# Classically Quantised Vacuum Electrodynamics, Part I: Maxwellian Vortices and an Emergent EM-Mass Gap

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**Abstract**

We introduce an algebraic vector closure $\mathfrak{M}$ set within a complex vector space $\mathcal{S} \subset \mathbb{C}^3$ equipped with a non-Hermitian symmetric bilinear product:

$$\mathfrak{M}[\mathbf{v},\mathbf{B},\mathbf{E}] := \left\{\mathbf{v},\mathbf{B},\mathbf{E} \in \mathcal{S} \;\middle|\; \frac{\mathbf{B}\times\mathbf{E}}{\mathbf{B}\cdot\mathbf{B}} - \mathbf{v} = 0, \quad \frac{\mathbf{E}\times\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}} - \mathbf{B} = 0, \quad \mathbf{v}\times\mathbf{B} - \mathbf{E} = 0\right\}.$$

Under a Maxwell-compatible transport law, this closure reconstructs the source-free Maxwell–Heaviside curl equations while leaving the transport speed $v$ as an initially free parameter. Consequently, $\mathfrak{M}$ acts as a structural constraint on the underlying d'Alembert wave operator $\square = \nabla^2 - v^{-2}\partial_t^2$. While the standard d'Alembert equation permits arbitrary linear wave packets, the non-linear algebraic requirements of $\mathfrak{M}$ restrict allowable field configurations to continuous 3D spatial-phase rotations $R = R_x R_y R_z$. This constraint unlocks non-trivial vortical and helical wave solutions across 1, 2, and 3 spatial dimensions—such as travelling vortices and helical centreline structures—that are unobtainable through d'Alembert's equation alone.

The vacuum itself is classically quantised: an elementary voxel $l_o^3$ with axial length $l_o$ and an elementary time $t_o$ carries a quantised disturbance, whose electric displacement is not a response to an applied field but the primitive excitation agent generating the fields. Defining a quantised displacement as $D_\mathrm{E} := e/l_o^2$ constrains a charge density and charge tensity, providing a propagation modulus $\mathcal{T} = l_o^2/t_o^2$ which autonomously establishes $c = \sqrt{\mathcal{T}}$ and thereby fixes the previously free transport speed in $\mathfrak{M}$ to $v = c$ before any constitutive parameter is derived. A parameterisation theorem then derives explicit expressions for the quantities $(\phi, \psi, \epsilon_0, \mu_0)$, and the elementary scales $l_o$ and $t_o$.

A second, structurally parallel construction introduces an action modulus $\mathcal{H}$ built from the same primitive charge variables to fix Planck's constant $h$. Consequently, both fundamental constants of vacuum electrodynamics ($c$ and $h$) emerge not as independent empirical inputs, but as numerical calibrations of closure-native moduli within the algebraic framework.

Furthermore, the elementary time scale bounds the excitation spectrum: the upper bound eliminates the classical Rayleigh–Jeans divergence while aligning with ultra-high-energy photon limits from the Pierre Auger and Telescope Array observatories, and the lower bound yields an emergent, strictly positive electromagnetic mass gap $\Delta_\mathrm{EM}$, read as a vortex-nucleation threshold above an otherwise gapless excitation branch. These results establish $\mathfrak{M}$ as a fundamental framework for vacuum electrodynamics.

## 1 Introduction

"The theoretical worker in the future will therefore have to proceed in a more indirect way. The most powerful method of advance that can be suggested at present is to em-

ploy all the resources of pure mathematics in attempts to perfect and generalise the mathematical formalism that forms the existing basis of theoretical physics, and *after* each success in this direction, to try to interpret the new mathematical features in terms of physical entities (by a process like Eddington's Principle of Identification)."
— P. A. M. Dirac [1]

In this work we introduce the cyclic algebraic vector closure $\mathfrak{M}$, defined over a complex space $\mathcal{S} \subset \mathbb{C}^3$ with a non-Hermitian symmetric bilinear product,

$$\mathfrak{M}[\mathbf{a},\mathbf{b},\mathbf{c}] := \left\{\mathbf{a},\mathbf{b},\mathbf{c} \in \mathcal{S} \;\middle|\; \frac{\mathbf{b}\times\mathbf{c}}{\mathbf{b}\cdot\mathbf{b}} - \mathbf{a} = 0, \quad \frac{\mathbf{c}\times\mathbf{a}}{\mathbf{a}\cdot\mathbf{a}} - \mathbf{b} = 0, \quad \mathbf{a}\times\mathbf{b} - \mathbf{c} = 0\right\}.$$

This article and its companion form a two-part study. Central to our approach is the ansatz that Maxwell's electric displacement acts not as a passive response to an applied field, but as the active excitation agent of a perturbed vacuum state—a localized disturbance such as a photon. The present article establishes the algebraic closure, its inseparable translational and gyrational components, the classically quantised vacuum, and the quantity derived from that construction. The companion article develops the corresponding quantised field equations through a recast of Maxwell's 1865 system, including potentials, displacement currents, induction, radiant-flux density, and loss dynamics. The closure relation $\mathfrak{M}$ is therefore not a substitute for an electromagnetic field equation: it determines the admissible algebraic architecture, whose field-equation realisation is developed in the companion article.

The closure $\mathfrak{M}$ is purely algebraic; its Maxwellian content enters through specific slot fillings. In the translational domain, inverse Lie transport of $\mathcal{M}[\mathbf{v},\mathbf{B},\mathbf{E}]$ reconstructs the source-free Maxwell–Heaviside curl equations while leaving the transport speed $v$ as an unconstrained parameter of the transport law. Although the Maxwell–Heaviside curl equations yield the standard d'Alembert wave operator $\Box = \nabla^2 - v^{-2}\partial_t^2$, their admissible field solutions remain strictly constrained by this closure.

Specifically, while the standard d'Alembert equation permits arbitrary linear wave packets, the non-linear algebraic demands of $\mathfrak{M}$ restrict physical field configurations to continuous 3D orthogonal rotations $R = R_x R_y R_z$. As detailed in Section 3.1, this structural constraint unlocks multi-dimensional vortical and helical wave solutions—including travelling vortices and helical centreline structures—that are unobtainable through d'Alembert's equation alone. The resulting gyrational domain is not an independent electromagnetic system: it completes the kinematics of the same translating disturbance.

Because the curl equations leave $v$ free, establishing how vacuum electrodynamics selects $v = c$ is a genuine structural question rather than a notational convention. To address this, we cast $\mathfrak{M}$ into a classically quantised model of the vacuum. The construction introduces an elementary length $l_o$ and elementary time $t_o$, together with a constructed vacuum transport modulus, termed the *transportivity*,

$$\mathcal{T} = \frac{l_o^2}{t_o^2} \qquad \Rightarrow \qquad c = \sqrt{\mathcal{T}},$$

whose positive square root gives the characteristic vacuum propagation speed $c$. In this sense, the speed of light is a property of the vacuum space–time structure rather than a quantity defined through its electromagnetic constitutive parameters.

Our construction treats the elementary charge $e$ as the sole primitive vacuum quantity. The

Planck constant $h$ and the vacuum propagation speed $c$ enter only as numerical calibrations of two closure-native moduli, the transportivity $\mathcal{T}$ and the action modulus $\mathcal{H}$, and the dimensionless Giorgi constant [2], here $g = 4\pi \times 10^{-7}$, or alternatively the fine-structure constant $\alpha$, likewise enters only through its numerical value as a retrospective calibration of the introduced electromagnetic to mechanical coupling constant $\varkappa$ — none of these numeric imports proceeds through prior electromagnetic constitutive relations.

Thus the axiomatic input is $e$ alone, together with the numeric calibrations $c$, $h$, and $g$ or $\alpha$. No previously established electrodynamic relations that connect the input quantities to other electromagnetic quantities enter as inputs.

With these inputs, and in a fully classically quantised setting, the construction yields explicit expressions for the translational and gyrational flux pair $(\phi, \psi)$, the constitutive pair $(\epsilon_0, \mu_0)$, and the elementary scales $l_o$ and $t_o$. As we show in Section 4 and Remark 5.4, fixing $v = c$ through this quantised vacuum framework establishes an intrinsic vacuum flux quantum $\phi = h/(\varkappa e)$, whose domain of validity stands in clear physical contrast to the condensed-matter flux quantum $\Phi_0 = h/(2e)$ [3, 4] and the single-particle Aharonov–Bohm period $\Phi_{\mathrm{AB}} = h/e$ [5].

The elementary time scale also bounds the excitation spectrum. Its reciprocal supplies a finite ultraviolet cutoff that removes the classical Rayleigh–Jeans divergence and yields a genuine observational consequence, checked below against the strongest available ultra-high-energy photon bounds from the Pierre Auger and Telescope Array observatories. The elementary phase quantum also fixes the lowest admissible gyrational frequency and hence a strictly positive electromagnetic mass gap $\Delta_{\mathrm{EM}}$: the energy separating the non-gyrating translational boundary configuration from the first complete translational–gyrational excitation.

The ordering is deliberately non-circular. The elementary scales $l_o$ and $t_o$ are primitive; their constructed transportivity $\mathcal{T} = l_o^2/t_o^2$ fixes $c = \sqrt{\mathcal{T}}$ before any constitutive parameter is obtained. The later identities $\epsilon_0\mu_0 c^2 = 1$ is therefore an internal consistency result, not definitions of $c$.

The remainder of the article is organised as follows. Section 3 develops the algebraic closure $\mathfrak{M}$ and provides the vortical solution ansatz. Section 4 casts $\mathfrak{M}$ into a classically quantised vacuum, identifying the elementary voxel and the vacuum's elementary quantities $e$, $\mathcal{T}$, and $\mathcal{H}$. Section 5 derives the flux pair, the constitutive pair, and the elementary scales $l_o, t_o$ from this construction, before turning, in its final subsection, to their empirical calibration against measured constants. Section 6 draws out two further consequences of the construction: an ultraviolet regularisation of the classical spectrum, and an electromagnetic mass gap. Section 7 concludes and points to a companion article.

**Typographical convention** As a general guide, this article adheres to the following typographic conventions to distinguish between different mathematical entities:

1. Scalars and geometric vectors: Scalar quantities are represented by italic Roman or Greek characters. When overset with an arrow or a circumflex $(\vec{a}, \hat{a})$, they indicate geometric vectors and unit vectors, respectively. For geometric vectors, $\times$ and $\cdot$ denote the cross and dot products.

