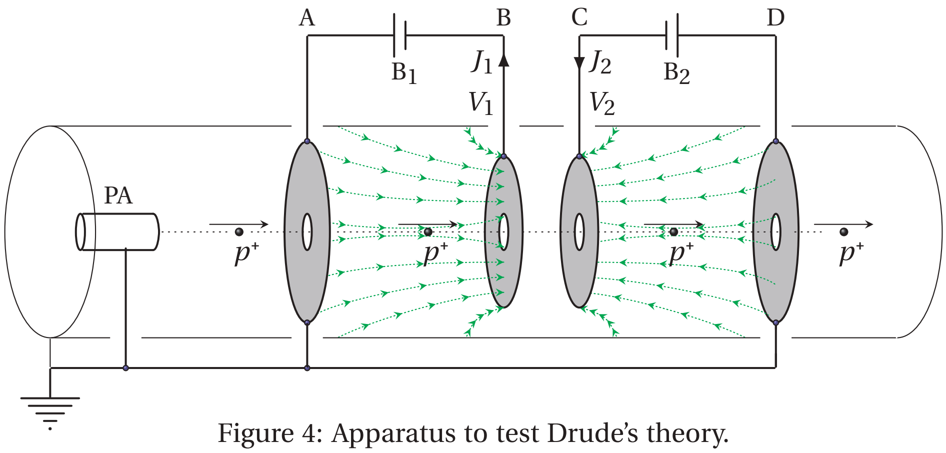

Figure 4: Apparatus to test Drude's theory.

fields in sections A–B and C–D. Crucially, points B and C are electrically isolated, and the potentials V_1 and V_2 are equal.

High-energy protons are emitted from the particle accelerator (PA) at ground potential. On the segment PA–A, no energy exchange occurs. From A to B, the protons are accelerated by the field $\vec{\mathbf{E}}_{AB}$, gaining kinetic energy. This energy must be drawn from the stored energy in battery B_1, which is discharged by a current J_1, in accordance with energy conservation.

In section B–C, with no potential difference, the proton energy remains unchanged.

From C to D, the decelerating field $\vec{\mathbf{E}}_{CD} = -\vec{\mathbf{E}}_{AB}$ reduces the protons' kinetic energy. This lost energy recharges battery B_2 via current J_2, mirroring the A–B process.

The protons exit with their initial energy, and the net energy change in the batteries is zero: one gains, the other loses equally. The process can be sustained indefinitely, implying that the currents J_1 and J_2 are also maintained indefinitely.

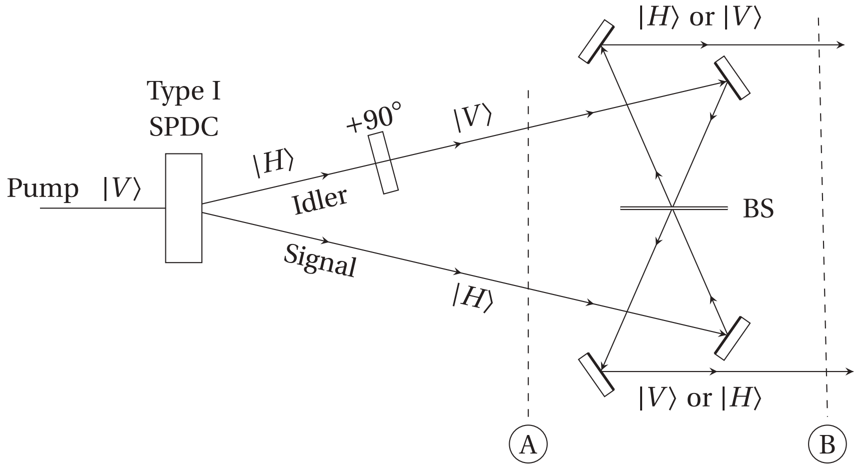

Figure 5: Ou amd Mandel's experimental setup successfully demonstrating the violation the Bell's inequality at position B from a single type I SPDC .

This outcome challenges Drude's electron drift model. If conduction were due to electrons physically drifting between terminals, one must explain how an infinite number of electrons continually enter and leave two electrically isolated regions—an impossibility in this configuration.

B Experimental Proposal: Causal or Probabilistic QM

We propose an experimental setup capable of discriminating between the Copenhagen interpretation of quantum mechanics and the deterministic, field-theoretic interpretation grounded in the $R(3)SO(3)$ framework developed in this work.

This experiment—designed to test whether quantum entanglement arises causally or probabilistically—is inspired by the pioneering work of Ou and Mandel.[2] As illustrated in Figure 5, their setup used a Type I spontaneous parametric down-conversion (SPDC) crystal to produce signal and idler photons, which were subsequently interfered at a beam splitter following a 90° rotation of the idler polarisation. Bell inequality violation was observed at position B, but not at the earlier station A.

Under the $R(3)SO(3)$ framework, however, this outcome demands reinterpretation. In contrast to post-processing schemes that induce effective entanglement through interference, the field-symmetric model asserts that entanglement is intrinsic to the SPDC process itself. The signal and idler solitons are created in an already entangled, nilpotent field configuration within the crystal. The question is then not *whether* they are entangled, but why the entanglement is not observable at position A.

We propose that two processes are simultaneously at work: (1) the production of entangled photon pairs via SPDC, and (2) their embedding in a coherent, linearly polarised macroscopic field structure (pump-polarisation-aligned). In this field-locked regime, the solitons cannot dynamically adjust their internal structure. The linear polarisation of both photons anchors them to a shared pilot wave originating

2 Ou, Z.Y. and Mandel, L. (1988) "Violation of Bell's Inequality and Classical Probability in a Two-Photon Correlation Experiment," *Physical Review Letters*, 61, pp. 50–53.

