Classically Quantised Vacuum Electrodynamics, Part II: Maxwell's 1865 Monopole Recast and a Classical Derivation of the Einstein–Planck Relation

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Abstract

Maxwell's 1865 field equations are built entirely on bipolar, laboratory-sourced quantities: displacement as a response to field, sourced and terminated at a remote potential difference. We show that Maxwell's own equations — prior to Heaviside's later reduction to curl-equation form — admit a second, monopole reading: an excitation sector in which displacement is itself the primitive, quantised excitation of a single quantised spatial volume (voxel), identified with the elementary disturbance of the companion article's algebraic closure $\mathfrak{M}$. Constructing this excitation sector requires, and thereby supplies, a Beltrami-field representation of $\mathfrak{M}$'s transverse solutions and a general electric–magnetic symmetry law absent from Maxwell's own system. Because the companion article already fixes the speed of light c and Planck's constant h as ratios of the vacuum's own primitive quantities, not as imported empirical constants, every quantum relation built on this recast system follows as a theorem rather than a postulate: the Einstein–Planck relation $\mathcal{E} = h\nu$ is derived, not assumed, and a Planck-shaped spectral envelope is obtained for resistive decay, without identifying that envelope with a temperature. The same resistive mechanism yields a specific, falsifiable prediction: a hyperbolic frequency redshift in a static, non-driven resistive medium, distinct from known plasma and collisional effects.

1 Introduction

In the companion article [1] — whose results are cited with the prefix I — we introduced the closure $\mathfrak{M}$ (Axiom I.3.2) and its Maxwellian instantiation on $(\mathbf{v}, \mathbf{B}, \mathbf{E})$,

$$\mathscr{M}[\mathbf{v}, \mathbf{B}, \mathbf{E}] := \left\{ \mathbf{v}, \mathbf{B}, \mathbf{E} \in \mathcal{S} \;\middle|\; \mathbf{v} = \frac{\mathbf{B} \times \mathbf{E}}{\mathbf{B} \cdot \mathbf{B}}, \quad \mathbf{B} = \frac{\mathbf{E} \times \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}}, \quad \mathbf{E} = \mathbf{v} \times \mathbf{B} \right\}, \tag{1}$$

over a complex space $\mathcal{S} \subset \mathbb{C}^3$ with a non-Hermitian symmetric bilinear product. Under the vacuum-transversality assumption of Assumption I.2.1,

$$\nabla \cdot \mathbf{v} = 0, \qquad \nabla \cdot \mathbf{B} = 0, \qquad \nabla \cdot \mathbf{E} = 0, \tag{2}$$

the closure $\mathfrak{M}$ forces the triple $(\mathbf{v}, \mathbf{B}, \mathbf{E})$ to be pairwise orthogonal, Corollary I.3.3

$$\mathbf{v} \cdot \mathbf{B} = 0, \qquad \mathbf{B} \cdot \mathbf{E} = 0, \qquad \mathbf{E} \cdot \mathbf{v} = 0, \tag{3}$$

not as a further assumption but as a consequence already established there. Under inverse Lie transport

$$\frac{\partial \mathbf{a}}{\partial t} = \nabla \times (\mathbf{a} \times \mathbf{v}) \equiv \mathcal{L}_{\mathbf{v}}\mathbf{a} = (\mathbf{v} \cdot \nabla)\mathbf{a} - (\mathbf{a} \cdot \nabla)\mathbf{v}. \tag{4}$$

the algebraic vector closure $\mathfrak{M}$ for the three vector fields $\mathbf{v}, \mathbf{B}, \mathbf{E}$ under inverse Lie transport reconstructs the source-free Maxwell equations as $\mathfrak{M}$ evolves, (see Proposition I.3.5):

$$\begin{aligned} \nabla \times \mathbf{B} &= \frac{1}{v^2}\, \nabla \times (\mathbf{E} \times \mathbf{v}) = \frac{1}{v^2}\, \partial_t \mathbf{E}, \\ \nabla \times \mathbf{E} &= \nabla \times (\mathbf{v} \times \mathbf{B}) = -\partial_t \mathbf{B}. \end{aligned} \tag{5}$$

Furthermore, that work [1] classically quantises $\mathcal{S}$: see Definition I.4.2, which defines an elementary length and time $\{l_o, t_o\} > 0$, and Definition I.4.9, which defines a transport modulus establishing $t_o = l_o/c$. Definition I.4.16 (Action modulus and the scale $\hbar^2$) likewise defines an action modulus $\mathcal{H}$, whose square root fixes h as a further vacuum-native quantity, on the same footing as c. Associated with these is the *voxel*: an elementary spatial unit $\mathscr{V} \subset \mathbb{R}^3$, defined as a quantised spatial element with volume l_o^3 and axial length l_o. Definition I.4.4 associates with $\mathscr{V}$ a quantised disturbance $\mathscr{W}$.

Here we recast Maxwell's original 1865 field equations [2] — his own potential-and-current formulation, prior to the reduction to curl-equation form recovered above — and not only the closure $\mathfrak{M}$ that motivates them. Because Maxwell's own equations are built on potentials rather than fields alone, the recast first requires a potential representation of the closure's transverse solutions: Section 4 shows these to be Beltrami fields — curl-eigenfunctions admitting self-referential vector potentials — supplying exactly the structure Maxwell's formulation demands. On this basis, Maxwell's equations admit two readings. In the *laboratory sector*, the one Maxwell himself worked with, displacement is a response to an applied field, sourced and terminated at a remote, potential-differenced boundary. In the *excitation sector* developed here, the same equations are read with displacement itself as the primitive, quantised excitation of a single voxel, identified with the disturbance $\mathscr{W}$ introduced above. Section 5 develops this correspondence equation by equation; doing so both requires and thereby delivers a general electric–magnetic symmetry law — pairing every electric quantity algebraically with its magnetic counterpart, absent from Maxwell's own system. From this recast system, and because c and h are already fixed, by the companion article's moduli, as ratios of the vacuum's own primitive quantities rather than imported constants, we derive independently the Einstein–Planck relation $\mathcal{E} = h\nu$ (Section 6) and a Planck-shaped spectral envelope for resistive decay (Section 7) — the latter obtained for monochromatic decay from a discrete-loss mechanism, without appeal to Boltzmann's statistical methods. The same mechanism also yields a specific, falsifiable prediction, stated in the Conclusion.

The nil-residue universe of Assumption I.2.4 supplies the global preservation principle for these resistive transfers: an excitation lost from one contribution to the universal ledger must be balanced elsewhere. The excitation-sector dynamics developed below identify the neighbouring voxel as the recipient and determine the resulting frequency decay and spectral form.

2 Maxwell's medium and the missing monopole structure

The unification of [2] is built entirely on laboratory fields: charges and currents fixed as boundary data, potentials and displacements that exist only in relation to a remote source and an oppositely-signed sink. A capacitor's D, a coil's B, a plane wave's (E, B) pair — every configuration the field equations (A)–(H) actually solve for is bipolar in this sense, sourced at one location and terminated at another, governed throughout by the potential difference between the two. Nothing in this apparatus describes a field structure that exists, self-contained, at a single point.

Yet Maxwell did not regard the field equations as the whole story of the medium's constitution. In the mechanical investigation that preceded them, he modelled the medium as a sea of molecular vortices [3]: "the excess of pressure in the equatorial direction arises from the centrifugal force of vortices or eddies in the medium having their axes in directions parallel to the lines of force." This was not an idle picture — it was the mechanism from which the displacement current, and with it the closed system of field equations, was derived. Maxwell credits the seed of the idea to Thomson [4] directly: "Professor Thomson has pointed out that the cause of the magnetic action on light must be a real rotation going on in the magnetic field" [3].

But the vortex, once made explicit, inherits the same polarity as the field equations it underlies. Maxwell states this himself: "every vortex is essentially dipolar, the two extremities of its axis being distinguished by the direction of its revolution as observed from those points" [3]. So neither of Maxwell's two pictures — the abstract field equations of 1865, or the mechanical vortex medium that motivated them — contains a genuinely localised, self-sourcing structure. One requires two potential-differenced surfaces; the other, even at the level of a single rotating cell, is dipolar by Maxwell's own description (See Remark 5.1). A monopole electromagnetic-field structure — an excitation existing at one place, sourced by nothing beyond itself — is missing from classical electromagnetism entire, not merely from its field-equation formulation.

This is the structure the excitation-sector hypothesis of the companion paper [1] proposes, identified there with the elementary voxel excitation $\mathcal{W}$. What follows recasts Maxwell's original 1865 formulation itself — not only the closure $\mathfrak{M}$ that motivates it — so that this monopole structure becomes explicit within Maxwell's own equations.

3 The algebraic closure $\mathfrak{M}$

The object of the present paper is to show that Maxwell's original 1865 formulation — prior to its later Heaviside recasting, and independently of any restriction to the bipolar reading — admits a second, equally legitimate instantiation of the same closure, on the monopole excitation quantities. From this point on, then, there are two sets of Maxwell 1865 equations under discussion: the original, laboratory-field set using the known notation $(\mathbf{B}, \mathbf{E}, \mathbf{D}, \mathbf{J}, \mathbf{A}, \Psi)$, and its monopole recast using the notation $(\widetilde{\mathbf{E}}, \widetilde{\mathbf{B}}, \widetilde{\mathbf{D}}, \widetilde{\mathbf{J}}, \widetilde{\mathbf{A}}, \widetilde{\Psi})$, with subscript modifies assigning the quantity to either the electric or magnetic domain, eg. $\widetilde{\mathbf{A}}_{\mathrm{E}}$ and $\widetilde{\mathbf{A}}_{\mathrm{B}}$ being the vector potentials for the electric and magnetic domains, respectively. We refer to these as the *laboratory sector* and the *excitation sector* throughout, and develop the correspondence between them equation by equation in Section 5.