2. Continuum fields and field products: Bold upright characters indicate continuum fields. For field products we use $\boldsymbol{\times}$ and $\boldsymbol{\cdot}$ (field cross and field dot products), while $\boldsymbol{\nabla\times}$ and $\boldsymbol{\nabla\cdot}$ denote the standard field curl and divergence operators in the same typographic style.

## 2 Assumptions

The article presents the above programme as a formal mathematical construction, using the following assumptions:

**Assumption 2.1** (Vacuum transversality)**.** Throughout we work in vacuum in the sense of no externally prescribed free charge and no magnetic monopoles, and impose the Gauss constraints

$$\nabla \cdot \mathbf{E} = 0, \qquad \nabla \cdot \mathbf{B} = 0.$$

Equivalently, for a travelling-wave ansatz with wave vector $\vec{k}$, the fields are transverse:

$$\vec{k} \cdot \mathbf{E} = 0, \qquad \vec{k} \cdot \mathbf{B} = 0.$$

Additionally, the velocity field is assumed to be divergence-free:

$$\nabla \cdot \mathbf{v} = 0.$$

**Assumption 2.2** (Maxwell-compatible transport and admissible sector)**.** Throughout we restrict attention to an *admissible sector* in which the temporal evolution of the continuum vector fields is compatible with Lie transport along the field $\mathbf{v}$ in the inverse-flow sense. Lie transport is standard differential-geometric machinery [6]; its application to electromagnetic fields and, in particular, to Faraday transport is well established [7–9]. Concretely, for each continuum vector field $\mathbf{a}$ appearing in the translational Maxwell reconstruction (e.g. $\mathbf{E}, \mathbf{B}$), we impose the transport constraint

$$\frac{\partial \mathbf{a}}{\partial t} = \nabla \times (\mathbf{a} \times \mathbf{v}) \equiv \mathcal{L}_{\mathbf{v}} \mathbf{a} = (\mathbf{v} \cdot \nabla)\mathbf{a} - (\mathbf{a} \cdot \nabla)\mathbf{v}. \tag{1}$$

For scalar fields $f$, this reduces to the advection $\partial_t f = (\mathbf{v} \cdot \nabla) f$.

**Assumption 2.3** (Fluid-like vacuum transport)**.** The vacuum is assumed to possess fluid-like local transport properties. Accordingly, the kinematic concepts of continuum and fluid mechanics may be applied to disturbances of the vacuum, including local density, tensity, transport modulus, and propagating elementary volumes. This assumption concerns the transport structure of the vacuum and does not identify it with an ordinary material fluid.

**Assumption 2.4** (Nil-residue universe)**.** The universe $\mathcal{U}$ constitutes a closed physical system for which every globally additive conserved quantity has zero net value at every instant. Let $\mathscr{C}$ denote the set of all such quantities, including energy–momentum, angular momentum, electric charge, and any further conserved Noether or topological charges. For each $\mathcal{Q} \in \mathscr{C}$, we assume

$$\mathcal{Q}_{\mathcal{U}}(t) = 0 \qquad \text{for all } t. \tag{2}$$

Equivalently, these quantities may be collected without dimensional conflation into the universal ledger

$$\mathfrak{L}_{\mathcal{U}}(t) := \left(P^{\mu}_{\mathcal{U}}(t), J^{\mu\nu}_{\mathcal{U}}(t), Q_{\mathcal{U}}(t), \{\mathcal{Q}_{\mathcal{U}}(t)\}_{\mathcal{Q} \in \mathscr{C}}\right), \qquad \mathfrak{L}_{\mathcal{U}}(t) = \mathbf{0}, \tag{3}$$

where the zero applies componentwise.

For every partition

$$\mathcal{U} = A \cup A^*, \qquad A \cap A^* = \varnothing,$$

each additive ledger component satisfies

$$Q_A(t) + Q_{A^*}(t) = 0. \tag{4}$$

Consequently, any interaction-induced change in one part is accompanied by the complementary change in the remainder:

$$\delta Q_A(t) + \delta Q_{A^*}(t) = 0. \tag{5}$$

This balance holds identically and without temporal delay. It is a global constraint on the admissible state of the universe, rather than a local transport law or the passage of a compensating signal between its parts.

Noether's theorem associates a continuous symmetry with a conserved current and its corresponding charge. For any such quantity $Q$ belonging to a closed system $\mathcal{U}$, it gives

$$\frac{\mathrm{d}}{\mathrm{d}t} Q_{\mathcal{U}}(t) = 0.$$

This establishes invariance in time but does not determine the value of the invariant. In particular, Noether's theorem does not require that value to be zero.

The choice of a vanishing universal total has precedent in the zero-energy universe hypothesis. Tryon proposed that the positive energy associated with matter may be balanced by the negative gravitational contribution, leaving the net energy of the universe equal to zero [10]. This proposal does not establish the present assumption, but it demonstrates that a conserved universal quantity may consistently be assigned a vanishing total even though its constituent contributions are individually nonzero.

Assumption 2.4 extends this principle beyond energy. It requires every globally additive conserved quantity to have zero net universal residue:

$$Q_{\mathcal{U}}(t) = 0 \qquad \text{for every } Q \in \mathscr{C}.$$

The condition applies componentwise; energy–momentum, angular momentum, electric charge, and other conserved charges retain their distinct dimensions and tensor characters and are not added to one another as scalars. They are collected only as separate components of the universal ledger.

For a partition $\mathcal{U} = A \cup A^*$, this assumption further requires every change in the ledger contribution of $A$ to be balanced by the corresponding change in its complement:

$$\delta Q_A(t) + \delta Q_{A^*}(t) = 0.$$

The balance is imposed upon the admissible state of the closed universe at each instant. It is therefore stronger than ordinary conservation: it specifies the conserved total to be zero and requires that zero residue to be preserved under every partition. It does not describe a compensating signal travelling from one region to another.

Assumption 2.4 is not derived from Noether's theorem or from Tryon's proposal. It is introduced as a physical restriction on the admissible configurations of the universe. Its particular consequences are developed in the companion articles.

## 3 Algebraic Closure with Maxwellian Instantiation

**Definition 3.1** (Phased Complex Vector Space and Bilinear Form)**.** Let $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3)$ denote the standard canonical basis of $\mathbb{R}^3$. For real phase angles $\theta_1, \theta_2 \in \mathbb{R}$ and

$$\theta_3 = \theta_1 + \theta_2 \pmod{2\pi},$$

define the phase-shifted basis vectors in $\mathbb{C}^3$:

$$\hat{x} = e^{i\theta_1}\hat{\mathbf{e}}_1, \qquad \hat{y} = e^{i\theta_2}\hat{\mathbf{e}}_2, \qquad \hat{z} = e^{i\theta_3}\hat{\mathbf{e}}_3.$$

We define $\mathcal{S} \subset \mathbb{C}^3$ as the 3-dimensional complex vector space given by the complex linear span of this unit frame:

$$\mathcal{S} = \mathrm{span}_{\mathbb{C}}\{\hat{x}, \hat{y}, \hat{z}\} = \left\{\vec{a}_x + \vec{a}_y + \vec{a}_z \;\middle|\; \vec{a}_x = c_1\hat{x},\ \vec{a}_y = c_2\hat{y},\ \vec{a}_z = c_3\hat{z},\ c_1, c_2, c_3 \in \mathbb{C}\right\}.$$

Since the three phase factors are non-zero, the phased frame $\{\hat{x}, \hat{y}, \hat{z}\}$ is a basis of $\mathbb{C}^3$. Thus $\mathcal{S} = \mathbb{C}^3$ as a vector space, with the notation $\mathcal{S}$ retained to emphasise the distinguished phased frame and bilinear structure used below.

Furthermore, we equip $\mathcal{S}$ with the non-Hermitian symmetric bilinear form

$$\langle \cdot, \cdot \rangle : \mathcal{S} \times \mathcal{S} \to \mathbb{C},$$

defined for any $\vec{u}, \vec{v} \in \mathcal{S}$ by

$$\vec{u} \cdot \vec{v} = \langle \vec{u}, \vec{v} \rangle = \vec{u}^T \vec{v} = \sum_{k=1}^{3} u_k v_k.$$

The cross product is the bilinear extension to $\mathcal{S}$ of the standard Euclidean cross product. Under these products, the phase condition preserves the right-handed cyclic relations

$$\hat{z} = \hat{x} \times \hat{y}, \qquad \hat{y} = \frac{\hat{z} \times \hat{x}}{\hat{x} \cdot \hat{x}}, \qquad \hat{x} = \frac{\hat{y} \times \hat{z}}{\hat{y} \cdot \hat{y}}. \tag{6}$$

The relations in (6) motivate the following cyclic closure, which is taken as a primitive algebraic structure rather than introduced as a consequence requiring a separate theorem.

**Axiom 3.2** (Cyclic algebraic closure)**.** *Let*

$$(\vec{a}, \vec{b}, \vec{c}) \in \mathcal{S}^3, \qquad \vec{a} \cdot \vec{a} \neq 0, \qquad \vec{b} \cdot \vec{b} \neq 0.$$

*The non-null conditions $\vec{a} \cdot \vec{a} \neq 0$ and $\vec{b} \cdot \vec{b} \neq 0$ are part of the definition of an admissible triple. Such a triple is required to satisfy the cyclic closure*

$$\mathfrak{M}\left[\vec{a}, \vec{b}, \vec{c}\right] \iff \left\{\frac{\vec{b} \times \vec{c}}{\vec{b} \cdot \vec{b}} - \vec{a} = 0, \quad \frac{\vec{c} \times \vec{a}}{\vec{a} \cdot \vec{a}} - \vec{b} = 0, \quad \vec{a} \times \vec{b} - \vec{c} = 0\right\}. \tag{7}$$

*The symbol $\mathfrak{M}\left[\vec{a}, \vec{b}, \vec{c}\right]$ denotes the complete cyclic system of constraints* (7)*, rather than an operator acting on the triple.*

The three members are correspondingly related by the standard bilinear extension of the three-dimensional vector identities. An admissible triple is, however, a simultaneous solution of all three relations in (7). Thus, although any one relation may be used to express one member in terms of the other two, this does not by itself establish closure. For example, if $\vec{c} = \vec{a} \times \vec{b}$ is formed from prescribed $\vec{a}$ and $\vec{b}$, the resulting triple belongs to $\mathfrak{M}$ only if it also satisfies

$$\frac{\vec{b} \times \vec{c}}{\vec{b} \cdot \vec{b}} - \vec{a} = 0, \qquad \frac{\vec{c} \times \vec{a}}{\vec{a} \cdot \vec{a}} - \vec{b} = 0.$$

Hence the members of an admissible triple are constrained collectively by the cyclic closure rather than generated independently from an arbitrary pair.