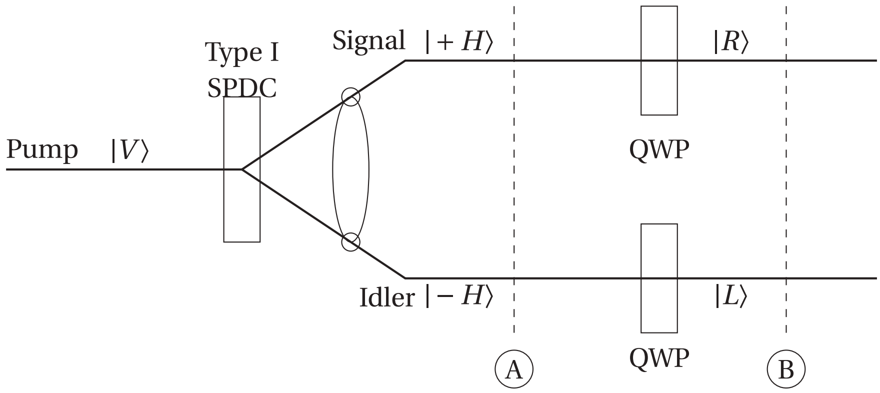

Figure 6: Schematic of the proposed experiment to distinguish field-based entanglement from collapse-based interpretations. A vertically polarised pump photon Υ_0 undergoes Type I spontaneous parametric down-conversion, producing two linearly polarised photons: the signal in state $|+H\rangle$ and the idler in $|-H\rangle$, differing by their solitonic rotation histories. Quarter-wave plates (QWPs), oriented relative to each soliton's internal polarisation frame, convert the signal and idler into right- and left-circular states $|R\rangle$ and $|L\rangle$, respectively. To preserve entanglement symmetry and ensure valid nilpotent alignment $\mathcal{U}+\overline{\mathcal{U}}=0$, the optical path lengths to the QWPs should be equalised as closely as possible to maintain temporal coherence.

from the pump, effectively suppressing entanglement observables at position A. Only when the fields are jointly transformed—via both the 90° phase shift of the idler and the subsequent field mixing at the beam splitter—do the solitons decouple from their rigid alignment with the macroscopic pump field. The phase shift alone is insufficient; it is the coherent superposition of the orthogonally polarised fields at the beam splitter that produces a circularly structured field environment. In this mixed-field regime, the solitons gain dynamic freedom to deform and adjust their internal configuration. This transition enables the observables required for a Bell-violating correlation to manifest at position B—whereas at position A, in the absence of circularisation, no such violation is observed.

The proposed refinement of this experiment, shown in Figure 6, circumvents the need for beam splitter interference. By introducing quarter-wave plates (QWPs) directly in the paths of the signal and idler, the fields are circularised while maintaining optical separation. If entanglement is now observed at station B without recombination, it implies that entanglement arises from soliton structure—not quantum superposition.

Station A: Linearly Polarised, Field-Locked Regime

Before encountering any optical elements, both signal and idler photons remain in linearly polarised $|H\rangle$ states. According to the $R(3)SO(3)$ interpretation, their solitonic structures are locked to a common origin Υ_0, preserving the macroscopic pilot wave of the pump.

Prediction: No Bell violation at detector A. Solitons remain rigid and dynamically inert. Entanglement is latent due to preserved topological coherence.

$$\text{At A:}\quad \text{Field-locked regime} \Rightarrow \text{No dynamic separability} \Rightarrow \text{No Bell violation}$$

Station B: Circularised, Dynamically Unlocked Regime

After passing through QWPs, the signal and idler fields transform into circular polarisation states $|R\rangle$ and $|L\rangle$, respectively. The solitons can now deform coherently, adjusting their pilot wave phase relationships to satisfy the nilpotent field constraint:

$$\mathcal{U} + \overline{\mathcal{U}} = 0.$$

Prediction: A Bell test will detect entanglement. Entanglement becomes dynamically accessible via structural deformation in the circularised field.

Interpretational Significance

Under the Copenhagen interpretation, entanglement is statistical and should manifest equally at stations A and B. By contrast, in the deterministic $R(3)SO(3)$-based framework, entanglement is geometric and topological. It manifests *only when the solitonic structures are dynamically accessible*, as made possible by circularisation.

A positive violation of the Bell inequality at station B—despite optical separation and without post-selection—would falsify collapse-based interpretations and provide strong evidence in favour of a soliton-guided, field-structured reality governed by nilpotent conservation laws.

REMARK B.1: *On Relative QWP Orientation and Soliton Frame Dependence.* The QWPs play a pivotal role in unlocking entanglement by transforming linearly polarised field-locked solitons into circularly polarised, dynamically accessible ones. However, their effect depends critically on the frame in which the transformation is applied.

While both photons may appear as $|H\rangle$ in the lab frame, their internal solitonic configurations—denoted $|+H\rangle$ and $|-H\rangle$—may differ due to their rotation histories. These distinctions are invisible to conventional optics but significant in the $R(3)SO(3)$ formalism.

If both QWPs are aligned according to the laboratory coordinate system, they may act asymmetrically with respect to each soliton's internal frame, resulting in two $|L\rangle$ states and violating nilpotent symmetry. Thus, each QWP must be oriented *relative to its corresponding soliton's internal polarisation basis.*

This ensures that the signal and idler acquire true opposite helicities and that their entangled state satisfies the nilpotent balance $\mathcal{U} + \overline{\mathcal{U}} = 0$, preserving field coherence.