We recall from [1] the closure's Maxwellian instantiation

$$\mathscr{M}[\mathbf{v},\mathbf{B},\mathbf{E}] := \left\{\mathbf{v},\mathbf{B},\mathbf{E}\in\mathcal{S}\;\middle|\;\mathbf{v}=\frac{\mathbf{B}\times\mathbf{E}}{\mathbf{B}\cdot\mathbf{B}},\quad \mathbf{B}=\frac{\mathbf{E}\times\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}},\quad \mathbf{E}=\mathbf{v}\times\mathbf{B}\right\}. \tag{6}$$

together with the result, established there, that Lie transport of (6) yields the source-free Maxwell equations in their modern, Heaviside curl-equation form.

Remark 3.1 (Uniform instantiation of the closure)**.** Referencing Axiom I.3.2 and Definition I.4.8 (Quantised electric displacement): The axiom places no restriction on the physical identity of its slot-fillers. The definition already exhibits a second instantiation of the closure, in normed (scalar) form, via $(v, D_{\mathrm{B}}, D_{\mathrm{E}})$; here we work with the underlying continuum vector fields directly, of which those normed quantities are the magnitudes. The closure applies unchanged to any admissible electromagnetic vector pair $(\mathbf{X}_{\mathrm{B}},\mathbf{X}_{\mathrm{E}})$ sharing the transport velocity $\mathbf{v}$:

$$\mathfrak{M}\{\mathbf{v},\mathbf{X}_{\mathrm{B}},\mathbf{X}_{\mathrm{E}}\}\;:\quad \mathbf{v}=\frac{\mathbf{X}_{\mathrm{B}}\times\mathbf{X}_{\mathrm{E}}}{\mathbf{X}_{\mathrm{B}}\cdot\mathbf{X}_{\mathrm{B}}},\quad \mathbf{X}_{\mathrm{B}}=\frac{\mathbf{X}_{\mathrm{E}}\times\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}},\quad \mathbf{X}_{\mathrm{E}}=\mathbf{v}\times\mathbf{X}_{\mathrm{B}}.$$

Below, $\mathbf{X}$ ranges over the monopole excitation triples $\widetilde{\mathbf{D}}$, $\widetilde{\mathbf{J}}$, and $\widetilde{\mathbf{A}}$, each recast from Maxwell's corresponding bipolar, laboratory-frame quantity.

4 Potentials and Beltrami Fields

The $\mathfrak{M}$ closure admits many solutions. Here we specifically adopt the simplest solution: writing in the shorthand $\mathrm{c}_\vartheta=\cos\vartheta$ and $\mathrm{s}_\vartheta=\sin\vartheta$,

$$\begin{bmatrix}\mathbf{v}\\ \widetilde{\mathbf{B}}\\ \widetilde{\mathbf{E}}\end{bmatrix}=\operatorname{diag}\begin{bmatrix}c\\ B\\ cB\end{bmatrix}\overbrace{\begin{bmatrix}0&0&1\\ \mathrm{c}_\vartheta&-\mathrm{s}_\vartheta&0\\ \mathrm{s}_\vartheta&\mathrm{c}_\vartheta&0\end{bmatrix}}^{R_{EB}}\begin{bmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{bmatrix}, \tag{7}$$

where for a travelling wave solution

$$\vartheta=\gamma\vec{s}-\nu t,\qquad \gamma=\frac{\nu}{c},\qquad \lambda=\frac{1}{\gamma},$$

where $\vec{s}$ is a distance along the z axis, and $d_t\vec{s}=\mathbf{v}$. Alternatively for the excited voxel solution, we use

$$\vartheta=\gamma\vec{s}+\nu t,\qquad \gamma=\frac{\nu}{c},\qquad \lambda=\frac{1}{\gamma},$$

The doubled phase in ϑ is explained in the companion article, Appendix I.A.2 (Analogy with rotating-field evolution), equation (56).

The triple $(\mathbf{v},\widetilde{\mathbf{B}},\widetilde{\mathbf{E}})$ is pairwise orthogonal at every point – a forced consequence of the closure itself, Corollary I.3.3 (Closure forces orthogonality), not an extra assumption – which confines $\widetilde{\mathbf{B}},\widetilde{\mathbf{E}}$ to the plane transverse to $\mathbf{v}$. Within that plane the two fields carry a further, stronger property, established directly below.

Proposition 4.1 (Beltrami eigenstructure of the transverse pair)**.** *For the solution* (7)*, the transverse* *fields $\widetilde{\mathbf{B}}, \widetilde{\mathbf{E}}$ are eigenfunctions of the curl operator sharing a common eigenvalue $\gamma = \nu/c$,*

$$\nabla \times \widetilde{\mathbf{B}} = \gamma\, \widetilde{\mathbf{B}}, \qquad \nabla \times \widetilde{\mathbf{E}} = \gamma\, \widetilde{\mathbf{E}}, \tag{8}$$

the defining property of a Beltrami field [5, 6]. This holds independently of the sign convention distinguishing the travelling-wave and excited-voxel branches of ϑ, since only $\partial_z \vartheta = \gamma$ enters the curl. By contrast, $\mathbf{v}$ is spatially uniform, so $\nabla \times \mathbf{v} = 0$ identically.

Proof. Write $\widetilde{\mathbf{B}} = B(\mathrm{c}_\vartheta, -\mathrm{s}_\vartheta, 0)$ and $\widetilde{\mathbf{E}} = cB(\mathrm{s}_\vartheta, \mathrm{c}_\vartheta, 0)$, functions of z alone through $\vartheta = \gamma z \mp \nu t$. For any planar field $(F_x(z), F_y(z), 0)$, $\nabla \times (F_x, F_y, 0) = (-\partial_z F_y, \partial_z F_x, 0)$. Applying this to $\widetilde{\mathbf{B}}$ and using $\partial_z \vartheta = \gamma$ gives $\nabla \times \widetilde{\mathbf{B}} = \gamma B(\mathrm{c}_\vartheta, -\mathrm{s}_\vartheta, 0) = \gamma \widetilde{\mathbf{B}}$; the identical computation on $\widetilde{\mathbf{E}}$ gives $\nabla \times \widetilde{\mathbf{E}} = \gamma \widetilde{\mathbf{E}}$. $\mathbf{v} = c\hat{z}$ has no spatial dependence, so $\nabla \times \mathbf{v} = 0$. □

Proposition 4.2 (Zero-vorticity boundary potentials)**.** *At the non-gyrational boundary $\nu = 0$, the self-referential Beltrami representation $\widetilde{\mathbf{A}} = \lambda \widetilde{\mathbf{B}}$, with $\lambda = c/\nu$, is not defined. Constant translational fields nevertheless possess the regular symmetric-gauge potentials*

$$\begin{bmatrix} \widetilde{\mathbf{A}}_v \\ \widetilde{\mathbf{A}}_{\mathrm{B}} \\ \widetilde{\mathbf{A}}_{\mathrm{E}} \end{bmatrix} = \frac{1}{2}\,\mathrm{diag}\begin{bmatrix} c \\ \lambda_0 B \\ c\lambda_0 B \end{bmatrix} \begin{bmatrix} -y & x & 0 \\ 0 & -z & y \\ z & 0 & -x \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}, \tag{9}$$

where λ_0 is a finite reference length, not c/ν. Direct differentiation gives constant curls $c\hat{z}$, $\lambda_0 B\hat{x}$, and $c\lambda_0 B\hat{y}$, respectively. Since the displacement currents are proportional to ν, both radiant-flux densities vanish at this boundary. It therefore supports the purely translational branch but is not a member of the positive-frequency Beltrami eigensector.

Because $\lambda = 1/\gamma$, Proposition 4.1 immediately supplies self-referential (Beltrami) vector potentials for the transverse pair,

$$\widetilde{\mathbf{A}}_{\mathrm{B}} := \lambda \widetilde{\mathbf{B}}, \qquad \widetilde{\mathbf{A}}_{\mathrm{E}} := \lambda \widetilde{\mathbf{E}}, \qquad \text{so that} \qquad \nabla \times \widetilde{\mathbf{A}}_{\mathrm{B}} = \widetilde{\mathbf{B}}, \qquad \nabla \times \widetilde{\mathbf{A}}_{\mathrm{E}} = \widetilde{\mathbf{E}}, \tag{10}$$

recovering Maxwell's equation group (B), that is $\mathbf{B} = \nabla \times \mathbf{A}$ – which is conceptually rooted in Michael Faraday's "electro-tonic state." Since $\mathbf{v}$ is curl-free but uniform, it instead takes the standard symmetric-gauge potential $\mathbf{A}_v := \frac{1}{2}(\mathbf{v} \times \vec{r})$, for which $\nabla \times \mathbf{A}_v = \mathbf{v}$ by the usual identity for a uniform field. Collecting all three,

$$\begin{bmatrix} \mathbf{v} \\ \widetilde{\mathbf{A}}_{\mathrm{B}} \\ \widetilde{\mathbf{A}}_{\mathrm{E}} \end{bmatrix} = \mathrm{diag}\begin{bmatrix} c \\ \lambda B \\ c\lambda B \end{bmatrix} \overbrace{\begin{bmatrix} 0 & 0 & 1 \\ \mathrm{c}_\vartheta & -\mathrm{s}_\vartheta & 0 \\ \mathrm{s}_\vartheta & \mathrm{c}_\vartheta & 0 \end{bmatrix}}^{R_{EB}} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}, \tag{11}$$

and

$$\begin{bmatrix} \widetilde{\mathbf{A}}_v \\ \widetilde{\mathbf{A}}_{\mathrm{B}} \\ \widetilde{\mathbf{A}}_{\mathrm{E}} \end{bmatrix} = \operatorname{diag}\begin{bmatrix} c \\ B \\ cB \end{bmatrix} \overbrace{\begin{bmatrix} -y/2 & x/2 & 0 \\ \lambda \mathrm{c}_\vartheta & -\lambda \mathrm{s}_\vartheta & 0 \\ \lambda \mathrm{s}_\vartheta & \lambda \mathrm{c}_\vartheta & 0 \end{bmatrix}}^{R_A} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}, \tag{12}$$