**Corollary 3.3** (Closure forces orthogonality). *For $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$ with $\vec{a} \cdot \vec{a} \neq 0$, $\vec{b} \cdot \vec{b} \neq 0$, closure* (7) *forces $\vec{a} \cdot \vec{b} = 0$ — orthogonality is a consequence of the closure, not an assumption fed into it.*

*Proof.* With $\vec{c} = \vec{a} \times \vec{b}$, the triple-product identity gives $\vec{b} \times \vec{c} = \vec{a}(\vec{b} \cdot \vec{b}) - \vec{b}(\vec{a} \cdot \vec{b})$; substituting into $\vec{a} = (\vec{b} \times \vec{c})/(\vec{b} \cdot \vec{b})$ forces $\vec{a} \cdot \vec{b} = 0$. Conversely, once $\vec{a} \cdot \vec{b} = 0$, the same identity and its mirror $(\vec{a} \times \vec{b}) \times \vec{a} = \vec{b}(\vec{a} \cdot \vec{a}) - \vec{a}(\vec{a} \cdot \vec{b})$ return $(\vec{b} \times \vec{c})/(\vec{b} \cdot \vec{b}) = \vec{a}$ and $(\vec{c} \times \vec{a})/(\vec{a} \cdot \vec{a}) = \vec{b}$ — the third closure relation alone regenerates the other two. □

**Hypothesis 3.4** (Maxwellian instantiation hypothesis). *We hypothesise that a coherent electromagnetic field theory may be constructed from the cyclic algebraic closure* (7)*, provided that the closure admits a Maxwellian instantiation and a compatible transport law on the admissible vacuum sector, Assumption 2.1.*

We return to this hypothesis in Section 7, once the paper's results are in hand, to assess what they do and do not establish about it.

Under the Maxwellian instantiation hypothesis we pass from the geometric closure (7) to its continuum-field realisation, filling the slots

$$(\vec{a}, \vec{b}, \vec{c}) \longmapsto (\mathbf{v}, \mathbf{B}, \mathbf{E}).$$

Thus, over the space $\mathcal{S}$, the continuum-field triple $(\mathbf{v}, \mathbf{B}, \mathbf{E})$ constitutes, pointwise, an admissible solution triple of the same closure $\mathfrak{M}$ — the slot-filling introduces no new algebraic structure, only a specific choice of what fills it. For this dedicated translational equation set we introduce the label $\mathscr{M}[\mathbf{v}, \mathbf{B}, \mathbf{E}]$:

$$\mathscr{M}[\mathbf{v}, \mathbf{B}, \mathbf{E}] := \left\{ \mathbf{v}, \mathbf{B}, \mathbf{E} \in \mathcal{S} \;\middle|\; \frac{\mathbf{B} \times \mathbf{E}}{\mathbf{B} \cdot \mathbf{B}} - \mathbf{v} = 0, \quad \frac{\mathbf{E} \times \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} - \mathbf{B} = 0, \quad \mathbf{v} \times \mathbf{B} - \mathbf{E} = 0 \right\}. \tag{8}$$

**Proposition 3.5** (Reconstruction of the source-free Maxwell equations). *Under Assumption 2.1 (divergence-free $\mathbf{v}, \mathbf{B}, \mathbf{E}$) and Assumption 2.2 (the Maxwell-compatible transport law), the continuum-field instantiation $\mathscr{M}[\mathbf{v}, \mathbf{B}, \mathbf{E}]$ reconstructs the source-free Maxwell curl equations in this reconstructed form, for a general transport speed $v$ as yet unconstrained:*

$$\begin{aligned} \nabla \times \mathbf{B} &= \frac{1}{v^2} \nabla \times (\mathbf{E} \times \mathbf{v}) = \frac{1}{v^2} \partial_t \mathbf{E}, \\ \nabla \times \mathbf{E} &= \nabla \times (\mathbf{v} \times \mathbf{B}) = -\partial_t \mathbf{B}. \end{aligned} \tag{9}$$

*These become the literal vacuum Maxwell equations once $v = c$, established independently in Proposition 4.13.*

*Proof.* Writing $v^2 := \mathbf{v} \cdot \mathbf{v}$ and applying the transport constraint (1) with $\mathbf{a} = \mathbf{E}$ and then $\mathbf{a} = \mathbf{B}$, the second and third equations of $\mathfrak{M}$ give

$$\nabla \times \mathbf{B} = \frac{1}{v^2} \nabla \times (\mathbf{E} \times \mathbf{v}) = \frac{1}{v^2} \partial_t \mathbf{E}, \qquad \nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B}) = -\partial_t \mathbf{B},$$

which is (9). $\square$

$\mathfrak{M}$ is the algebraic closure; its translational Maxwellian instantiation $\mathscr{M}[\mathbf{v}, \mathbf{B}, \mathbf{E}]$ ((8)) describes the interaction of the velocity and magnetic fields with the translational electric field, while leaving the transport speed $v$ unfixed. It is the first, not the complete, Maxwellian realisation of the closure.

## 3.1 Solutions as gyrations

The Maxwell–Heaviside curl equations reconstructed above imply the d'Alembert operator

$$\square = \nabla^2 - v^{-2} \partial_t^2,$$

but the linear wave equation alone does not impose the cyclic, non-linear relations of $\mathfrak{M}$. It therefore admits a wider solution class than the closure: satisfying d'Alembert's equation is necessary for propagation in the present construction, but is not sufficient for membership of $\mathfrak{M}$. The closure supplies the additional restriction that the velocity and electromagnetic fields form an admissible rotating triple.

An admissible triple has the rotating representation

$$\begin{bmatrix} \mathbf{v} \\ \mathbf{B} \\ \mathbf{E} \end{bmatrix} = \operatorname{diag} \begin{bmatrix} v \\ B \\ vB \end{bmatrix} R \begin{bmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{bmatrix}, \tag{10}$$

where $v$ is the transport speed, $B$ is a reference magnetic amplitude, and $R = R_{\mathrm{z}} R_{\mathrm{y}} R_{\mathrm{x}}$, with

$$R_{\mathrm{z}} = \begin{bmatrix} 0 & 0 & 1 \\ \mathrm{c_z} & -\mathrm{s_z} & 0 \\ \mathrm{s_z} & \mathrm{c_z} & 0 \end{bmatrix}, \quad R_{\mathrm{y}} = \begin{bmatrix} \mathrm{c_y} & 0 & \mathrm{s_y} \\ 0 & 1 & 0 \\ -\mathrm{s_y} & 0 & \mathrm{c_y} \end{bmatrix}, \quad R_{\mathrm{x}} = \begin{bmatrix} 0 & \mathrm{c_x} & -\mathrm{s_x} \\ 0 & \mathrm{s_x} & \mathrm{c_x} \\ 1 & 0 & 0 \end{bmatrix}. \tag{11}$$

For each $\mathrm{i} \in \{\mathrm{x}, \mathrm{y}, \mathrm{z}\}$, introduce the normalised phase $\zeta_{\mathrm{i}} \in \mathbb{R}/\mathbb{Z}$ and its radian representation $\theta_{\mathrm{i}}$ by

$$\begin{aligned} &\mathrm{c_i} = \cos \theta_{\mathrm{i}}^{(\mathfrak{g},\mathfrak{d})} = \cos\!\big(2\pi \zeta_{\mathrm{i}}^{(\mathfrak{g},\mathfrak{d})}\big), \qquad \mathrm{s_i} = \sin \theta_{\mathrm{i}}^{(\mathfrak{g},\mathfrak{d})} = \sin\!\big(2\pi \zeta_{\mathrm{i}}^{(\mathfrak{g},\mathfrak{d})}\big), \\ &\zeta_{\mathrm{i}}^{(\mathfrak{g},\mathfrak{d})} = \gamma_{\mathrm{i}} s_{\mathrm{i}} + \mathfrak{g}_{\mathrm{i}} \nu_{\mathrm{i}} t \pmod 1, \qquad \theta_{\mathrm{i}}^{(\mathfrak{g},\mathfrak{d})} = 2\pi \zeta_{\mathrm{i}}^{(\mathfrak{g},\mathfrak{d})}, \qquad \gamma_{\mathrm{i}} = \frac{\mathfrak{g}_{\mathrm{i}} \nu_{\mathrm{i}}}{\mathfrak{d}_{\mathrm{i}} v_{\mathrm{i}}}, \qquad \mathfrak{g}_{\mathrm{i}}, \mathfrak{d}_{\mathrm{i}} \in \{-1, +1\}. \end{aligned} \tag{12}$$

Here $\zeta_{\mathrm{i}}$ records the fraction of a complete revolution, $\nu_{\mathrm{i}}$ is the cyclic-frequency parameter, $\omega_{\mathrm{i}} = 2\pi\nu_{\mathrm{i}}$ is its angular counterpart, and $s_{\mathrm{i}} = s_{\mathrm{i}}(t)$ is the distance travelled along the centreline, parametrised by

$$\dot{s}_{\mathrm{i}} = v_{\mathrm{i}}, \qquad s_{\mathrm{i}}(t) = s_{\mathrm{i}}(t_0) + \int_{t_0}^{t} v_{\mathrm{i}}(\tau)\, \mathrm{d}\tau.$$

The corresponding signed centreline displacement is $\mathfrak{d}_{\mathrm{i}} s_{\mathrm{i}} \hat{\mathbf{e}}_{\mathrm{i}}$. The signs are independent: $\mathfrak{g}_{\mathrm{i}}$ is the gyricity, defined by the signed angular frequency; $\mathfrak{d}_i$ is the directicity, selecting propagation along $+\mathbf{v}$ or $-\mathbf{v}$. Their product defines the helicity,

$$\mathfrak{h}_i := \mathfrak{g}_i\mathfrak{d}_i \in \{-1, +1\}.$$

which records whether the gyration is aligned or anti-aligned with the directed centreline. Thus each axis admits four states,

$$(\mathfrak{g}_i, \mathfrak{d}_i) \in \{(+,+), (+,-), (-,+), (-,-)\}.$$

For a transverse rotation about the propagation axis, the two gyricities may be written

$$\mathbf{B}_{\mathfrak{g}} = B\big(\hat{x}\cos\omega t - \mathfrak{g}\,\hat{y}\sin\omega t\big), \qquad \mathfrak{g} \in \{-1, +1\},$$

and either gyricity may propagate in either direction $\mathfrak{d} = \pm 1$. The handedness of the reference system is unchanged. The appendix records the $(\mathfrak{g}, \mathfrak{d}) = (+,+)$ example and establishes its doubled centreline phase in Appendix A.2, equation (56).

### 3.2 Physical interpretation: gyrational momentum

Since every factor $R_i$ is orthogonal, $R$ preserves dot products, cross products, and norms. It therefore preserves the orthogonality and norm relations imposed by $\mathfrak{M}$: a rotated admissible triple remains admissible. This restriction admits non-trivial electromagnetic field dynamics within the d'Alembert solution class.

A one-axis gyration produces helical fields along an open centreline. Two- and three-axis compositions produce topologically structured gyrational disturbances, including closed circular configurations and knotted fields. A three-axis composition may close around a sphere. We call this centreline *spherular*: it is a sphere-enclosing gyrational path that remains exterior to the enclosed sphere and does not cross its poles. These solutions are not selected by the d'Alembert equation alone; they arise when wave propagation is required simultaneously to satisfy the algebraic closure.

The rotational degrees of freedom introduce an additional electromagnetic property, which we call *gyrational momentum*. Relative to the propagation-adapted frame, we distinguish three forms:

- GAM: Gyrational angular momentum, whose signed scaling is determined by $\mathfrak{g}_z\nu_z$.
- GOM: Gyrational orbital momentum, whose signed scaling is determined by $\mathfrak{g}_y\nu_y$. GOM describes an orbital gyration of the disturbance itself and is distinct from the orbital angular momentum of structured light [11];
- GNM: Gyrational nutational momentum, whose signed scaling is determined by $\mathfrak{g}_x\nu_x$.

No counterpart to GOM and GNM as defined here exists in standard classical electrodynamics or quantum mechanics, because neither framework contains the finite-volume, three-axis gyrational degree of freedom introduced by this construction.

The factors $\mathfrak{g}_i\nu_i$ specify the signed frequency scaling of these momenta; their dimensional normalisation follows from the action scale derived below. This classification allows an emitted gyrational disturbance to carry rotational momentum lost by a source excitation, in addition to the momentum associated with centreline translation.

For the present work, we adopt the following physical identification.

**Assumption 3.6** (The photon as a travelling gyration)**.** The one-axis solution (10), with $R = R_z$, provides the basis for describing a photon. Gyricity $\mathfrak{g}_z$ specifies the sense of gyration in the reference frame, directicity $\mathfrak{d}_z$ specifies the propagation direction, and their product

$$\mathfrak{h}_z := \mathfrak{g}_z \mathfrak{d}_z \in \{-1, +1\}$$

defines the dimensionless photon helicity. Thus, under this identification, spin angular momentum is a manifestation of gyrational angular momentum.

This ansatz is developed further in the companion papers [12, 13].

# 4 Classically Quantised Vacuum

We now cast $\mathfrak{M}$ into a classically quantised model of the vacuum, identifying first the elementary voxel that fixes the vacuum's length and time scales, and then the vacuum's own elementary quantities.

**Axiom 4.1** (Vacuum dynamics)**.** *This is the paper's central vacuum ansatz. The elementary length $l_o$ and elementary time $t_o$ are independent primitive scales. Their constructed ratio $\mathcal{T} = l_o^2/t_o^2$ relates them and fixes the vacuum's propagation speed $c = \sqrt{\mathcal{T}}$, without using the constitutive identity $c^2 = (\epsilon_0 \mu_0)^{-1}$. Electric displacement is then taken as the primitive energetic variable of the source-free disturbance $\mathscr{W}$; the closure supplies its magnetic conjugate. From the same elementary quantities the action modulus $\mathcal{H}$ fixes the quantum of action. The constitutive parameters are derived only afterward, so their product recovers $c^2$ as a consistency result rather than defining it.*

## 4.1 Elementary Length and Time: The Voxel

**Definition 4.2** (Elementary length and time: the voxel)**.** We propose that there exist an elementary length $l_o > 0$ and an elementary time $t_o > 0$. Associated with these is the *voxel*: an elementary spatial unit $\mathscr{V} \subset \mathbb{R}^3$, defined as a quantised spatial element with volume $l_o^3$ and axial length $l_o$. Their ratio $l_o/t_o$ is identified with the transport speed as a consequence, not assumed here, once Definition 4.9 and Proposition 4.13 are in hand.

**Remark 4.3** (Historical context of discrete space and time)**.** The introduced elementary scales $(l_o, t_o)$ align with a long-standing tradition of addressing ultraviolet divergences and fundamental constants by discretising the continuum [14–16]. Heisenberg early on advocated for a fundamental length $l_0$ (a *Gitterwelt* or lattice world) to cut off field infinities and derive scale-dependent couplings [14], while Dirac repeatedly argued that the continuous space-time assumption must break down to resolve quantum electrodynamic divergences and account for dimensionless physical constants [15, 17]. Heisenberg's own conviction ran deeper still: writing to Pauli in November 1926, he held it "more than ever as completely out of the question that the world is continuous," arguing that velocity itself becomes meaningless at a point in a discontinuous world, since its definition requires a second point infinitely close to the first [18, p. 88]. Hermann, too, argued for a discretisation of physical interactions [19]. Subsequent formalisations, notably Snyder's non-commutative space-time, sought to embed discrete units into Lorentz-covariant frameworks [16]. Each of these routes to discreteness, however, proceeds through quantum mechanics or a quantum-motivated formalism; the present construction reaches $(l_o, t_o)$ by classical means alone, and the citations above are placed here as historical and philosophical context, not as inputs to the derivation given in this article. The construction above frames the voxel $(\mathscr{V}, l_o, t_o)$ not merely as a high-energy cutoff, but as an elementary energetic unit through which vacuum parameters and field bounds dynamically emerge.

**Definition 4.4** (Elementary vacuum disturbance)**.** The quantised disturbance of the vacuum associated with the elementary voxel $\mathscr{V}$ is denoted by

$$\mathscr{W}.$$

The voxel $\mathscr{V}$ denotes the elementary spatial support of $\mathscr{W}$, whereas $\mathscr{W}$ denotes the disturbance itself.

**Remark 4.5** (Understanding the voxel $\mathscr{V} \subset \mathbb{R}^3$)**.** Space is not divided into a fixed pixelated grid, from which the term "voxel" is conventionally derived. Rather, $\mathscr{V}$ represents the dynamic elementary volume occupied by the quantised disturbance $\mathscr{W}$.

## 4.2 The Vacuum's Elementary Quantities: $e$, $\mathcal{T}$, and $\mathcal{H}$

Three elementary quantities characterise the vacuum: the elementary charge $e$, a genuine primitive with nothing to calibrate; the transportivity (transport modulus) $\mathcal{T}$; and the action modulus $\mathcal{H}$. We introduce $e$ first, as a primitive axiom in its own right, before constructing $\mathcal{T}$ and $\mathcal{H}$ from the elementary voxel quantities introduced above.

**Axiom 4.6** (Elementary charge, a vacuum primitive)**.** *The elementary charge $e$, in coulombs, is accepted as the sole primitive vacuum quantity of the present construction: unlike the transportivity $\mathcal{T}$ and the action modulus $\mathcal{H}$ introduced below, $e$ is not a constructed algebraic ratio and admits no independent numerical calibration.*

**Hypothesis 4.7** (Primitive-displacement hypothesis)**.** *Definition 4.4 identifies a quantised disturbance $\mathscr{W}$, whose elementary spatial support is the voxel $\mathscr{V}$. In the absence of externally prescribed laboratory sources, we propose that electric displacement is not a response to an antecedent applied field but the primitive energetic variable of $\mathscr{W}$. Its nontrivial quantised state displaces the neutral voxel into a polarised state. This reversed ordering is termed the* excitation sector*; the electromagnetic fields are generated from the displacement carried by $\mathscr{W}$.*

What follows develops an explicit excitation sector consistent with the assumed dynamics of $\mathscr{W}$.

We return to this hypothesis in Section 7, once the paper's results are in hand, to assess what it does and does not establish about it.

**Definition 4.8** (Quantised electric displacement)**.** The excitation $\mathscr{W}$ is a binary state of the voxel: unexcited or excited, $\epsilon_\mathrm{E} \in \{0, 1\}$. We identify $\mathscr{W}$ with the nontrivial state, $\epsilon_\mathrm{E} := 1$. Conjugate to this excitation indicator is the voxel's elementary displacement modulus $\iota e / l_o^2$, distributed over its cross-sectional area $l_o^2$; together these define the voxel's quantised electric displacement, where $\iota > 0$ is an initially undetermined dimensionless normalisation constant,

$$D_\mathrm{E} := \frac{\iota e}{l_o^2}\, \epsilon_\mathrm{E}. \tag{13}$$

**Definition 4.9** (Transportivity and the propagation scale $c^2$)**.** From the voxel's own elementary quantities $e, l_o, t_o$, the same excitation indicator $\epsilon_{\mathrm{E}} := 1$ fixes a charge-density and a charge-tensity,

$$M_\eta := \frac{\iota e}{l_o^3}, \qquad K_\eta := \frac{\iota e}{l_o\, t_o^2}. \tag{14}$$

Both are analytical tools: bookkeeping combinations built directly from the voxel's own elementary quantities $e, l_o, t_o$, with no standing of their own beyond that computation.