Lemma 4.3 (Existence of Beltrami vector potentials for $\mathfrak{M}$)**.** *For the one-axis case R_{EB}, the field components of $\mathfrak{M}$ are recovered from vector potentials $\widetilde{\mathbf{A}}_{\mathrm{v}}$, $\widetilde{\mathbf{A}}_{\mathrm{B}}$, and $\widetilde{\mathbf{A}}_{\mathrm{E}}$ by the curl operator:*

$$\begin{bmatrix} \mathbf{v} \\ \widetilde{\mathbf{B}} \\ \widetilde{\mathbf{E}} \end{bmatrix} = \nabla \times \begin{bmatrix} \widetilde{\mathbf{A}}_{\mathrm{v}} \\ \widetilde{\mathbf{A}}_{\mathrm{B}} \\ \widetilde{\mathbf{A}}_{\mathrm{E}} \end{bmatrix} \quad \text{with} \quad \overbrace{\begin{bmatrix} 0 & 0 & 1 \\ \mathrm{c}_\vartheta & -\mathrm{s}_\vartheta & 0 \\ \mathrm{s}_\vartheta & \mathrm{c}_\vartheta & 0 \end{bmatrix}}^{R_{EB}} = \nabla \times \overbrace{\begin{bmatrix} -y/2 & x/2 & 0 \\ \lambda \mathrm{c}_\vartheta & -\lambda \mathrm{s}_\vartheta & 0 \\ \lambda \mathrm{s}_\vartheta & \lambda \mathrm{c}_\vartheta & 0 \end{bmatrix}}^{R_A}. \tag{13}$$

Proof. Immediate from Proposition 4.1 and the symmetric-gauge potential for $\mathbf{v}$: the $\widetilde{\mathbf{B}}, \widetilde{\mathbf{E}}$ rows follow from (10), and the $\mathbf{v}$ row from $\nabla \times \left[\frac{1}{2}(\mathbf{v} \times \vec{r})\right] = \mathbf{v}$. □

The recovery of $(\mathbf{v}, \widetilde{\mathbf{B}}, \widetilde{\mathbf{E}})$ as the curl of $(\widetilde{\mathbf{A}}_v, \widetilde{\mathbf{A}}_{\mathrm{B}}, \widetilde{\mathbf{A}}_{\mathrm{E}})$ places the excitation sector's wave structure in the same class as the Aharonov–Bohm potential [7]: a wave carried by $\widetilde{\mathbf{A}}$, not by $\widetilde{\mathbf{B}}, \widetilde{\mathbf{E}}$ themselves.

This shared structure, however, exposes a genuine tension, worth recording explicitly rather than passing over.

Remark 4.4 (Radiant-flux ordering and the classical Poynting form)**.** For either gyricity and either signed centreline direction, the closure fixes

$$\widetilde{\mathbf{E}} = \mathbf{v} \times \widetilde{\mathbf{B}}.$$

Together with $\widetilde{\mathbf{B}} \cdot \mathbf{v} = 0$, this gives

$$\widetilde{\mathbf{B}} \times \widetilde{\mathbf{E}} = \widetilde{\mathbf{B}} \times (\mathbf{v} \times \widetilde{\mathbf{B}}) = |\widetilde{\mathbf{B}}|^2 \mathbf{v},$$

whereas

$$\widetilde{\mathbf{E}} \times \widetilde{\mathbf{B}} = -|\widetilde{\mathbf{B}}|^2 \mathbf{v}.$$

These orientations are independent of gyricity; reversing the signed centreline velocity reverses both.

The radiant-flux density defined in Grouping 5.G follows the first ordering. Indeed, $\widetilde{\mathbf{J}}_{\mathrm{E}} \parallel \widetilde{\mathbf{B}}$, and hence

$$\widetilde{\mathbf{\Phi}}_{\mathrm{E}} = \widetilde{\mathbf{J}}_{\mathrm{E}} \times \widetilde{\mathbf{E}} \parallel \widetilde{\mathbf{B}} \times \widetilde{\mathbf{E}} \parallel \mathbf{v}.$$

The magnetic contribution has the same direction.

This quantity should be distinguished from the classical Poynting vector

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

[8]. The distinction is not merely one of cross-product ordering:

$$[\mathbf{S}] = \mathrm{W\,m^{-2}}, \qquad [\widetilde{\mathbf{\Phi}}_{\mathrm{E}}] = \mathrm{W\,m^{-3}}.$$

The Poynting vector is the areal energy flux derived from the classical electromagnetic energy-balance equation. By contrast, $\widetilde{\mathbf{\Phi}}_{\mathrm{E}}$ is the volumetric excitation-flux density defined here and is directed along the disturbance's centreline. The opposite cross-product orientations therefore do not constitute a disagreement about the direction of one common quantity; they reflect two distinct constructions.

5 Maxwell 1865: Laboratory and excitation sectors

Maxwell's 1865 formulation [2] characterises an electromagnetic–mechanical interface and admits plane-wave solutions. While robust for electromechanical coupling, the original framework does not by itself supply the algebraic closure of the transported field triad developed here. In contemporary notation (with the constitutive relation $\mathbf{B} = \mu\mathbf{H}$ understood), the Maxwellian system may be summarised as

$$\begin{array}{llll}
\text{(A)} & \mathbf{J}_{\text{tot}} = \mathbf{J} + \partial_t \mathbf{D} & \text{(E)} & \mathbf{D} = \epsilon\mathbf{E} \\
\text{(B)} & \mathbf{B} = \nabla \times \mathbf{A} & \text{(F)} & \mathbf{E} = -\rho\,\mathbf{J} \\
\text{(C)} & \mu\,\mathbf{J}_{\text{tot}} = \nabla \times \mathbf{B} & \text{(G)} & q + \nabla \cdot \mathbf{D} = 0 \\
\text{(D)} & \mathbf{E} = \mathbf{v} \times \mathbf{B} - \partial_t \mathbf{A} - \nabla\Psi & \text{(H)} & \partial_t q + \nabla \cdot \mathbf{J} = 0
\end{array}$$

where $\epsilon = \epsilon_r\epsilon_0$ and $\mu = \mu_r\mu_0$, and ϵ_r and μ_r are the dimensionless relative permittivity and permeability, respectively.

Remark 5.1. The requisite $\mathfrak{M}$ closure is not supplied by Maxwell's 1865 formulation in isolation. Instead, it is realised by enforcing an electric–magnetic symmetry induced by $\mathfrak{M}$. Earlier symmetry-based approaches, most notably the monopole-based dual formulations [9–13], moved in this direction, but do not furnish the algebraic triadic closure required in the present framework. We therefore introduce a symmetric excitation sector as a formal structural extension of the 1865 equations, developed throughout within the vacuum sector of Assumption I.2.1, under which $\nabla \cdot \mathbf{B} = 0$ remains in force — the extension completes $\mathfrak{M}$'s electric–magnetic symmetry; it does not relax this constraint. The sector is identified with the excitation sector introduced in Hypothesis I.4.7 (Primitive-displacement hypothesis) and its elementary voxel excitation $\mathcal{W}$.

What follows recasts Maxwell's original 1865 formulation itself — not merely the closure $\mathfrak{M}$ that motivates it — by applying the Maxwell-compatible inverse Lie transport of Assumption I.2.2 (Maxwell-compatible transport and admissible sector),

$$\frac{\partial \mathbf{a}}{\partial t} = \nabla \times (\mathbf{a} \times \mathbf{v}) \equiv \mathcal{L}_{\mathbf{v}}\mathbf{a}, \tag{14}$$

thereby making the excitation sector's monopole structure explicit within Maxwell's own equations.

Lemma 5.2 (Electric–magnetic symmetry and Lie-transport substitutions)**.** *Within the structure of $\mathfrak{M}$, each electric quantity is paired algebraically with a corresponding magnetic quantity. Writing* $\widetilde{\square}_{\mathrm{E}}$

and $\widetilde{\square}_{\mathrm{B}}$ *for any such pair, one has*

$$\widetilde{\square}_{\mathrm{E}} = \mathbf{v} \times \widetilde{\square}_{\mathrm{B}}, \qquad \widetilde{\square}_{\mathrm{B}} = \epsilon_0\mu_0\, \widetilde{\square}_{\mathrm{E}} \times \mathbf{v}. \tag{15}$$

Consequently, the transport law (14) *gives*

$$\begin{aligned} \partial_t\widetilde{\square}_{\mathrm{E}} &= \nabla \times (\widetilde{\square}_{\mathrm{E}} \times \mathbf{v}) = (\epsilon_0\mu_0)^{-1}\, \nabla \times \widetilde{\square}_{\mathrm{B}}, \\ \partial_t\widetilde{\square}_{\mathrm{B}} &= \nabla \times (\widetilde{\square}_{\mathrm{B}} \times \mathbf{v}) = -\,\nabla \times \widetilde{\square}_{\mathrm{E}}. \end{aligned} \tag{16}$$

Remark 5.3 (Symmetry with no counterpart in Maxwell 1865)**.** Lemma 5.2 supplies the missing counterpart whenever Maxwell defines a quantity on only one side of the electric–magnetic pair. Maxwell's own 1865 equations are asymmetric in exactly this sense: for example, group (B), $\mathbf{B} = \nabla \times \mathbf{A}$, has no electric-potential analogue producing $\mathbf{E}$ via a curl. In what follows, the missing halves are supplied by Lemma 5.2, not added by hand: $\mathfrak{M}$'s closure requires both an electric and a magnetic member of every such pair, so the symmetry is forced throughout, not asserted case by case. As a notational consequence, equation numbers are sub-tagged "e" and "m" for electric and magnetic identification, respectively.

Proposition 5.4 (Symmetric transformer induction)**.** *Within the transverse Beltrami eigensector with non-zero eigenvalue γ, the Maxwell–Heaviside curl equations and the symmetric potential definitions*

$$\widetilde{\mathbf{B}} = \nabla \times \widetilde{\mathbf{A}}_{\mathrm{B}}, \qquad \widetilde{\mathbf{E}} = \nabla \times \widetilde{\mathbf{A}}_{\mathrm{E}}$$

determine the induced pair

$$\widetilde{\mathbf{E}}^{\mathrm{ind}} := \partial_t\widetilde{\mathbf{A}}_{\mathrm{B}} = -\widetilde{\mathbf{E}}, \qquad \widetilde{\mathbf{B}}^{\mathrm{ind}} := \epsilon_0\mu_0\, \partial_t\widetilde{\mathbf{A}}_{\mathrm{E}} = \widetilde{\mathbf{B}}. \tag{17}$$

Proof. Using $\widetilde{\mathbf{B}} = \nabla \times \widetilde{\mathbf{A}}_{\mathrm{B}}$, the Maxwell–Faraday equation gives

$$\nabla \times \widetilde{\mathbf{E}} = -\partial_t\widetilde{\mathbf{B}} = -\partial_t\, \nabla \times \widetilde{\mathbf{A}}_{\mathrm{B}},$$

and therefore

$$\nabla \times \big(\widetilde{\mathbf{E}} + \partial_t\widetilde{\mathbf{A}}_{\mathrm{B}}\big) = 0.$$

Similarly, using $\widetilde{\mathbf{E}} = \nabla \times \widetilde{\mathbf{A}}_{\mathrm{E}}$, the Maxwell–Ampère equation gives

$$\nabla \times \widetilde{\mathbf{B}} = \epsilon_0\mu_0\, \partial_t\widetilde{\mathbf{E}} = \epsilon_0\mu_0\, \partial_t\, \nabla \times \widetilde{\mathbf{A}}_{\mathrm{E}},$$

and hence

$$\nabla \times \big(\widetilde{\mathbf{B}} - \epsilon_0\mu_0\, \partial_t\widetilde{\mathbf{A}}_{\mathrm{E}}\big) = 0.$$

Each expression in parentheses remains in the same Beltrami eigenspace, since $\nabla\times$ commutes with ∂_t. Each is therefore simultaneously curl-free and an eigenfield of $\nabla\times$ with non-zero eigenvalue γ, and hence must vanish:

$$\partial_t\widetilde{\mathbf{A}}_{\mathrm{B}} = -\widetilde{\mathbf{E}}, \qquad \epsilon_0\mu_0\, \partial_t\widetilde{\mathbf{A}}_{\mathrm{E}} = \widetilde{\mathbf{B}},$$

which is the stated induced pair. $\square$

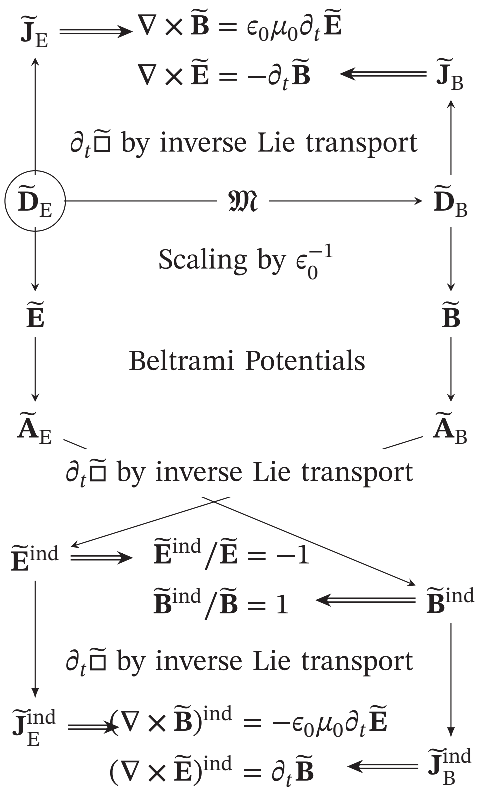

Figure 1: Construction and closure of the excitation-sector field equations. The primitive electric displacement $\widetilde{\mathbf{D}}_{\mathrm{E}}$ fixes $\widetilde{\mathbf{D}}_{\mathrm{B}}$ through $\mathfrak{M}$. Inverse Lie transport of the displacement pair produces the currents $\widetilde{\mathbf{J}}_{\mathrm{E}}$ and $\widetilde{\mathbf{J}}_{\mathrm{B}}$, from which the upper Maxwell–Heaviside curl equations are constructed. Constitutive scaling by ϵ_0^{-1} gives $\widetilde{\mathbf{E}}$ and $\widetilde{\mathbf{B}}$ (Maxwell's group (E)); their static Beltrami definitions give $\widetilde{\mathbf{A}}_{\mathrm{E}}$ and $\widetilde{\mathbf{A}}_{\mathrm{B}}$. Inverse Lie transport of the potentials gives the parity-related induction pair $\widetilde{\mathbf{E}}^{\mathrm{ind}}$ and $\widetilde{\mathbf{B}}^{\mathrm{ind}}$. The displayed ratios compare their components across the parity change and are not same-frame substitutions. A second inverse Lie transport produces the induced currents and hence the induced curl equations. These are opposite to the upper pair, so the sums of corresponding curls vanish. This current-generated cancellation of curl equations is the field-equation closure; no corresponding cancellation condition is imposed on the underlying fields.

Lemma 5.5 (Parity covariance under inverse Lie transport)**.** *By Proposition 5.4, inverse Lie transport produces*

$$\widetilde{\mathbf{E}}^{\mathrm{ind}} = -\widetilde{\mathbf{E}}, \qquad \widetilde{\mathbf{B}}^{\mathrm{ind}} = \widetilde{\mathbf{B}}.$$

These are component correspondences under the parity change $\hat{z}^{\mathrm{ind}} = -\hat{z}$, not same-frame identities. Accordingly, the parity reversal does not break the closure: $\mathfrak{M}\{\mathbf{v}, \widetilde{\mathbf{B}}^{\mathrm{ind}}, \widetilde{\mathbf{E}}^{\mathrm{ind}}\}$ retains the full algebraic form of Lemma 5.2 in the induced orientation.

Corollary 5.6 (Single-generator closure of the excitation-sector field equations)**.** *With reference to Figure 1, the entire excitation sector, $\widetilde{\mathbf{D}}_{\mathrm{B}}$, $\widetilde{\mathbf{E}}$, $\widetilde{\mathbf{B}}$, $\widetilde{\mathbf{A}}_{\mathrm{E}}$, $\widetilde{\mathbf{A}}_{\mathrm{B}}$, and the induced pair $\widetilde{\mathbf{E}}^{\mathrm{ind}}, \widetilde{\mathbf{B}}^{\mathrm{ind}}$, is generated from the single primitive $\widetilde{\mathbf{D}}_{\mathrm{E}}$ by a fixed sequence of maps, none introducing an independent degree of freedom:*

$$\widetilde{\mathbf{D}}_{\mathrm{E}} \xrightarrow{\mathfrak{M}} \widetilde{\mathbf{D}}_{\mathrm{B}} \xrightarrow{\times\epsilon_0^{-1}} \widetilde{\mathbf{E}}, \widetilde{\mathbf{B}} \xrightarrow{\textit{Beltrami}} \widetilde{\mathbf{A}}_{\mathrm{E}}, \widetilde{\mathbf{A}}_{\mathrm{B}} \xrightarrow{\partial_t\widetilde{\mathbf{A}}\ \textit{inv. Lie transport}} \widetilde{\mathbf{E}}^{\mathrm{ind}}, \widetilde{\mathbf{B}}^{\mathrm{ind}}.$$

Here $\mathfrak{M}$ supplies $\widetilde{\mathbf{D}}_{\mathrm{B}}$ from $\widetilde{\mathbf{D}}_{\mathrm{E}}$, scaling by ϵ_0^{-1} supplies $\widetilde{\mathbf{E}}$ and $\widetilde{\mathbf{B}}$, and Proposition 4.1 and Lemma 4.3 supply the Beltrami potentials. Inverse Lie transport then gives the parity-related induced pair

$$\widetilde{\mathbf{E}}^{\mathrm{ind}} = \partial_t\widetilde{\mathbf{A}}_{\mathrm{B}} = -\widetilde{\mathbf{E}}, \qquad \widetilde{\mathbf{B}}^{\mathrm{ind}} = \epsilon_0\mu_0\,\partial_t\widetilde{\mathbf{A}}_{\mathrm{E}} = \widetilde{\mathbf{B}}.$$