We then define the vacuum's constructed propagation modulus, or *transportivity*, by

$$\mathcal{T} := \frac{K_\eta}{M_\eta} = \frac{\iota e/(l_o\, t_o^2)}{\iota e/l_o^3} = \frac{l_o^2}{t_o^2}. \tag{15}$$

Accordingly, the characteristic propagation speed is defined by

$$c := \sqrt{\mathcal{T}} = \frac{l_o}{t_o}, \tag{16}$$

so that

$$\mathcal{T} \equiv c^2. \tag{17}$$

Equivalently,

$$l_o = c\, t_o, \tag{18}$$

and hence $t_o = l_o/c$. In this ordering, $\epsilon_0$ and $\mu_0$ are recovered later as derived effective constitutive parameters of the voxel model.

**Corollary 4.10** (The vacuum's own length and time scales)**.** *Neither $l_o$ nor $t_o$ is defined by the other; both are primitive scales, and $\mathcal{T}$ fixes their relation. The same charge-density and charge-tensity give their reciprocal squared representations*

$$\Gamma_\eta := \frac{l_o}{e} K_\eta = \frac{1}{t_o^2}. \tag{19}$$

*and*

$$\Lambda_\eta := \frac{l_o}{e} M_\eta = \frac{1}{l_o^2}. \tag{20}$$

*Their ratio reproduces the transportivity itself,*

$$\frac{\Gamma_\eta}{\Lambda_\eta} = \frac{K_\eta}{M_\eta} = \mathcal{T} \equiv c^2, \tag{21}$$

*while their positive square roots recover the primitive cyclic frequency and inverse length,*

$$\nu_o := \sqrt{\Gamma_\eta} = \frac{1}{t_o}, \tag{22}$$

*and*

$$l_o = \frac{c}{\nu_o} = \frac{1}{\sqrt{\Lambda_\eta}}. \tag{23}$$

*These are representations of the posited scales, not an independent derivation of them. In the normalised-phase coordinate introduced in Section 3.1, one elementary revolution has $\nu_o = 1/t_o$; its radian repre-*

*sentation is $\omega_o = 2\pi\nu_o = 2\pi/t_o$, so that $\omega_o^2 = 4\pi^2\Gamma_\eta$. The factor $4\pi^2$ is the coordinate conversion between normalised-cycle and radian representations, not a second independently posited vacuum modulus.*

**Definition 4.11** (Elementary angle: the vacuum's phase quantum)**.** Let $t_{\mathrm{SI}} := 1$ second denote the SI second, the metrological reference against which the vacuum's own elementary time $t_o$ is measured. We propose that this same elementary time, read against $t_{\mathrm{SI}}$ rather than against itself, fixes an elementary angle

$$\varpi_o := 2\pi\,\frac{t_o}{t_{\mathrm{SI}}}. \tag{24}$$

**Remark 4.12** (Reading $\varpi_o$)**.** Unlike $l_o$ and $t_o$, $\varpi_o$ carries no physical dimension: $t_o$ and $t_{\mathrm{SI}}$ are both times, so their ratio is a pure number, and the factor $2\pi$ reads that number as an angle rather than leaving it as a bare ratio. $\varpi_o$ answers a definite question: how much phase would a reference rotation of exactly one cycle per SI second, $2\pi/t_{\mathrm{SI}}$, sweep out during a single elementary tick $t_o$? Equivalently, $\varpi_o$ is the vacuum's own tick $t_o$ re-expressed on an angular rather than a temporal scale, just as Corollary 4.10 re-expresses $t_o$ as the reciprocal of the primitive frequency $\nu_o$.

**Proposition 4.13** (Magnetic displacement and transport speed)**.** *The magnetic displacement is defined by the excitation-sector instantiation of Axiom 3.2's* (7) *itself, filling its slots with the scalars* $(\upsilon, D_{\mathrm{B}}, D_{\mathrm{E}})$ *rather than the continuum-field triple* $(\mathbf{v}, \mathbf{B}, \mathbf{E})$ *— the normed/scalar reduction of the vector closure to an admissible orthogonal triple, under which the cross and dot products of* (7) *collapse to ordinary multiplication and division:*

$$\mathfrak{M}\,[\upsilon, D_{\mathrm{B}}, D_{\mathrm{E}}] \quad\Longleftrightarrow\quad \left\{\frac{D_{\mathrm{E}}}{D_{\mathrm{B}}} - \upsilon = 0, \quad \frac{D_{\mathrm{E}}}{\upsilon} - D_{\mathrm{B}} = 0, \quad \upsilon\, D_{\mathrm{B}} - D_{\mathrm{E}} = 0\right\}.$$

*The three relations are not independent — each is the single statement $D_{\mathrm{E}} = \upsilon\, D_{\mathrm{B}}$, rearranged — and it is the second of them,*

$$D_{\mathrm{B}} := \frac{D_{\mathrm{E}}}{\upsilon}, \tag{25}$$

*that defines $D_{\mathrm{B}}$: no independent physical grounding is claimed for it, unlike $D_{\mathrm{E}}$ — it is a closure-native quantity, fixed entirely by what $\mathfrak{M}$ requires once $D_{\mathrm{E}}$ and $\upsilon$ are given. Here $\upsilon$ is the characteristic transport speed of the medium, here the vacuum, and therefore satisfies*

$$\upsilon = c. \tag{26}$$

*Proof.* Definition 4.9 independently constructs the vacuum's own characteristic propagation speed, $c := \sqrt{\mathcal{T}} = l_o/t_o$, from $K_\eta/M_\eta$ alone — built purely from the voxel's elementary quantities $e, l_o, t_o$, with no dependence on $D_{\mathrm{B}}$, $D_{\mathrm{E}}$, or $\upsilon$ itself. The vacuum admits only one such characteristic speed: $\upsilon$ and $c$ are two descriptions of the same quantity, identified here rather than solved for algebraically — (25) cannot itself be solved for $\upsilon$, since it defines $D_{\mathrm{B}}$ in terms of $\upsilon$, not the other way round. With $\upsilon = c$, $D_{\mathrm{B}}$ closes on the voxel's elementary quantities alone,

$$D_{\mathrm{B}} = \frac{D_{\mathrm{E}}}{c} = \frac{\iota e/l_o^2}{l_o/t_o} = \frac{\iota e\, t_o}{l_o^3}.$$

The instantiation $\mathfrak{M}[\upsilon, D_{\mathrm{B}}, D_{\mathrm{E}}]$ above is thus resolved into $\mathfrak{M}[\upsilon{=}c, D_{\mathrm{B}}, D_{\mathrm{E}}]$. □

Thus $D_{\mathrm{E}}$ is the posited primitive displacement of the excitation sector, whereas $D_{\mathrm{B}} = D_{\mathrm{E}}/c$ is its closure-required magnetic conjugate.

**Definition 4.14** (Electromagnetic action and coupling constants)**.** Associated with the elementary excitation $\mathscr{W}$, we define four further quantities:

$\not{p}$ *Elementary electric momentum:*

$$\not{p} := e\,c \;\left[\mathrm{C\,m\,s^{-1}}\right].$$

$\hbar$ *Elementary electric action:*

$$\hbar := \not{p}\,l_o = e\,c\,l_o \;\left[\mathrm{C\,m^2\,s^{-1}}\right].$$

$\not{f}$ *Electric-to-mechanical conversion factor:*

$$\not{f} := 1 \;\left[\mathrm{kg\,C^{-1}}\right].$$

$\varkappa$ *Electric-to-mechanical coupling constant:* $\varkappa$ is the dimensionless scalar relating the elementary electric action to mechanical action. Its value is fixed independently, via one of the two calibration routes of Axiom 5.5 (through $\not{g}$ or $\alpha$); see Definition 4.16 below for its role in constructing the action modulus $\mathcal{H}$.

**Remark 4.15** (Dimensional hierarchy of the elementary action)**.** The quantities of Definition 4.14 sit in the dimensional hierarchy

$$e \xrightarrow{\times c} \not{p} \xrightarrow{\times l_o} \hbar \xrightarrow{\times \not{f}\,\varkappa} h, \qquad \not{p} = ec, \quad \hbar = ecl_o,$$

with $\not{f}$ converting dimension and $\varkappa$ fixing scale:

$$h = \not{f}\,\varkappa\,\hbar = \not{f}\,\varkappa\,ecl_o. \tag{27}$$

Definition 4.16 below promotes this relation to a closure-native modulus $\mathcal{H}$, calibrated directly to $h$.

**Definition 4.16** (Action modulus and the scale $\hbar^2$)**.** From the charge-tensity $K_\eta$ of Definition 4.9 we define the *action-tensity*

$$\mathcal{K}_\hbar := (e\,l_o)^2\,K_\eta. \tag{28}$$

This construction mirrors action as momentum times distance, $\hbar = \not{p}\,l_o$ (Definition 4.14): $K_\eta$ carries the charge but not the transport rate; the missing factor is supplied only once we divide by $M_\eta$, whose ratio $K_\eta/M_\eta = \mathcal{T}$ (Definition 4.9) equals $v^2$ by Proposition 4.13. Dividing $\mathcal{K}_\hbar$ by that same $M_\eta$, we define the vacuum's action modulus

$$\mathcal{H} := \frac{\mathcal{K}_\hbar}{M_\eta} = (e\,l_o)^2\,\frac{K_\eta}{M_\eta} = (e\,l_o)^2\,\mathcal{T} = (e\,c\,l_o)^2 = \hbar^2. \tag{29}$$

$\mathcal{H}$ is built purely from the vacuum's elementary quantities, of dimension $Q^2L^4T^{-2}$, matching $[\hbar^2]$; conversion to mechanical action by (27) gives $h = \not{f}\,\varkappa\sqrt{\mathcal{H}}$.

## 5 Derived Electromagnetic Quantities

From the vacuum's elementary quantities above, the closure $\mathfrak{M}$ facilitates the determination of the flux quanta, the vacuum constitutive parameters, and the elementary length and time scales.

**Definition 5.1** (Magnetic-flux coupling)**.** The magnetic flux associated with the elementary disturbance $\mathscr{W}$ is defined by its proportionality to the elementary electric momentum $\wp$ (Definition 4.14); we write

$$\phi := \chi \wp = \chi \kappa e c,$$

where $\chi$ is a dimensional proportionality.

**Theorem 5.2** (Constructive parameterisation of electromagnetic quantities)**.** *Within the quantised voxel framework specified by Axiom 4.6, Definition 4.2, Hypothesis 4.7, Definitions 5.1, 4.8, 4.9, 4.14, and 4.16, and Proposition 4.13, the following quantities admit explicit algebraic expressions:*

- $\phi$ *the magnetic flux quantum,*
- $\psi$ *the electric flux quantum,*
- $\epsilon_0$ *the vacuum permittivity,*
- $\mu_0$ *the vacuum permeability,*
- $t_o$ *the elementary time unit,*
- $l_o$ *the elementary length unit.*

*These quantities are obtained within a single quantised Maxwellian framework determined by e, $h = \ell\, \kappa \sqrt{\mathcal{H}}$, and the propagation scale $c = \sqrt{\mathcal{T}}$, with the remaining dimensionless coupling*

- $\kappa$ *the electric-to-mechanical coupling constant,*

*fixed retrospectively by either calibration route of Axiom 5.5. The proof additionally invokes three identifications not listed as separate hypotheses: the flux product $\phi\psi$ as an action-density carrier, the closure's normed divisors as accumulators for the physical constants they multiply, and a quantisation principle fixing each dimensionless accumulator to unity once the elementary quantities are set to their unit values.*

*Proof.* Within a quantised voxel, and in a quantised unit system, we identify the fluxes with the fields acting on the elementary surface $l_o^2$ through the one-to-one correspondence

$$\vec{\phi} \longmapsto \mathbf{B}, \qquad \vec{\psi} \longmapsto \mathbf{E}.$$

Expressing the first equation of $\mathfrak{M}$ in norm form then gives

$$\mathbf{v} = \frac{\mathbf{B} \times \mathbf{E}}{\mathbf{B} \cdot \mathbf{B}} \quad \longmapsto \quad \mathbf{v} = \frac{\vec{\phi} \times \vec{\psi}}{\vec{\phi} \cdot \vec{\phi}} \quad \Rightarrow \quad c = \frac{\phi\psi}{\phi^2}.$$

By interpreting $\vec{\phi} \times \vec{\psi}$ as an action-density carrier and multiplying the norm expression by $h/c$, we obtain

$$\left[\frac{h}{\phi^2 c}\right] \phi\psi = h. \tag{30}$$

In the normed representation of $\mathfrak{M}$, the divisor may be treated as a physical constant. Accordingly, the bracketed factor in (30) acts as an accumulator for a physical constant to be determined.

Thus, (30) states that a physical constant multiplying the product of the electric and magnetic flux yields an action.

By Definition 4.8, Proposition 4.13, and $\mathfrak{M}$, the propagating voxel carries the elementary charge $e$ at velocity $c$. Definitions 4.14 and 4.16, together with Remark 4.15, give the translational electric action as momentum times distance. We write

$$h = \ell \kappa e c l_o. \tag{31}$$

By Definition 5.1, the magnetic flux $\phi$ associated with the elementary excitation is proportional to the elementary electric momentum,

$$\phi = \chi \kappa e c.$$

Substituting this relation into (30) gives

$$\left[\frac{h}{\phi^2 c}\right] [\chi]\, \kappa e c\, \psi = h.$$

Using the third equation of $\mathfrak{M}$ in norm form gives $\psi = c\phi$, and by substitution the above becomes

$$\left[\frac{h}{\phi^2 c}\right] [\chi]\, \kappa e c^2 \phi = h. \tag{32}$$

In the quantised unit representation, the numerical values of the elementary quantities are normalised to unity,

$$\bar{\phi} = \bar{e} = \bar{h} = \bar{c} = 1.$$

The dimensionless accumulator multiplying the elementary product must therefore satisfy $[\bullet][\circ]\bar{c}^2 = 1$. Restoring the corresponding dimensional elementary units gives $\kappa e \phi = h$, hence

$$\phi = \frac{h}{\kappa e}, \qquad \psi = \frac{ch}{\kappa e}. \tag{33}$$

Substituting $\phi$ into (32) gives

$$\left[\frac{\kappa^2 e^2}{hc}\right] [\chi]\, c^2 = 1, \tag{34}$$

from which

$$[\chi] = \frac{h}{\kappa^2 c e^2}$$

The constants appearing in the quantisation condition are therefore identified as the vacuum constitutive parameters

$$\epsilon_0 := \frac{\kappa^2 e^2}{hc}, \qquad \mu_0 := \frac{h}{\kappa^2 c e^2}, \tag{35}$$

namely the vacuum permittivity and permeability. The opening action-carrier relation (30) therefore takes the derived form

$$\epsilon_0 \phi \psi = h. \tag{36}$$

Moreover, (34) gives

$$\epsilon_0 \mu_0 c^2 = 1.$$

The numerical value of $\kappa$ is then fixed retrospectively by one of the two routes of Axiom 5.5: either by setting the dimensionless numerical representation of $\mu_0$ equal to the Giorgi calibration constant $\mathcal{g}$, or by setting $\kappa^2 = 1/(2\alpha)$ from the measured fine-structure constant $\alpha$. The numerical values reported in Section 5.1 below use the latter route, with CODATA 2022 inputs.

Finally, from (31) we recover the elementary length and time scales,

$$l_o = \frac{h}{\ell \kappa^2 ec}, \qquad t_o = \frac{l_o}{c}. \tag{37}$$

$\square$

**Proposition 5.3** (Displacement normalisation)**.** *Maxwell's* equations of electric elasticity, *group (E), fix the displacement normalisation of Definition 4.8 as*

$$\iota = \kappa, \qquad D_{\mathrm{E}} = \frac{\kappa e}{l_o^2}. \tag{38}$$

*Proof.* In contemporary notation, Maxwell's group (E) is $\mathbf{D} = \epsilon_0 \mathbf{E}$. With $E = \psi/l_o^2$, together with (33) and (35), it gives

$$D_{\mathrm{E}} = \epsilon_0 E = \left(\frac{\kappa^2 e^2}{hc}\right)\left(\frac{ch}{\kappa e\, l_o^2}\right) = \frac{\kappa e}{l_o^2}.$$

Comparison with $D_{\mathrm{E}} = \iota e/l_o^2$ fixes $\iota = \kappa$. Thus Maxwell's constitutive relation determines the previously free normalisation; it is not used in the derivation of $\epsilon_0$, $\psi$, or $\kappa$. $\square$

Thus, given the elementary charge $e$ and the quantised vacuum specified above, the closure yields the electromagnetic flux, constitutive relation, and elementary space–time scale rather than taking them as phenomenological inputs. In this precise sense, electrodynamics emerges from $\mathfrak{M}$ within the stated construction.

**Remark 5.4** (Comparison with established electromagnetic constants)**.** At first sight, the magnetic flux quantum obtained in (33),

$$\phi = \frac{h}{\kappa e},$$

appears to be in tension with standard established constants, namely the superconducting flux quantum $\Phi_0 = h/(2e)$ resulting from Cooper pairs [3, 4] and the fundamental Aharonov–Bohm phase periodicity $\Phi_{\mathrm{AB}} = h/e$ [5]. However, these quantities arise in fundamentally different physical contexts and from distinct theoretical inputs; their numerical divergence is therefore not a contradiction, but a reflection of their distinct domains of applicability.

While $\Phi_0$ and $\Phi_{\mathrm{AB}}$ govern condensed-matter bound states and single-particle transport respectively, the flux quantum $\phi$ derived here is an intrinsic property of the bare vacuum. It is obtained constructively from our classically quantised vacuum ansatz (Section 4) together with the algebraic closure $\mathfrak{M}$. Here, the electromechanical coupling constant $\kappa$ ties $\phi$ directly to the fine-structure constant $\alpha$ in a pure vacuum, entirely devoid of any condensed matter.

## 5.1 Empirical Calibration

The construction above is entirely symbolic: no measured numerical constants have yet been inserted into it. The elementary charge $e$ remains a symbolic primitive throughout; the unit-conversion constant $\ell := 1$ is fixed by convention, not measurement; and the transportivity $\mathcal{T}$, the action modulus $\mathcal{H}$, and the coupling $\varkappa$ remain calibrated only symbolically, awaiting the measured values of $c$, $h$, and $\text{\textit{g}}$ or $\alpha$. We now import these measured constants as a single empirical calibration, the first point in the article at which any measured number enters.

**Axiom 5.5** (Empirical calibration). *The transportivity $\mathcal{T}$ (Definition 4.9) is calibrated numerically as*

$$\mathcal{T} = (299792458)^2\ [L^2T^{-2}],$$

*fixing $c = 299792458\ [LT^{-1}]$. The action modulus $\mathcal{H}$ (Definition 4.16) is calibrated jointly with the unit-conversion constant $\ell$ and the coupling $\varkappa$ so that $h = \ell\varkappa\sqrt{\mathcal{H}}$ reproduces the measured Planck constant $h = 6.626\,070\,15 \times 10^{-34}$ J s. The elementary charge itself enters the numerical evaluations below at its exact defined SI value (post-2019 redefinition), $e = 1.602\,176\,634 \times 10^{-19}$ C, alongside c, h, and $g$ or $\alpha$.*

*For the dimensionless electric-to-mechanical coupling constant $\varkappa$ (Definition 4.14), we admit either of two independent calibration routes:*

- $g$ — *the Giorgi magnetic calibration scalar $4\pi \times 10^{-7}$ (pre-2019 SI convention, under which the dimensionless numerical representation of $\mu_0$ equals $g$ exactly); or*
- $\alpha$ — *the measured fine-structure constant, via $\varkappa^2 = 1/(2\alpha)$ (CODATA, independent of any SI convention).*

*Since the 2019 SI redefinition, $\mu_0$ is no longer exactly $g$, so the two routes need not agree beyond current measurement precision. The present article adopts the latter throughout, using CODATA 2022 values. This calibration route for $\varkappa$ is independent of $h$: it fixes $\varkappa$ from $g$ or $\alpha$ alone, so that $\varkappa$ is already available before $\mathcal{H}$ is calibrated above, and the calibration of $\mathcal{H}$ to $h$ does not form a circular loop. The unit-conversion constant $\ell$ (Definition 4.14) is independent for the same reason, but trivially so: it is fixed by convention, not measurement.*

With the calibration choices $\ell = 1$ and $\varkappa^2 = 1/(2\alpha)$, the symbolic relations (37) take the explicit form

$$l_o = \frac{2\alpha h}{ec}, \qquad t_o = \frac{2\alpha h}{ec^2}. \tag{39}$$

Quantifying the newly defined elementary quantities using 2022 CODATA values [20], with $\varkappa$ fixed via the fine-structure calibration route of Axiom 5.5, yields

$$\begin{aligned}
\varkappa^2 &= 68.517\,999\,5888(103),\\
\varkappa &= 8.277\,560\,002\,13(62),\\
l_o &= 2.013\,354\,536\,97(39) \times 10^{-25} \quad \text{metres},\\
t_o &= 6.715\,827\,844\,39(102) \times 10^{-34} \quad \text{seconds}, \qquad \text{with } l_o = c\,t_o.
\end{aligned}$$

The elementary time scale defines the angular frequency corresponding to one complete cycle during one elementary propagation interval,

$$\omega_{\max} := \frac{2\pi}{t_o} = \frac{2\pi c}{l_o} = 9.355\,786\,736\,60(141) \times 10^{33} \quad \text{radians per second}. \tag{40}$$

Thus, during one period $2\pi/\omega_{\max} = t_o$, a disturbance propagating at $c$ advances by exactly one elementary length $l_o$. If $t_o$ is taken as the minimum physically resolvable temporal interval of the quantised vacuum, $\omega_{\max}$ constitutes the corresponding upper bound on physically meaningful rotational frequency.