As established by Lemma 5.5, these are correspondences between opposite orientations, with both pairs remaining admissible instantiations of $\mathfrak{M}$; a second inverse Lie transport, followed by the $\mathfrak{M}$ mapping, yields the induced displacement currents:

$$\partial_t\widetilde{\mathbf{E}}^{\mathrm{ind}} = \nabla\times\left(\widetilde{\mathbf{E}}^{\mathrm{ind}}\times\mathbf{v}\right) = (\epsilon_0\mu_0)^{-1}\,\nabla\times\widetilde{\mathbf{B}}^{\mathrm{ind}}, \qquad \partial_t\widetilde{\mathbf{B}}^{\mathrm{ind}} = \nabla\times\left(\widetilde{\mathbf{B}}^{\mathrm{ind}}\times\mathbf{v}\right) = -\,\nabla\times\widetilde{\mathbf{E}}^{\mathrm{ind}},$$

and after rearrangement

$$\nabla \times \widetilde{\mathbf{B}}^{\text{ind}} = \epsilon_0 \mu_0 \partial_t \widetilde{\mathbf{E}}^{\text{ind}}, \qquad \nabla \times \widetilde{\mathbf{E}}^{\text{ind}} = -\partial_t \widetilde{\mathbf{B}}^{\text{ind}}.$$

To express these equations in terms of $\widetilde{\mathbf{E}}$ *and* $\widetilde{\mathbf{B}}$*, while preserving the induced origin of each curl, we use* $\widetilde{\mathbf{E}}^{\text{ind}} = -\widetilde{\mathbf{E}}$ *and* $\widetilde{\mathbf{B}}^{\text{ind}} = \widetilde{\mathbf{B}}$ *and introduce the notation*

$$\nabla \times \widetilde{\mathbf{B}}^{\text{ind}} =: (\nabla \times \widetilde{\mathbf{B}})^{\text{ind}}, \qquad \nabla \times \widetilde{\mathbf{E}}^{\text{ind}} =: -(\nabla \times \widetilde{\mathbf{E}})^{\text{ind}},$$

which then gives

$$(\nabla \times \widetilde{\mathbf{B}})^{\text{ind}} = -\epsilon_0 \mu_0\, \partial_t \widetilde{\mathbf{E}}, \qquad (\nabla \times \widetilde{\mathbf{E}})^{\text{ind}} = \partial_t \widetilde{\mathbf{B}}. \tag{18}$$

These are therefore the sign-reversed counterparts of the Maxwell–Heaviside curl equations (5)*. The nil-residue requirement of Assumption I.2.4 is realised in this excitation sector by balancing each current-generated curl with its induced counterpart. Consequently,*

$$(\nabla \times \widetilde{\mathbf{B}}) + (\nabla \times \widetilde{\mathbf{B}})^{\text{ind}} = 0 \qquad (\nabla \times \widetilde{\mathbf{E}}) + (\nabla \times \widetilde{\mathbf{E}})^{\text{ind}} = 0 \tag{19}$$

is the nil-residue closure of the field equations. It is a closure of the two current-generated curl equations, not a cancellation of $\widetilde{\mathbf{E}}$ *with* $\widetilde{\mathbf{E}}^{\text{ind}}$ *or of* $\widetilde{\mathbf{B}}$ *with* $\widetilde{\mathbf{B}}^{\text{ind}}$.

Proof. The first four maps are respectively the defining closure property of $\mathfrak{M}$, the constitutive definition, the Beltrami construction of Lemma 4.3, and inverse Lie transport. Applying inverse Lie transport once more produces the induced currents. Their parity-reversed curls give (18); adding these to the original current-generated curl equations (5) gives (19). Thus the displayed zero curl residue follows from the sign-reversed induced equations; it is the specific field-equation balance through which the excitation sector satisfies Assumption I.2.4. □

We now develop the field equations for the excitation sector in an unoccupied vacuum. Rather than mapping Maxwell's 1865 formulation of the laboratory sector into the excitation sector, we follow the construction laid out in Figure 1 and Corollary 5.6.

5.A Electric and magnetic displacement fields The electromagnetic displacement of the quantised disturbance $\mathscr{W}$ — see Definition I.4.4 — is defined by the primitive electric displacement of Definition I.4.8; its magnetic conjugate is supplied by Proposition I.4.13. We write

$$\widetilde{\mathbf{D}}_{\text{E}} = \frac{1}{\sqrt{2}} \frac{\kappa e}{l_o^2} \,\widehat{\text{E}} \quad \left[Q\, L^{-2}\right], \tag{20.e}$$

$$\widetilde{\mathbf{D}}_{\text{B}} = \frac{1}{\sqrt{2}} \frac{\kappa e}{c l_o^2} \,\widehat{\text{B}} \quad \left[Q\, T\, L^{-3}\right], \tag{20.m}$$

where the orthonormal vectors $\widehat{\text{E}}$ and $\widehat{\text{B}}$ are defined by the rotation matrix R_{EB} in (7).

5.B Electric and magnetic field intensities The field intensities are obtained by scaling the displacements by ϵ_0^{-1}, as established by Proposition I.5.3.

$$\widetilde{\mathbf{E}} = \frac{\widetilde{\mathbf{D}}_{\mathrm{E}}}{\epsilon_0} = Z_0\, c\, \widetilde{\mathbf{D}}_{\mathrm{E}} = \frac{1}{\sqrt{2}} \frac{ch}{\kappa e l_o^2} \quad \big[H\, Q^{-1}\, L^{-1}\, T^{-1}\big], \tag{21.e}$$

$$\widetilde{\mathbf{B}} = \frac{\widetilde{\mathbf{D}}_{\mathrm{B}}}{\epsilon_0} = Z_0\, c\, \widetilde{\mathbf{D}}_{\mathrm{B}} = \frac{1}{\sqrt{2}} \frac{h}{\kappa e l_o^2} \quad \big[H\, Q^{-1}\, L^{-2}\big]. \tag{21.m}$$

Here $[H] = \big[M\, L^2\, T^{-1}\big]$ denotes the dimension of action.

5.C Displacement currents: Maxwell–Ampère law The displacement currents are defined as the time derivatives of the corresponding displacement fields.

By Assumption I.2.2 (Maxwell-compatible transport and admissible sector), the temporal evolution of the continuum vector fields is compatible with transport along the field $\mathbf{v}$ in the inverse-flow sense. Before evaluating the curl, we reduce the triple cross product using the symmetry law (15). Using $|\mathbf{v}| = c$, we obtain

$$\begin{aligned} \widetilde{\mathbf{J}}_{\mathrm{E}} = \partial_t \widetilde{\mathbf{D}}_{\mathrm{E}} &= \nabla \times \big(\widetilde{\mathbf{D}}_{\mathrm{E}} \times \mathbf{v}\big) \\ &= (\epsilon_0 \mu_0)^{-1}\, \nabla \times \widetilde{\mathbf{D}}_{\mathrm{B}} \\ &= \nu c\, \widetilde{\mathbf{D}}_{\mathrm{B}} = \widehat{\mathrm{B}}\, \nu \big|\widetilde{\mathbf{D}}_{\mathrm{E}}\big|. \end{aligned} \tag{22.e}$$

$$\begin{aligned} \widetilde{\mathbf{J}}_{\mathrm{B}} = \partial_t \widetilde{\mathbf{D}}_{\mathrm{B}} &= \nabla \times \big(\widetilde{\mathbf{D}}_{\mathrm{B}} \times \mathbf{v}\big) \\ &= -\, \nabla \times \widetilde{\mathbf{D}}_{\mathrm{E}} \\ &= -\frac{\nu}{c}\, \widetilde{\mathbf{D}}_{\mathrm{E}} = -\widehat{\mathrm{E}}\, \nu \big|\widetilde{\mathbf{D}}_{\mathrm{B}}\big|. \end{aligned} \tag{22.m}$$

Here ν denotes the gyration frequency of the displacement field pair $\widetilde{\mathbf{D}}_{\mathrm{E}}$ and $\widetilde{\mathbf{D}}_{\mathrm{B}}$ about the propagation centreline defined by $\widehat{\mathrm{v}}$. Rearrangement gives

$$\begin{aligned} \nabla \times \widetilde{\mathbf{D}}_{\mathrm{B}} &= \epsilon_0 \mu_0\, \widetilde{\mathbf{J}}_{\mathrm{E}} = \epsilon_0 \mu_0\, \partial_t \widetilde{\mathbf{D}}_{\mathrm{E}}, \\ \nabla \times \widetilde{\mathbf{D}}_{\mathrm{E}} &= -\widetilde{\mathbf{J}}_{\mathrm{B}} = -\partial_t \widetilde{\mathbf{D}}_{\mathrm{B}}. \end{aligned}$$

Dividing by ϵ_0 yields the source-free Maxwell–Heaviside curl equations:

$$\nabla \times \widetilde{\mathbf{B}} = \mu_0\, \widetilde{\mathbf{J}}_{\mathrm{E}} = \epsilon_0 \mu_0\, \partial_t \widetilde{\mathbf{E}}, \tag{23.e}$$

$$\nabla \times \widetilde{\mathbf{E}} = -\frac{1}{\epsilon_0}\, \widetilde{\mathbf{J}}_{\mathrm{B}} = -\partial_t \widetilde{\mathbf{B}}. \tag{23.m}$$

5.D Electric and magnetic vector potentials The electric and magnetic vector potentials are defined as Beltrami vector potentials, as established by Lemma 4.3.

$$\widetilde{\mathbf{E}} = \nabla \times \widetilde{\mathbf{A}}_{\mathrm{E}} = \gamma\, \widetilde{\mathbf{A}}_{\mathrm{E}}, \tag{24.e}$$

$$\widetilde{\mathbf{B}} = \nabla \times \widetilde{\mathbf{A}}_{\mathrm{B}} = \gamma\, \widetilde{\mathbf{A}}_{\mathrm{B}}. \tag{24.m}$$