# 6 Spectral Consequences

The principal structural result above is that an admissible disturbance through $\mathfrak{M}$ has translational and gyrational components. These components belong to the same disturbance and occur together; neither is a second instantiation of the closure. The Einstein–Planck relation converts the frequency bound of the resulting classically quantised vacuum into an energy bound. We now consider two spectral consequences: an ultraviolet regularisation of the classical spectrum and an electromagnetic mass gap.

## 6.1 Ultraviolet Regularisation

**Remark 6.1** (Ultraviolet regularisation)**.** The existence of the elementary time $t_o$, and hence of the finite upper angular frequency

$$\omega_{\max} = \frac{2\pi}{t_o},$$

also removes the ultraviolet divergence of classical electromagnetic mode counting. In the Rayleigh–Jeans limit [21], the electromagnetic mode density per unit angular frequency is

$$g(\omega) = \frac{\omega^2}{\pi^2 c^3},$$

and classical equipartition assigns the mean energy $k_{\mathrm{B}}T$ to each mode. With an unbounded frequency spectrum the resulting energy density diverges. In the present quantised vacuum, however,

$$u(T) = \int_0^{\omega_{\max}} k_{\mathrm{B}}T\, g(\omega)\, d\omega = \frac{k_{\mathrm{B}}T}{3\pi^2 c^3}\omega_{\max}^3,$$

which is finite. Thus the elementary space–time scale provides a natural ultraviolet cutoff for the classical electromagnetic spectrum. This regularisation removes the Rayleigh–Jeans divergence; the derivation of the corresponding black-body spectral distribution lies beyond the scope of the present article.

**Remark 6.2** (Testable prediction: ultra-high-energy photon bound)**.** The finite ultraviolet bound $\omega_{\max}$ has an immediate observational consequence: no source-free electromagnetic excitation $\mathscr{W}$ — no photon — should exist with energy exceeding

$$\hbar\omega_{\max} \approx 0.987 \text{ J} \approx 6.16 \times 10^{18} \text{ eV}.$$

This places the present construction within reach of existing ultra-high-energy cosmic-ray observatories. The Pierre Auger Observatory reports a 95% CL upper limit on the diffuse photon flux of $1.72\times10^{-4}$ km$^{-2}$sr$^{-1}$yr$^{-1}$ above $4\times10^{19}$ eV, from data spanning January 2004 to June 2020 [22]. The Telescope Array Collaboration reports a 95% CL upper limit of $1.3\times10^{-3}$ km$^{-2}$sr$^{-1}$yr$^{-1}$ above $10^{20}$ eV, from nine years of surface-detector data [23]. Neither collaboration reports a confirmed photon primary at these energies; both figures are non-detection upper limits.

These thresholds lie above $\hbar\omega_{\text{max}}$ by roughly 0.8 (Pierre Auger) and 1.2 (Telescope Array) orders of magnitude, so the absence of observed ultra-high-energy photons is consistent with, though not yet a stringent test of, the cutoff predicted here. This comparison carries a caveat: the classification of an air shower as photon- rather than hadron-induced is inferred statistically, from observables such as the depth of shower maximum, muon content, and shower-front curvature, rather than from direct identification of the primary particle, so the reported limits could in principle be affected by misidentified massive-particle events.

Even so, this constitutes a hard, falsifiable observational target of the present work: a confirmed ultra-high-energy photon primary substantially above $\hbar\omega_{\text{max}}$ would directly falsify the construction proposed here, whereas improved photon/hadron discrimination reaching closer to $\hbar\omega_{\text{max}}$ itself would provide the sharpest available test.

## 6.2 The Electromagnetic Mass Gap

The single closure describes a disturbance with inseparable translational and gyrational components. Its solution at zero gyrational frequency, $\nu = 0$, remains mathematically admissible. In that limit the displacement current and radiant flux vanish, so the solution is not a radiative electromagnetic quantum of the positive-frequency branch.

Definition 4.11 gives the elementary phase quantum

$$\varpi_o = 2\pi \frac{t_o}{t_{\text{SI}}}.$$

We hypothesise that the first gyrational excitation advances by at least this phase during the reference interval $t_{\text{SI}}$.

**Hypothesis 6.3** (Lowest vortex-nucleation frequency under SI quantisation)**.** *Its lowest admissible cyclic and angular frequencies are*

$$\nu_{\text{min}} := \frac{\varpi_o}{2\pi t_{\text{SI}}} = \frac{t_o}{t_{\text{SI}}^2} > 0, \qquad \omega_{\text{min}} := 2\pi\nu_{\text{min}} = \frac{2\pi t_o}{t_{\text{SI}}^2}. \tag{41}$$

Thus $\nu = 0$ is a boundary solution, not a member of the hypothesised positive-frequency gyrational spectrum.

Using the Einstein–Planck relation, provisionally adopted here and derived in the companion article as Corollary II.6.1 (Energy content of a quantised disturbance $\mathcal{W}$), the energy of a gyrational electromagnetic excitation is

$$\mathcal{E} = h\nu.$$

Associated with the lowest admissible gyrational frequency is therefore the irreducible onset energy

$$\Delta_{\text{EM}} := h\nu_{\text{min}} = \frac{h\, t_o}{t_{\text{SI}}^2}. \tag{42}$$

The quantity $\Delta_{\text{EM}}$ is the hypothesised first vortex-nucleation energy. It is in this specific sense that $\Delta_{\text{EM}}$ constitutes an electromagnetic mass gap.

The first admissible state has

$$\mathcal{E}_{\text{min}} = \Delta_{\text{EM}}. \tag{43}$$

Further excitation occurs in integer multiples of the elementary frequency interval. Define

$$\delta\nu_n := n\nu_{\min}, \qquad n \in \mathbb{N}_{>0}. \tag{44}$$

The corresponding frequencies and energies above the first state are

$$\nu_n = \nu_{\min} + \delta\nu_n = (n+1)\nu_{\min}, \qquad \mathcal{E}_n = \Delta_{\mathrm{EM}} + h\delta\nu_n = (n+1)\Delta_{\mathrm{EM}}. \tag{45}$$

The hypothesised lower bound and the derived upper bound therefore delimit the proposed positive-frequency gyrational spectrum:

$$0 < \nu_{\min} \leq \nu \leq \nu_{\max}.$$

Using the elementary time obtained in Section 5.1 gives

$$\Delta_{\mathrm{EM}} = 4.449\,954\,6412 \times 10^{-67} \quad \text{joule}, \tag{46}$$

and

$$\omega_{\min} = 4.219\,679\,0837 \times 10^{-33} \quad \text{radians per second}. \tag{47}$$

The very small value of this threshold does not distinguish radio radiation from higher-frequency electromagnetic radiation: every radiative quantum has non-zero frequency and lies within the same proposed gyrational spectrum.

**Remark 6.4** (The zero-frequency solution and a possible EM phonon)**.** The admissible $\nu = 0$ solution has no displacement current and zero radiant flux. We may hypothesise that this non-radiative configuration is an EM phonon. We may further hypothesise that its quantised energy content equals the first vortex-nucleation energy $\Delta_{\mathrm{EM}}$. Neither physical identification follows from the closure or from the vanishing- current and vanishing-flux result, and neither is claimed as proved here.

**Remark 6.5** (Plane waves, polarisation, and electromagnetic quanta)**.** The plane-wave representation describes the collective field profile of electromagnetic radiation; it does not introduce a second species of quantum associated with translation. In particular, radio transmission is electromagnetic radiation and is represented here by quantised translational–gyrational excitations, not by a succession of phonons.

The transverse fields may be complex-valued, with their relative complex phases encoding the rotation within the transverse plane. The observable real projection determines the corresponding polarisation:

$$\mathbf{B}_{\mathrm{phys}} := \operatorname{Re}\mathbf{B}, \qquad \mathbf{E}_{\mathrm{phys}} := \operatorname{Re}\mathbf{E},$$

Linear, elliptical, and circular polarisation arise from the relative amplitudes and phases of the projected transverse components. This supplies a physical motivation for constructing the closure on $\mathcal{S} \subset \mathbb{C}^3$: the complex space retains the phase information required by the rotating field, while its real projection supplies the observable polarisation.

Propagation and transverse rotation are respectively the translational and gyrational components of the same disturbance through $\mathfrak{M}$. A macroscopic radio signal contains many such electromagnetic quanta. At fixed carrier frequency, modulation of the radiated energy flux changes their occupation, or mean number, within the field envelope. The familiar continuous plane wave is consequently the many-quantum field representation, rather than evidence for an unquantised or phonon-like propagating branch. Likewise, a discrete atomic transition at radio frequency emits an electromagnetic quantum with the same inseparable components.

## 7 Conclusion

This article has shown that the algebraic closure $\mathfrak{M}$ restricts electromagnetic disturbances to admissible rotating triples. The d'Alembert equation supplies their wave propagation but does not, by itself, impose the non-linear closure that selects their gyration. Within that restricted solution class, the translational and gyrational components occur together and admit non-trivial vortical and helical field configurations. We then cast the closure into a classically quantised model of the vacuum, obtaining an explicit electromagnetic parameterisation and fixing the transport speed $v$ left free by the curl equations. These aims have been met within the stated assumptions.