5.E Electric and magnetic transformer induction The present construction separates transformer induction from motional induction and distinguishes the primary curl equations (23) from their parity-reflected induced counterparts (18). Proposition 5.4 establishes $\partial_t \widetilde{\mathbf{A}}_{\mathrm{B}}$ and $\partial_t \widetilde{\mathbf{A}}_{\mathrm{E}}$

as the local drivers of electric and magnetic transformer induction, respectively. The electric member is the Maxwell–Faraday transformer-induction law; the magnetic member is its symmetric excitation-sector counterpart.

$$\widetilde{\mathbf{E}}^{\mathrm{ind}} := \partial_t \widetilde{\mathbf{A}}_{\mathrm{B}} = -\widetilde{\mathbf{E}}, \tag{25.e}$$

$$\widetilde{\mathbf{B}}^{\mathrm{ind}} := \epsilon_0 \mu_0\, \partial_t \widetilde{\mathbf{A}}_{\mathrm{E}} = \widetilde{\mathbf{B}}. \tag{25.m}$$

5.F Equation of continuity Maxwell's equation of continuity, group (H), together with group (G), $q + \nabla \cdot \mathbf{D} = 0$, gives $-\partial_t(\nabla \cdot \mathbf{D}) + \nabla \cdot \mathbf{J} = 0$. Its electric–magnetic completion in the excitation sector is

$$-\frac{\partial}{\partial t}\big(\nabla \cdot \widetilde{\mathbf{D}}_{\mathrm{E}}\big) + \nabla \cdot \widetilde{\mathbf{J}}_{\mathrm{E}} = 0, \tag{26.e}$$

$$-\frac{\partial}{\partial t}\big(\nabla \cdot \widetilde{\mathbf{D}}_{\mathrm{B}}\big) + \nabla \cdot \widetilde{\mathbf{J}}_{\mathrm{B}} = 0. \tag{26.m}$$

By (22), $\widetilde{\mathbf{J}}_{\mathrm{E}} = \partial_t \widetilde{\mathbf{D}}_{\mathrm{E}}$ and $\widetilde{\mathbf{J}}_{\mathrm{B}} = \partial_t \widetilde{\mathbf{D}}_{\mathrm{B}}$, so both relations follow identically from the transport construction rather than constituting additional closure conditions. For the transverse solution (7),

$$\nabla \cdot \widetilde{\mathbf{D}}_{\mathrm{E}} = \nabla \cdot \widetilde{\mathbf{D}}_{\mathrm{B}} = \nabla \cdot \widetilde{\mathbf{J}}_{\mathrm{E}} = \nabla \cdot \widetilde{\mathbf{J}}_{\mathrm{B}} = 0,$$

and the vacuum transversality constraint is therefore preserved throughout the evolution.

5.G Radiant flux density We define the radiant flux densities by

$$\widetilde{\mathbf{\Phi}}_{\mathrm{E}} := \widetilde{\mathbf{J}}_{\mathrm{E}} \times \widetilde{\mathbf{E}}, \tag{27.e}$$

$$\widetilde{\mathbf{\Phi}}_{\mathrm{B}} := c^2\, \widetilde{\mathbf{J}}_{\mathrm{B}} \times \widetilde{\mathbf{B}}. \tag{27.m}$$

Each has units $\left[\mathrm{W\,m^{-3}}\right]$, and both are directed parallel to the transport vector. Their magnitudes are

$$|\widetilde{\mathbf{\Phi}}_{\mathrm{E}}| = |\widetilde{\mathbf{J}}_{\mathrm{E}}|\,|\widetilde{\mathbf{E}}| = \nu\,|\widetilde{\mathbf{D}}_{\mathrm{E}}|\,\frac{|\widetilde{\mathbf{D}}_{\mathrm{E}}|}{\epsilon_0} = \frac{1}{2}\,\nu\,\frac{\kappa^2 e^2}{l_o^4}\,\frac{hc}{\kappa^2 e^2} = \frac{1}{2}\,h\nu\,\frac{c}{l_o^4}, \tag{28}$$

$$|\widetilde{\mathbf{\Phi}}_{\mathrm{B}}| = c^2\,|\widetilde{\mathbf{J}}_{\mathrm{B}}|\,|\widetilde{\mathbf{B}}| = c^2\,\nu\,|\widetilde{\mathbf{D}}_{\mathrm{B}}|\,\frac{|\widetilde{\mathbf{D}}_{\mathrm{B}}|}{\epsilon_0} = \frac{1}{2}\,h\nu\,\frac{c}{l_o^4}. \tag{29}$$

Hence

$$|\widetilde{\mathbf{\Phi}}_{\mathrm{tot}}| = |\widetilde{\mathbf{\Phi}}_{\mathrm{E}}| + |\widetilde{\mathbf{\Phi}}_{\mathrm{B}}| = h\nu\left(\frac{1}{t_o l_o^3}\right). \tag{30}$$

By (22), one has $\widetilde{\mathbf{J}}_{\mathrm{E}} \parallel \widehat{\mathrm{B}}$ and $\widetilde{\mathbf{J}}_{\mathrm{B}} \parallel -\widehat{\mathrm{E}}$. Therefore

$$\widetilde{\mathbf{\Phi}}_{\mathrm{E}} = \widetilde{\mathbf{J}}_{\mathrm{E}} \times \widetilde{\mathbf{E}} \parallel \widehat{\mathrm{B}} \times \widehat{\mathrm{E}}, \qquad \widetilde{\mathbf{\Phi}}_{\mathrm{B}} = c^2\,\widetilde{\mathbf{J}}_{\mathrm{B}} \times \widetilde{\mathbf{B}} \parallel (-\widehat{\mathrm{E}}) \times \widehat{\mathrm{B}} = \widehat{\mathrm{B}} \times \widehat{\mathrm{E}}.$$

Since $\widehat{\mathrm{B}} \times \widehat{\mathrm{E}} = \hat{\mathrm{v}}$, both radiant flux densities are aligned with the transport velocity $\mathbf{v}$.

5.H Losses by resistivity Maxwell's group (F) implements Ohm's law in the macroscopic laboratory sector. In the excitation sector, relative resistivity is instead treated as a medium-specific property governing excitation loss. We define the characteristic impedance scale by

$$\varrho_o := Z_0 = \mu_0 c = \frac{h}{\kappa^2 e^2} \qquad \left[H\, Q^{-2}\right]. \tag{31}$$

The corresponding travel-loss coefficient is

$$\varrho := \varrho_r \varrho_o, \qquad \varrho_r \geq 0. \tag{32}$$

Here ϱ_r is its dimensionless, relative resistivity. The value $\varrho_r = 0$ gives $\varrho = 0$ and hence no resistive transfer, while ϱ_o supplies the universal characteristic scale. Here ϱ is not the bulk resistivity of Drude's electron theory of metals [14], in which resistance arises from collisions between charge carriers and a lattice – no such carriers or lattice exist in the vacuum sector considered here. Instead, ϱ is a medium-specific coefficient for contactless travel loss, under which energy lost by an excited voxel is gained by another voxel — excited or not — near the position $\vec{s}$, where $\dot{\vec{s}} = \mathbf{v}$.

The electric- and magnetic-sector travel-loss coefficients are therefore

$$\varrho = \varrho_r \varrho_o = \varrho_r \frac{h}{\kappa^2 e^2} \quad \left[H\, Q^{-2}\right], \tag{33.e}$$

$$\varrho_{\mathrm{B}} = \frac{\varrho}{\epsilon_0 \mu_0}, \tag{33.m}$$

so that ϱ is measured in ohms. The travel loss of radiant flux density is then

$$\mathscr{L}_{\mathrm{E}} = \varrho\, \widetilde{\mathbf{J}}_{\mathrm{E}} \cdot \widetilde{\mathbf{J}}_{\mathrm{E}}, \tag{34.e}$$

$$\mathscr{L}_{\mathrm{B}} = \frac{\varrho}{\epsilon_0 \mu_0} \widetilde{\mathbf{J}}_{\mathrm{B}} \cdot \widetilde{\mathbf{J}}_{\mathrm{B}}. \tag{34.m}$$

and

$$\mathscr{L} = \mathscr{L}_{\mathrm{E}} + \mathscr{L}_{\mathrm{B}} = 2\varrho\, \widetilde{\mathbf{J}}_{\mathrm{E}} \cdot \widetilde{\mathbf{J}}_{\mathrm{E}}. \tag{35}$$

Thus $\mathscr{L}$ is the radiant-flux-density loss per unit distance travelled. Assumption I.2.4 (Nil-residue universe) requires this loss to be compensated rather than dissipated to nothing. Within the excitation-sector mechanism defined here, the energy removed from one voxel's excitation is gained, in full, by the neighbouring voxel along $\mathbf{v}$.