The transport speed is fixed not by the constitutive identity $c^2 = (\epsilon_0\mu_0)^{-1}$, but by the vacuum's own constructed propagation modulus, the transportivity $\mathcal{T} = l_o^2/t_o^2$ (Definition 4.9): the magnetic displacement $D_{\mathrm{B}}$ (Proposition 4.13) is defined directly by the excitation-sector closure's own second relation, $D_{\mathrm{B}} := D_{\mathrm{E}}/v$ — a closure-native quantity, not an independently-grounded one like $D_{\mathrm{E}}$; $v$ here is the medium's characteristic transport speed. Definition 4.9 independently constructs the vacuum's own characteristic propagation speed, $c = \sqrt{\mathcal{T}}$, from $M_\eta$ and $K_\eta$ alone, with no dependence on $D_{\mathrm{B}}$, $D_{\mathrm{E}}$, or $v$ itself. Since the vacuum admits only one such characteristic speed, $v$ is identified with $c$ (Proposition 4.13) — not solved for algebraically, which would be circular given how $D_{\mathrm{B}}$ is defined, but identified as the same quantity reached by two independent constructions. The familiar relation $c^2 = (\epsilon_0\mu_0)^{-1}$ then reappears only as an internal consistency check on the subsequently derived constitutive parameters, not as the premise defining $c$. In the same way, Planck's constant $h$ is fixed not by an independent axiomatic input, but by the vacuum's own constructed action modulus $\mathcal{H}$ (Definition 4.16), built from the same charge-tensity and charge-density that give $\mathcal{T}$: $h = \ell\,\kappa\sqrt{\mathcal{H}} = \ell\,\kappa\, ecl_o$, with $\ell$ carrying only the unit convention fixing the kilogram and $\kappa$ carrying the sole physical calibration, fixed independently via $\alpha$ or $g$.

Casting $\mathfrak{M}$ into the elementary voxel framework further yields, from the primitive quantity $e$, the two calibrated moduli $\mathcal{T}$ and $\mathcal{H}$, and the independently calibrated dimensionless coupling $\kappa$, the flux quanta $\phi = h/(\kappa e)$ and $\psi = ch/(\kappa e)$, the constitutive parameters $\epsilon_0$ and $\mu_0$, and the elementary scales $l_o$ and $t_o$ (Theorem 36). This coupling $\kappa$ is fixed by one of two retrospective calibration routes, the Giorgi constant or the fine-structure constant (Axiom 5.5); this article adopts the latter throughout.

The elementary time scale bounds the excitation spectrum at both ends, but the two bounds have different logical status. $\omega_{\mathrm{max}} = 2\pi/t_o$ caps the positive-frequency gyrational excitation spectrum: it removes the Rayleigh–Jeans divergence and, as noted in Remark 6.2, corresponds to a photon energy already within about an order of magnitude of the strongest ultra-high-energy photon bounds reported by the Pierre Auger and Telescope Array observatories. The emergent electromagnetic mass gap $\Delta_{\mathrm{EM}} > 0$, by contrast, fixes the threshold frequency $\omega_{\mathrm{min}}$ at which a topologically distinct vortex excitation can nucleate (Hypothesis 6.3). The admissible $\nu = 0$ solution has neither displacement current nor radiant flux; its possible interpretation as an EM phonon, and the assignment to it of a quantised energy equal to the first nucleation energy, remain explicit hypotheses (Remark 6.4).

Two hypotheses introduced along the way remain, by design, open. The Maxwellian instantiation hypothesis (Hypothesis 3.4) is supported in the sense that a consistent Maxwell-compatible transport law exists, compatible with the transportivity construction's identification of $\upsilon = c$ (Proposition 4.13); it is not established that this is the unique, or physically necessary, instantiation of the closure. The excitation-sector hypothesis (Hypothesis 4.7) — that displacement, not field, is the elementary energetic excitation of the vacuum — is supported by the fact that a complete electromagnetic parameterisation follows from it, but the hypothesis itself remains a physical postulate rather than a derived necessity.

The present construction should accordingly be read as a self-consistent mathematical model, not a uniqueness proof: the speed of light, electromagnetic flux quanta, constitutive parameters, elementary space–time scales, and spectral bounds all arise within one algebraically closed structure, and the ultraviolet bound in particular yields a genuine, falsifiable observational target.

The remark on ultraviolet regularisation above (Remark 6.1) explicitly deferred the derivation of the black-body spectral distribution. The flux norm itself is no longer without a stated meaning: Remark 5.4 shows $e^2\phi^2 = \ell^2\mathcal{H}$, so the squared flux quantum is already a direct algebraic image of the action modulus $\mathcal{H}$ (Definition 4.16). What remains open here is the physical content of the photon energy relation $E = hf$ itself, and the black-body spectrum it implies. The companion article (Part II) [12], revisiting Maxwell's original 1865 formulation, closes both: Theorem II.7.1 (Planck-shaped decay envelope from relative resistivity) derives the black-body spectrum from a single characteristic vacuum resistivity, Corollary II.6.1 (Energy content of a quantised disturbance $\mathcal{W}$) obtains $E = hf$ as a theorem rather than a heuristic postulate, and Lemma II.5.2 (Electric–magnetic symmetry and Lie-transport substitutions) identifies the flux pair $(\phi, \psi)$ as one instance of a general electric–magnetic symmetry law within $\mathfrak{M}$. All three results are deferred to that subsequent work.

# A Trajectory-Level Travelling Whirl Precursor

This appendix records a one-dimensional construction that anticipates the rotating-field evolution

$$\begin{bmatrix} \mathbf{v} \\ \mathbf{B} \\ \mathbf{E} \end{bmatrix} = \operatorname{diag}\begin{bmatrix} \upsilon \\ B \\ \upsilon B \end{bmatrix} R(t) \begin{bmatrix} \hat{\mathrm{x}} \\ \hat{\mathrm{y}} \\ \hat{\mathrm{z}} \end{bmatrix}. \tag{48}$$

## A.1 One-dimensional d'Alembert equation and centreline consistency

Consider the one-dimensional d'Alembert wave equation

$$\frac{\partial^2 \mathcal{W}}{\partial x^2} - \frac{1}{\upsilon^2}\frac{\partial^2 \mathcal{W}}{\partial t^2} = 0, \tag{49}$$

where $\upsilon$ is the signed centreline speed in the selected direction. We study a travelling centreline $x = x(t)$ and impose a *centreline-consistency* requirement: the wave admits compatible representations when restricted to that centreline. Specifically, we require the logical conjunction

$$\mathcal{W} = \begin{cases} \text{(a):} \quad F(x) = f(x)^2 & \text{AND} \\ \text{(b):} \quad G(t) = g(t)^2 & \text{AND} \\ \text{(c):} \quad H(x,t) = f(x)g(t). \end{cases} \tag{50}$$

Here (a) and (b) are the spatial and temporal descriptions, while (c) is their separable spacetime representation.

For (50)(c),

$$\frac{\partial^2 \mathcal{W}}{\partial x^2} = g(t)\frac{d^2 f(x)}{dx^2}, \qquad \frac{\partial^2 \mathcal{W}}{\partial t^2} = f(x)\frac{d^2 g(t)}{dt^2}.$$

Substitution in (49), followed by division by $f(x)g(t)$, gives

$$\frac{v^2}{f(x)}\frac{d^2 f(x)}{dx^2} - \frac{1}{g(t)}\frac{d^2 g(t)}{dt^2} = 0. \tag{51}$$

The two terms depend separately on $x$ and $t$, so each equals a constant. Introducing the angular frequency $\omega$, let

$$\frac{v^2}{f(x)}\frac{d^2 f(x)}{dx^2} = \frac{1}{g(t)}\frac{d^2 g(t)}{dt^2} = -\frac{\omega^2}{4}, \tag{52}$$

or equivalently

$$\frac{1}{f(x)}\frac{d^2 f(x)}{dx^2} = -\frac{\omega^2}{4v^2}, \qquad \frac{1}{g(t)}\frac{d^2 g(t)}{dt^2} = -\frac{\omega^2}{4}. \tag{53}$$

A convenient complex-form solution is

$$f(x) = \exp\left(\frac{i\omega x}{2v}\right), \qquad g(t) = \exp\left(\frac{i\omega t}{2}\right). \tag{54}$$

To enforce centreline consistency between the spatial and temporal descriptions in (50)(a)–(b), we impose $f(x(t)) = g(t)$, which yields the travelling condition $x(t) = vt$. Squaring and introducing an amplitude $B$, we obtain the centreline-consistent family

$$\mathcal{W}_x = \begin{cases} \text{(a):} \quad B\exp(ikx) & \text{AND} \\ \text{(b):} \quad B\exp(i\omega t) & \text{AND} \\ \text{(c):} \quad B\exp\left[i\left(\frac{kx}{2} + \frac{\omega t}{2}\right)\right]. \end{cases} \quad \text{AND} \quad x = vt, \tag{55}$$

where

$$k := \frac{\omega}{v}, \qquad \nu := \frac{\omega}{2\pi}, \qquad \gamma := \frac{\nu}{v}, \qquad k = 2\pi\gamma.$$

### A.2 Analogy with rotating-field evolution

The travelling condition $x = vt$ is the one-dimensional analogue of fixing a centreline direction by the constant-velocity field $\mathbf{v}$, while the complex phase in $\mathcal{W}_x$ provides an internal rotation along that centreline. This admits the rotating-field representation

$$R(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix},$$

which mirrors the role of $R(t)$ in (48): a travelling centreline together with an internal rotation encoded by the time-dependent field evolution. The logical conjunction in (55) allows the transverse rotation to be represented by the single phase

$$\theta := kx + \omega t. \tag{56}$$

On the centreline $x = vt$, where $\omega = kv$, this gives

$$\theta = 2\omega t, \qquad \exp(i\theta/2) = \exp(i\omega t).$$

Equivalently, in the normalised phase convention used in Section 3.1,

$$\zeta := \frac{\theta}{2\pi} = \gamma x + \nu t, \qquad \zeta\big|_{x=vt} = 2\nu t.$$

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