5.I Decay by resistivity The total radiant flux density is $\widetilde{\mathbf{\Phi}} = \widetilde{\mathbf{\Phi}}_{\mathrm{E}} + \widetilde{\mathbf{\Phi}}_{\mathrm{B}} = 2\widetilde{\mathbf{\Phi}}_{\mathrm{E}}$. We therefore write

$$|\widetilde{\mathbf{\Phi}}| = \Phi = \frac{2\nu \widetilde{D}_{\mathrm{E}}^2}{\epsilon_0}, \qquad \left[H\, L^{-3}\, T^2\right] \tag{36}$$

and

$$\mathscr{L} = 2\varrho\, \widetilde{\mathbf{J}}_{\mathrm{E}} \cdot \widetilde{\mathbf{J}}_{\mathrm{E}} = 2\varrho\, \nu^2 \widetilde{D}_{\mathrm{E}}^2, \qquad \left[H\, L^{-4}\, T^2\right] \tag{37}$$

Assuming a decay law of the form

$$\frac{\mathrm{d}\Phi}{\mathrm{d}s} = -\mathscr{L}, \qquad s = ct, \quad \Rightarrow \quad \frac{\mathrm{d}\Phi}{\mathrm{d}t} = -c\mathscr{L} \tag{38}$$

we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\nu\widetilde{D}_{\mathrm{E}}^{2}}{\epsilon_0}\right) = -\varrho c\nu^2\widetilde{D}_{\mathrm{E}}^{2}. \tag{39}$$

As $\widetilde{D}_{\mathrm{E}}$ is preserved, this reduces to

$$\frac{\widetilde{D}_{\mathrm{E}}^{2}}{\epsilon_0}\,\dot{\nu} = -\varrho c\nu^2\widetilde{D}_{\mathrm{E}}^{2}. \tag{40}$$

Separating variables and cancelling $\widetilde{D}_{\mathrm{E}}^{2}$ gives

$$\frac{\mathrm{d}\nu}{\nu^2} = -\varrho c\epsilon_0\,\mathrm{d}t, \tag{41}$$

which integrates to

$$-\frac{1}{\nu} = -\varrho c\epsilon_0\, t + C. \tag{42}$$

Hence, the frequency as a function of time is

$$\nu(t) = \frac{1}{\varrho c\epsilon_0\, t + C'}. \tag{43}$$

Here $C' = 1/\nu_0$ is fixed by the initial condition $\nu(0) = \nu_0$. Therefore,

$$\nu(t) = \frac{\nu_0}{1 + \varrho c\epsilon_0\nu_0 t}. \tag{44}$$

Thus, with $\widetilde{D}_{\mathrm{E}}$ preserved, the frequency, rather than the amplitude, carries the resistive loss. For $\varrho > 0$, a single quantised disturbance redshifts at the hyperbolic rate (44) as it propagates through a resistive medium. The accumulated redshift results from successive contactless excitation transfers: at each step, the energy lost by one voxel is gained by a neighbouring voxel along the disturbance's path (Grouping 5.H), thereby preserving the energy component of the universal ledger.

6 The Einstein-Planck relation

The Einstein–Planck relation ($\mathcal{E} = hf$, writing $f = \nu$ for the frequency used throughout this paper) was historically posited as a heuristic assumption [15, 16] to reconcile light quanta with empirical radiation spectra, lacking a first-principles derivation from classical field dynamics. The following corollary demonstrates that this quantisation condition naturally emerges from classical vacuum electrodynamics using displacement as the excitation agent.

Corollary 6.1 (Energy content of a quantised disturbance $\mathcal{W}$)**.** *The total radiant flux density of the disturbance $\mathcal{W}$ is given by* (30)*,*

$$|\widetilde{\mathbf{\Phi}}_{\mathrm{tot}}| = h\nu\left(\frac{1}{t_o\, l_o^3}\right).$$

Integrating this over one occupied voxel volume l_o^3 and one elementary transport step t_o yields the energy associated with a single excited site:

$$\mathcal{E}_{\mathrm{voxel}} = h\nu.$$

By Definition I.4.16 (Action modulus and the scale $\hbar^2$) we established a vacuum electromagnetic action modulus $\mathcal{H}$, whose square root recovers h and is retrospectively calibrated against the CODATA numeric value of Planck's constant. Thus, the construction here annuls all elements of circular argumentation.

The Einstein-Planck relation $\mathcal{E} = hf$ is now proven as a formal mathematical construction.

7 Planck-shaped spectrum from discrete loss events

Status of the spectral reconstruction. The spectral form obtained below is recovered within the discrete-loss model introduced in this section. It describes the progress of an initially monochromatic excitation as resistive decay redistributes it over lower frequencies. It is not assigned a temperature and is not presented as an equilibrium black-body distribution.

Theorem 7.1 (Planck-shaped decay envelope from relative resistivity)**.** *Under the discrete-loss model developed below, contactless, voxel-to-voxel excitation loss governed by $\varrho = \varrho_r \varrho_o$, with $\varrho_r > 0$, (Grouping 5.H) redistributes an initially monochromatic excitation into a spread spectrum whose envelope has the mathematical form of the Planck radiation law, parameterised by the relative resistivity ϱ_r.*

Proof. We adopt a phenomenological discrete-loss model for the excitation sector and interpret the smooth decay law (44),

$$\nu(t) = \frac{\nu_0}{1 + \varrho c \epsilon_0 \nu_0 t},$$

as the coarse-grained envelope of an underlying discrete process. In resistive media, the loss of radiant flux proceeds through elementary loss events separated by the characteristic time t_o, equivalently, through one transport-loss event for each elementary distance l_o traversed.

Using (37), namely

$$\mathscr{L} = 2\,\varrho \nu^2 \widetilde{D}_{\mathrm{E}}^2,$$

and assuming that $\widetilde{D}_{\mathrm{E}}$ is preserved, we interpret $\mathscr{L}$ as the travel loss of radiant-flux density and $l_o \mathscr{L}$ as the loss accumulated over one elementary path. We decompose the latter into the product of ν^2 (vorticity squared) and the action-loss density per elementary step, $\eta_{\mathscr{L}}$. Thus the loss spectrum scales as

$$\Gamma(\nu) = \nu^2. \tag{45}$$

Using (20) and (33), we evaluate $\eta_{\mathscr{L}}$ as

$$\eta_{\mathscr{L}} := 2\,\varrho l_o\, \widetilde{D}_{\mathrm{E}}^2 = 2\left(\varrho_r \frac{h l_o}{\kappa^2 e^2}\right)\left(\frac{1}{\sqrt{2}}\,\frac{\kappa e}{l_o^2}\right)^2 = \frac{\varrho_r h}{l_o^3}. \tag{46}$$

Moreover, from (28) we have

$$\Phi = \frac{2\nu \widetilde{D}_{\mathrm{E}}^2}{\epsilon_0}, \tag{47}$$

and hence

$$\mathscr{L} = \varrho \epsilon_0 \nu\, \Phi. \tag{48}$$

We postulate that elementary loss events are discretised per elementary distance travelled, that is, for the path l_o traversed in the resistive medium. During this path segment the frequency ν is held fixed, so the discretised decay law, derived from (38), is

$$\frac{\mathrm{d}\Phi}{\mathrm{d}s} = -\varrho \epsilon_0 \nu\, \Phi, \qquad 0 \le s \le l_o. \tag{49}$$

Integrating over one elementary length step gives

$$\Phi \longmapsto \Phi \exp(-\varrho\epsilon_0 \nu l_o). \tag{50}$$

Hence the statistical weight of one elementary emission event at frequency ν is

$$q(\nu) = e^{-\varrho\epsilon_0 \nu l_o}. \tag{51}$$

Assuming that repeated emission events are independent, the occupation number is

$$n(\nu) = \sum_{k=1}^{\infty} q(\nu)^k = \sum_{k=1}^{\infty} e^{-k\varrho\epsilon_0 \nu l_o} = \frac{1}{e^{\varrho\epsilon_0 \nu l_o} - 1}. \tag{52}$$

Each term $q(\nu)^k$ is the weight of a chain of k successive elementary loss events. By Grouping 5.H, what one voxel's excitation loses, its neighbour along $\mathbf{v}$ gains in full, as required by the energy component of Assumption I.2.4 (Nil-residue universe). Thus $n(\nu)$ is a gain density built out of the loss mechanism already defined, rather than an independent statistical postulate.

Multiplying the action-loss density per mode, $\eta_{\mathscr{L}}\nu$, by the quadratic loss spectrum (45) and the occupation number (52), the energy density spectrum becomes

$$u(\nu) = \eta_{\mathscr{L}}\nu\, \Gamma(\nu) n(\nu) = \frac{\eta_{\mathscr{L}}\nu^3}{e^{\varrho\epsilon_0 \nu l_o} - 1} = \frac{\varrho_r}{l_o^3} \frac{h\nu^3}{e^{\varrho\epsilon_0 \nu l_o} - 1}. \tag{53}$$

Evaluating the exponent gives

$$\varrho\epsilon_0 \nu l_o = \left(\varrho_r \frac{h}{\kappa^2 e^2}\right)\left(\frac{\kappa^2 e^2}{hc}\right)\nu l_o = \frac{\varrho_r h\nu}{h/t_o}.$$

Accordingly, for $\varrho_r > 0$, the decay spectrum assumes the Planck-shaped form

$$u(\nu) = \frac{\varrho_r}{l_o^3} \frac{h\nu^3}{\exp(\frac{\varrho_r h\nu}{h/t_o}) - 1}. \tag{54}$$

This does not make the spectrum a thermal equilibrium distribution. The relative resistivity ϱ_r remains in both the action-loss density and the exponential and governs the progress of the decay that populates the spectrum. When $\varrho_r = 0$, the action-loss density vanishes and no loss event occurs. The geometric occupation series (52), derived for $\varrho_r > 0$, is then inapplicable; the corresponding loss-generated spectrum is defined by the separate trivial branch

$$u_{\mathscr{L}}(\nu; 0) := 0. \tag{55}$$

□

7.1 Adiabatic cavity interpretation

Consider an initially monochromatic photon gas enclosed by ideal mirrors in an adiabatic cavity. The mirrors return incident radiation to the cavity and add the resistive decay mechanism, but neither absorb energy nor supply an independent energy reservoir. Successive returns therefore advance the monochromatic population through the decay law and populate lower frequencies, producing the spread spectrum (54).

The resemblance of its envelope to Planck radiation can look like cooling, but no temperature has been introduced and the total radiant energy density has not changed. Instead, the conserved cavity energy may be written schematically as

$$U = Nh\bar{\nu} = \text{constant}, \tag{56}$$

where N is the photon number and $\bar{\nu}$ is the mean frequency of the evolving spectrum. As resistive decay lowers $\bar{\nu}$, conservation of U requires the photon population to increase. This preservation of total radiant energy is the cavity realisation of the energy balance imposed by Assumption I.2.4; the discrete-loss model, rather than the assumption alone, determines the recipients, event weights, and spectral shape. The Planck-shaped envelope thus records the progress of monochromatic decay into a larger population of lower-energy photons; it is not a thermodynamic temperature distribution.

7.2 Balanced absorption and radiation

Consider a material body that continuously absorbs monochromatic electromagnetic energy while radiating the same mean power. Such equality establishes a stationary energy balance, although it does not by itself establish thermal equilibrium. If the absorbed excitation is repeatedly redistributed and re-emitted through the loss mechanism developed above, the outgoing radiation may approach the Planck-shaped envelope of (54) while the body's mean stored energy remains constant.

This suggests a possible electromagnetic mechanism underlying the spectral redistribution associated with black-body radiation: absorption supplies monochromatic excitation, whereas repeated loss and re-emission populate a spread of lower frequencies. On this reading, the conventional statistical treatment describes the occupation probabilities of the resulting balanced state, while the present construction proposes a field-level process by which that state might be approached.

This interpretation remains provisional. The present model does not derive thermodynamic detailed balance, the electromagnetic mode density, or a relation between ϱ_r and temperature. It therefore establishes a Planck-shaped redistribution mechanism, not an alternative derivation of the complete equilibrium black-body law.

8 Beltrami phase interaction

The excitation sector admits an interaction that is distinct from both gravitational attraction and the electrostatic Coulomb force. It is not a central force generated by a scalar potential. Rather, it is a field–vector-potential interaction between two quantised disturbances, directed by the transverse geometry of their Beltrami fields. We call this the *Beltrami phase interaction*. It is an interaction at a distance: the effect of the disturbance $\mathscr{W}_2$ on $\mathscr{W}_1$, written

$$\mathscr{W}_2 \rightsquigarrow \mathscr{W}_1.$$

The interaction is analysed pointwise at voxel level: the excited voxel of $\mathscr{W}_1$ interacts with the potentials of $\mathscr{W}_2$. By Theorem I.36 (Derived Electromagnetic Quantities),

$$\epsilon_0 := \frac{\kappa^2 e^2}{hc}, \qquad \phi = \frac{h}{\kappa e}, \qquad \psi = \frac{ch}{\kappa e}, \quad \Rightarrow \quad \epsilon_0 \phi \psi = h.$$

Accordingly, we convert the fields to flux quantities pointwise,

$$(\widetilde{\mathbf{B}}, \widetilde{\mathbf{E}}) \longmapsto \left(\vec{\phi}, \vec{\psi}\right) \qquad (\widetilde{\mathbf{A}}_{\mathrm{B}}, \widetilde{\mathbf{A}}_{\mathrm{E}}) \longmapsto \left(\frac{\vec{\phi}}{l_o}, \frac{\vec{\psi}}{l_o}\right)$$

The potential mapping is displayed at the elementary voxel scale. At a general Beltrami scale it acquires the corresponding dimensionless factor λ/l_o; restoring this factor changes the interaction magnitude, but neither its existence nor its direction.

Let $\mathscr{W}_1$ and $\mathscr{W}_2$ be two disturbances, and let their separation be resolved on the quantised radial stack

$$r_{12} := n_r l_o, \qquad n_r \in \mathbb{N}. \tag{57}$$

The electromagnetic vector potentials of $\mathscr{W}_2$, sampled at the position of $\mathscr{W}_1$, are assigned the inverse-distance envelope

$$\vec{\widetilde{\mathrm{A}}}_{\mathrm{E},2}(n_r) := \frac{1}{n_r} \vec{\widetilde{\mathrm{A}}}_{\mathrm{E},2}, \qquad \vec{\widetilde{\mathrm{A}}}_{\mathrm{B},2}(n_r) := \frac{1}{n_r} \vec{\widetilde{\mathrm{A}}}_{\mathrm{B},2} \tag{58}$$

where $\vec{\widetilde{\mathrm{A}}}_{\mathrm{E},i}$ and $\vec{\widetilde{\mathrm{A}}}_{\mathrm{B},i}$ are the pointwise vector potentials within the voxel of disturbance $\mathscr{W}_i$. Outside the voxel they decay by the inverse law $1/n_r$, in quantised steps of l_o. Similarly, $\vec{\widetilde{\mathrm{E}}}_i$ and $\vec{\widetilde{\mathrm{B}}}_i$ are the pointwise vector fields within the voxel of the disturbance.

Definition 8.1 (Beltrami phase interaction)**.** At time τ, the pointwise interaction of the electromagnetic Beltrami potentials of $\mathscr{W}_2$ with $\mathscr{W}_1$ is represented by

$$\begin{aligned} \vec{\widetilde{\mathrm{I}}}_{12}(\tau) :=& \frac{\epsilon_0}{2n_r(\tau)} \left(\vec{\phi}_1 \times \frac{\vec{\psi}_2}{l_o} + \frac{\vec{\phi}_2}{l_o} \times \vec{\psi}_1\right), \\ |\vec{\widetilde{\mathrm{I}}}_{12}(\tau)|_{\max} =& \frac{h}{l_o n_r(\tau)} \qquad \mathrm{kg\,m\,s^{-1}}. \end{aligned} \tag{59}$$

The first term is the interaction of $\vec{\phi}_1$ with the electric potential of $\mathscr{W}_2$; the second is the symmetric interaction of $\vec{\psi}_1$ with its magnetic potential. The factor $1/2$ incorporates the two $1/\sqrt{2}$ normalisations in each pointwise product, as in the radiant-flux construction. Equation (59) identifies the maximum interaction with a momentum. The corresponding instantaneous force is

$$\vec{\widetilde{\mathrm{F}}}_{12}(\tau) := \frac{\vec{\widetilde{\mathrm{I}}}_{12}(\tau)}{t_o}. \tag{60}$$

Its direction is fixed solely by the pointwise vector products in (59) and is not necessarily aligned with $\vec{r}_{12}$, as gravitational and Coulomb forces are.

The Beltrami phase interaction does not alter the invariant transport speed c. A force component parallel or antiparallel to the propagation centreline therefore changes the disturbance's energy and frequency rather than its speed. A transverse component instead changes the propagation direction and relative phase. The interaction may consequently redistribute phase, frequency and direction while conserving the total energy and momentum of the interacting disturbances.

9 Conclusion

This paper has developed a second, monopole reading of Maxwell's original 1865 equations [2] — the excitation sector — alongside the bipolar laboratory reading Maxwell himself worked with (Section 5). Doing so required, and thereby supplied, a potential representation of the closure's transverse solutions as Beltrami fields (Section 4) and a general electric–magnetic symmetry law absent from Maxwell's own system (Lemma 5.2). Because the companion article's transport and action moduli fix c and h as ratios of the vacuum's own primitive quantities (Definition I.4.16 (Action modulus and the scale $\hbar^2$)), not as imported constants, every quantum-flavoured relation built on this recast system is a derived consequence of it rather than a separate empirical postulate: the independent Einstein–Planck derivation $\mathcal{E} = h\nu$ (Corollary 6.1), historically introduced as a heuristic assumption [15, 16], is here a theorem, and a Planck-shaped decay envelope ((54)) is obtained from a discrete-loss mechanism acting on an initially monochromatic photon gas and built on the characteristic impedance scale $\varrho_o = Z_0$ and a relative resistivity ϱ_r (Grouping 5.H), without assigning a temperature to the decay envelope or invoking thermodynamic detailed balance. In an adiabatic mirrored cavity this redistribution conserves total radiant energy density: the mean photon energy falls while the photon number rises. The universal preservation requirement follows from Assumption I.2.4; the excitation-sector transfer mechanism supplies the voxel-to-voxel compensation and the Planck-shaped spectral form. For a material body in stationary balance between absorption and radiation, the same mechanism suggests a possible field-level route by which absorbed monochromatic excitation may approach a black-body-like spectral envelope; conventional statistical methods would then describe the occupation probabilities of the balanced state. This interpretation remains provisional, since the present construction does not derive the electromagnetic mode density, thermodynamic detailed balance, or a relation between ϱ_r and temperature. Together with the electric–magnetic symmetry law developed here, these results complete the programme set out for the present paper. To our knowledge, no prior classical construction has placed either derived result on this footing.

This construction makes a specific, falsifiable prediction: coherent, weak-intensity light of frequency ν_0 propagating a path length $s = ct$ through a static, quasi-steady medium with travel-loss coefficient ϱ – not an actively driven, time-varying one – should red-shift according to (44), $\nu(t) = \nu_0/(1 + \varrho c \epsilon_0 \nu_0 t)$, with its amplitude preserved rather than attenuated. In a linear, time-independent medium, standard electrodynamics permits no such shift at all: frequency components propagate independently, so ordinary collisional absorption attenuates intensity at fixed frequency without shifting it, and the known plasma photon-deceleration effect [17] requires an actively time-varying density structure (a driven wake) that a static plasma does not have. A frequency shift observed under these controlled, static conditions would not be explainable by either known mechanism, making a laser propagated through a steady, well-characterised plasma a clean experimental test.

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